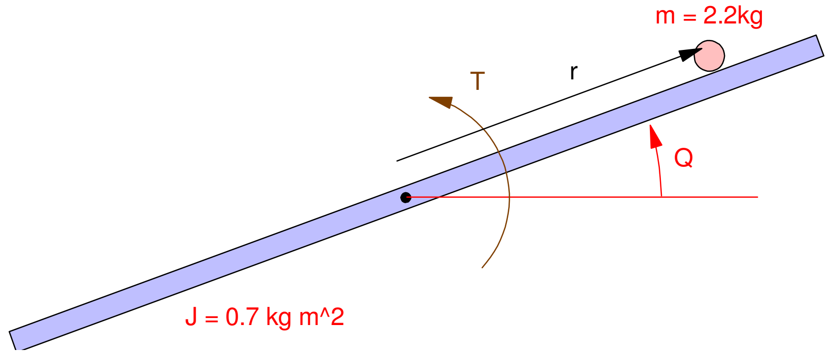
ECE 463/663 - Test #2: Name ________________

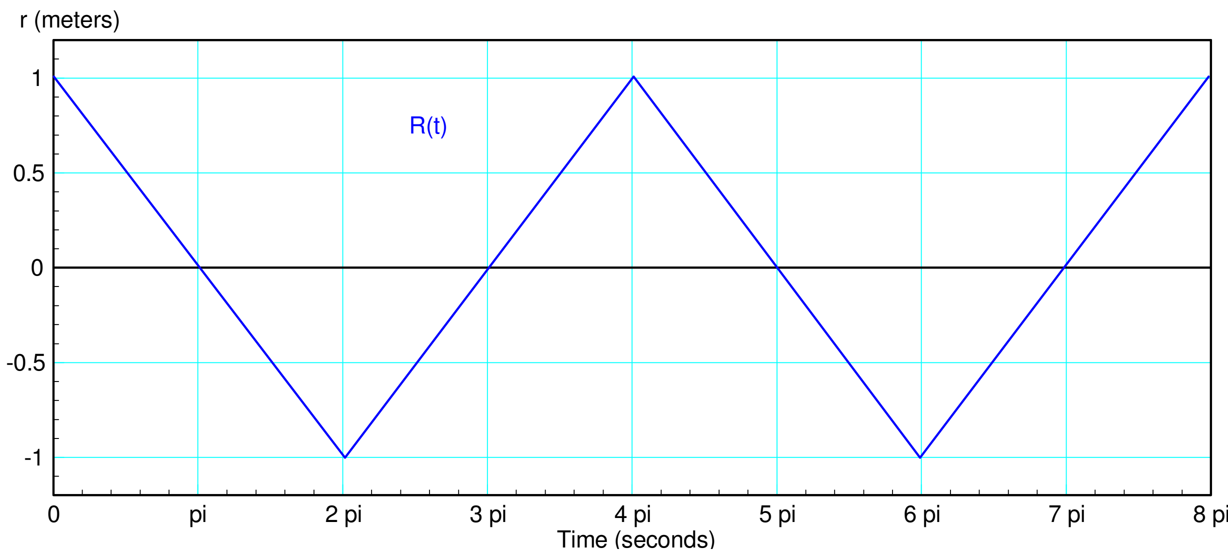
Due midnight Sunday, March 30th. Individual Effort Only (no working in groups)



The linearized dynamics for a ball and beam system (homework #4) are:

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.434 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} T$$

The ball is to move back and forth following R(t) from -1m to +1m and back, repeating every 4π seconds:



You can approximate R(t) using the first-three terms in its Fourier series expansion:

$$R(t) = \left(\frac{8}{\pi^2}\right)\cos(0.5t) + \left(\frac{8}{9\pi^2}\right)\cos(1.5t) + \left(\frac{8}{25\pi^2}\right)\cos(2.5t)$$

Problem 1) Controller Design (50 points)

Design a feedback controller to track R(t) (pick one method)

- (30 points partial credit) Using full-state feedback (no servo compensator)

  $$U = K_r R - K_x X$$

- (40 points partial credit) Using a servo compensator that tracks the 0.5 rad/sec term for R(t)

  $$U = -K_z Z - K_x X$$

- (50 points full credit) Using a servo compensator that tracks the {0.5, 1.5, 2.5} rad/sec terms for R(t)

  $$U = -K_z Z - K_x X$$

Provide in your solutions

- A block diagram of your plant and controller
- Calculations (Matlab code and results) for computing your feedback control gains
- The response of the linear system to R(t),
- The response of the nonlinear simulation to R(t) with your control law with
  - m = 2.2kg (nominal case)
  - m = 2.5kg (disturbance)
- The Matlab code for the main routine in the nonlinear simulation, including your control law.

Problem 2: Observer Design (50 points)

Assume you can measure position (r) and angle (θ). Design a full-order observer assuming (pick one)

- (30 points partial credit) No disturbance
- (40 points partial credit) An input disturbance at 0.5 rad/sec,
- (50 points full credit) An input disturbance at {0.5, 1.5, 2.5} rad/sec

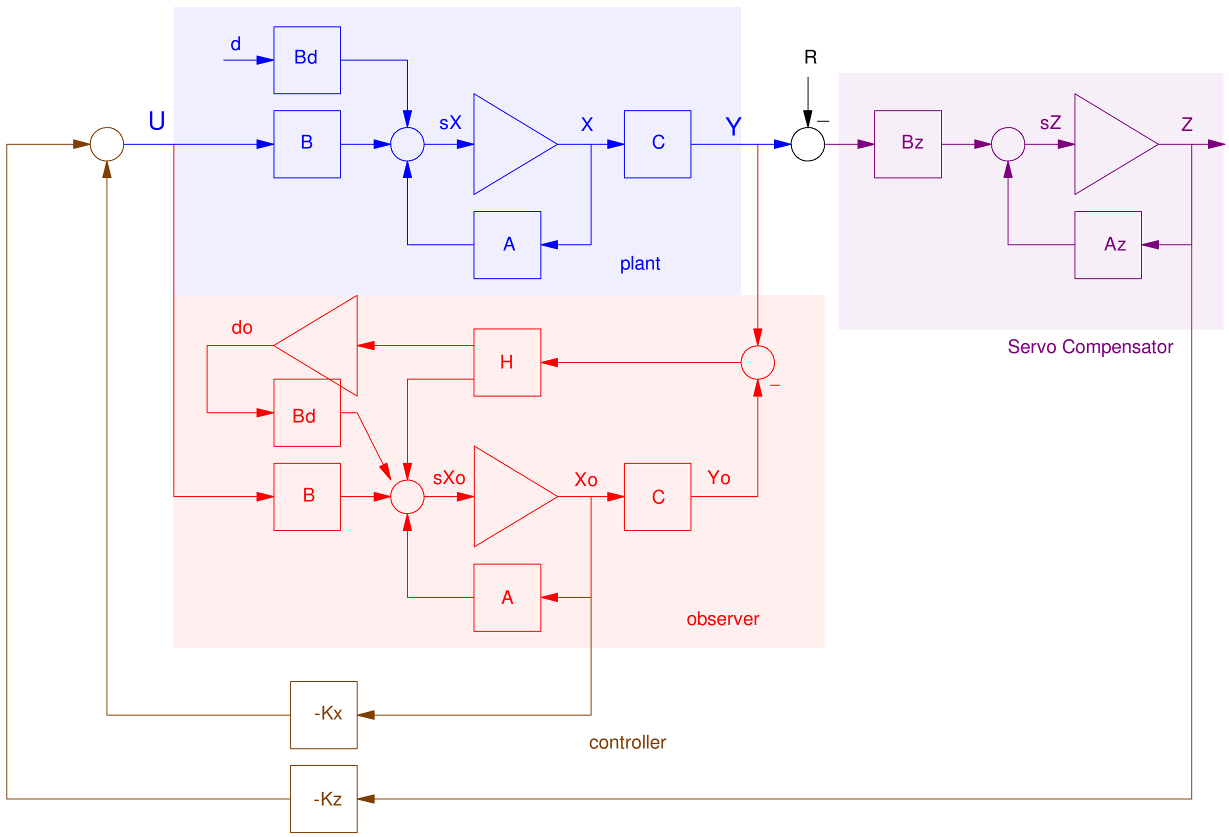
Provide in your solutions

- A block diagram of the plant, controller (from problem 1), and full-order observer
- Matlab code for computing the observer and observer gains
- The response of the nonlinear simulation with your observer to R(t) using the actual states to compute the input, and
- The response of the nonlinear simulation with your observer to R(t) using the observer states to compute the input with
  - m = 2.2kg (nominal case)
  - m = 2.5kg (disturbance)
- Your final code for the nonlinear simulation

Note: Due to the nonlinearities in the plant (which the linear observer doesn't account for), you'll probably need to use the actual states to stabilize the system as in problem #1. For problem #2, just show that the observer is tracking the plant

- **When m = 2.2kg (nominal case) and**
- **When m = 2.5kg (disturbanc)**

while U continues to use the actual states (U = -Kz*Z - Kx*X)



Block diagram for the Plant, Servo Compensator, Disturbance, Observer, and Full-State Feedback

Starting Code

```
% Ball & Beam System
% Test #2 (Spring 2025)

X = [-1,0,0,0]';
dt = 0.01;
t = 0;
n = 0;
y = [];

% Plant
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.434,0,0,0];
B = [0;0;0;0.345];
C = [1,0,0,0];

% Control Law  (needs to change)
Kx = [-31   101   -21    29];
Kr = -21;

% Full-Order Observer (needs to change)
Ae = A;
Be = B;
Ce = C;
H = [0 0 0 0]';
Xe = X;

% Ramp Input
dR = dt/pi;
Ref = -1;

while(t < 40)
 Ref = Ref + dR;
 if(abs(Ref) > 1) dR = -dR; end

 U = Kr*Ref - Kx*X;

 dX = BeamDynamics(X, U);
 dXe = Ae*Xe + Be*U + H*(C*X - Ce*Xe);

 X = X + dX * dt;
% Observer (cheating for now)
 Xe = X + [0.1,0,0,0]';
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay3(X, Xe, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```