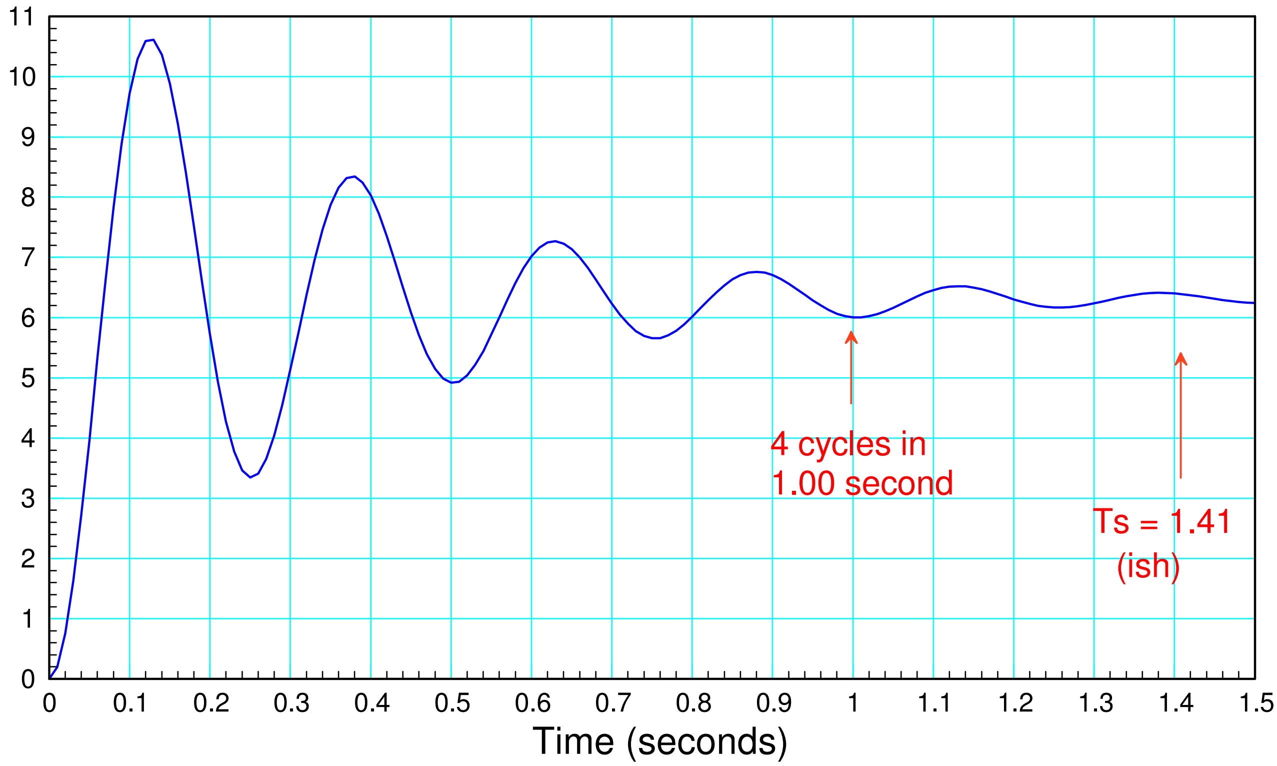
ECE 463/663: Test #1. Name _______________

Spring 2025. Calculators allowed. Individual Effort

1) Find the transfer funciton for a system with the following step response



The oscillations tell you this is a second-order system

$$G(s) = \left(\frac{k}{(s+a+jb)(s+a-jb)}\right)$$

a: The 2% settling time is about 1.41 seconds (give or take)

$$a \approx \frac{4}{1.41} = 2.83$$

b: The frequency of oscillation (in rad/sec)

$$b \approx \left(\frac{4 \text{ cycles}}{1.00s}\right) 2\pi = 25.13$$

k: The DC gain is about 6.2

$$\left(\frac{k}{(s+2.83+j25.13)(s+2.83-j25.13)}\right)_{s=0} = 6.2$$

k = 3965

$$G(s) \approx \left(\frac{3965}{(s+2.83+j25.13)(s+2.83-j25.13)}\right)$$

answers will vary

2) Determine a 2nd-order system which has approximately the same step response as the following system

$$Y = \left(\frac{500(s+5)(s+6)(s+40)}{(s+2)(s+4)(s+30)(s+45)(s+50)} \right) X$$

Keep the dominant pole: (s+2)

Keep all poles and zeros within 10x of the dominant pole

$$Y \approx \left(\frac{k(s+5)(s+6)}{(s+2)(s+4)} \right)$$

Pick 'k' to match the DC gain

$$DC = \left(\frac{500(s+5)(s+6)(s+40)}{(s+2)(s+4)(s+30)(s+45)(s+50)} \right)_{s=0} = 1.1111$$

$$\left(\frac{k(s+5)(s+6)}{(s+2)(s+4)} \right)_{s=0} = 1.1111$$

$$k = 0.2963$$

so

$$Y \approx \left(\frac{0.2963(s+5)(s+6)}{(s+2)(s+4)} \right)$$

(not required) - If you plot the two step responses, they're similar

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>> G5 = zpk([-5,-6,-40],[-2,-4,-30,-45,-50],500);
>> G2 = zpk([-5,-6],[-2,-4],0.2963);
>> t = [0:0.01:4]';
>> y5 = step(G5,t);
>> y2 = step(G2,t);
>> plot(t,y5,'b',t,y2,'r')
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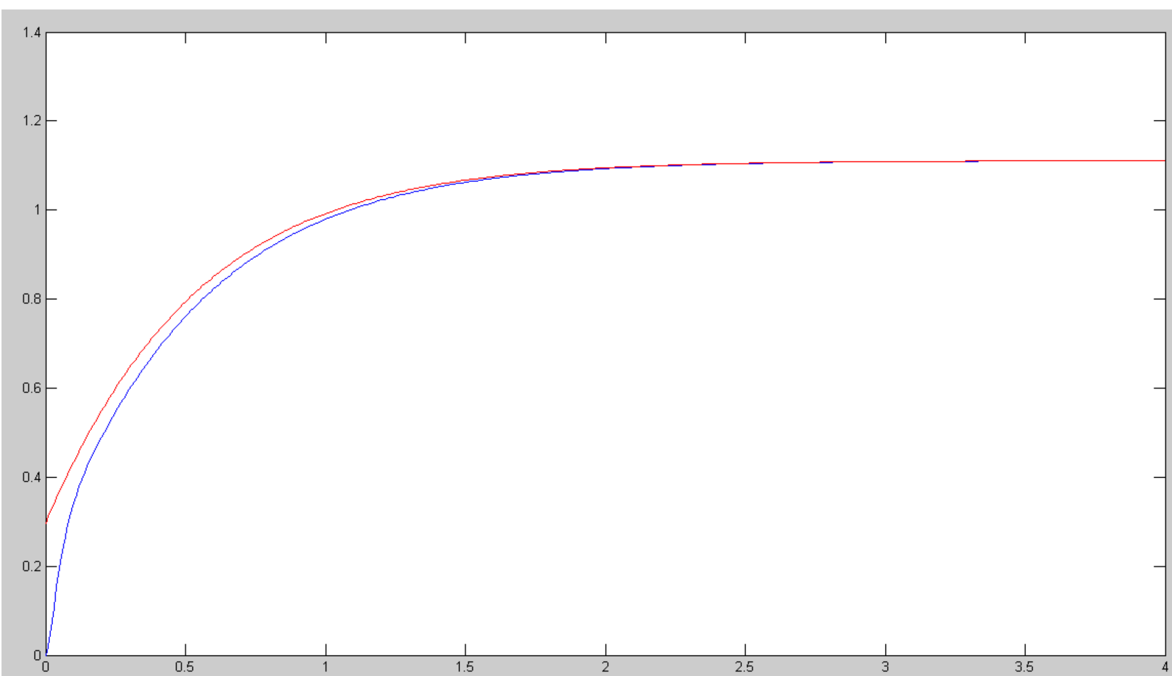
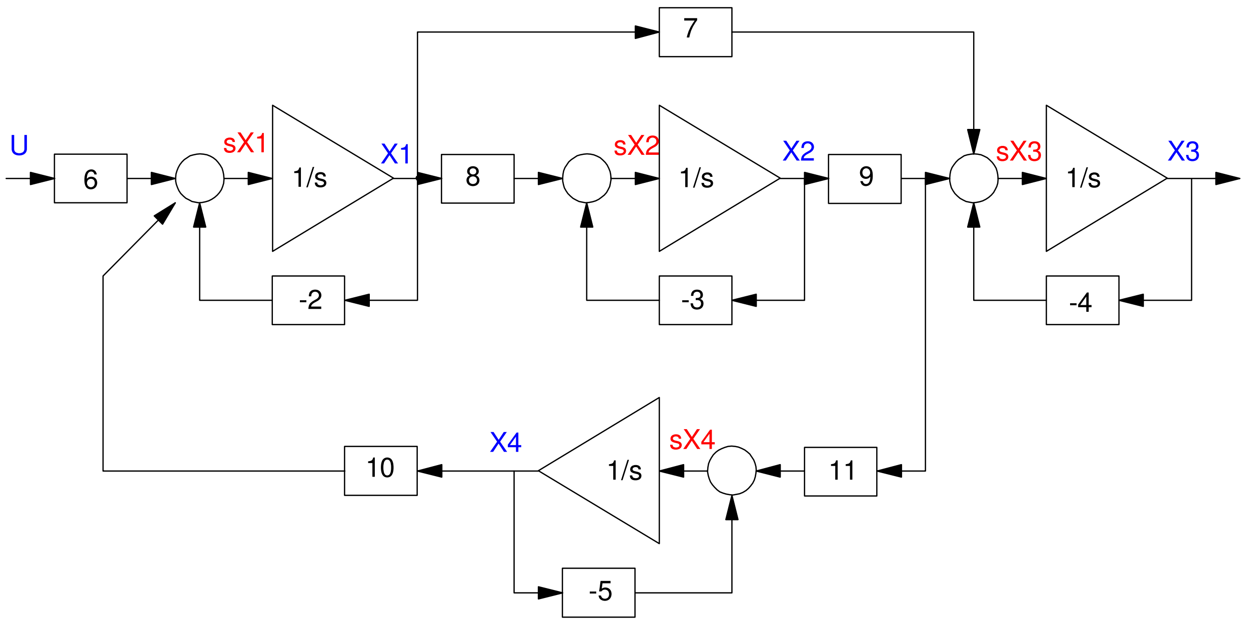


3) Give {A and B} for the the state-space model for the following system



Writing the equations:

$$sX_1 = 6U - 2X_1 + 10X_4$$

$$sX_2 = 8X_1 - 3X_2$$

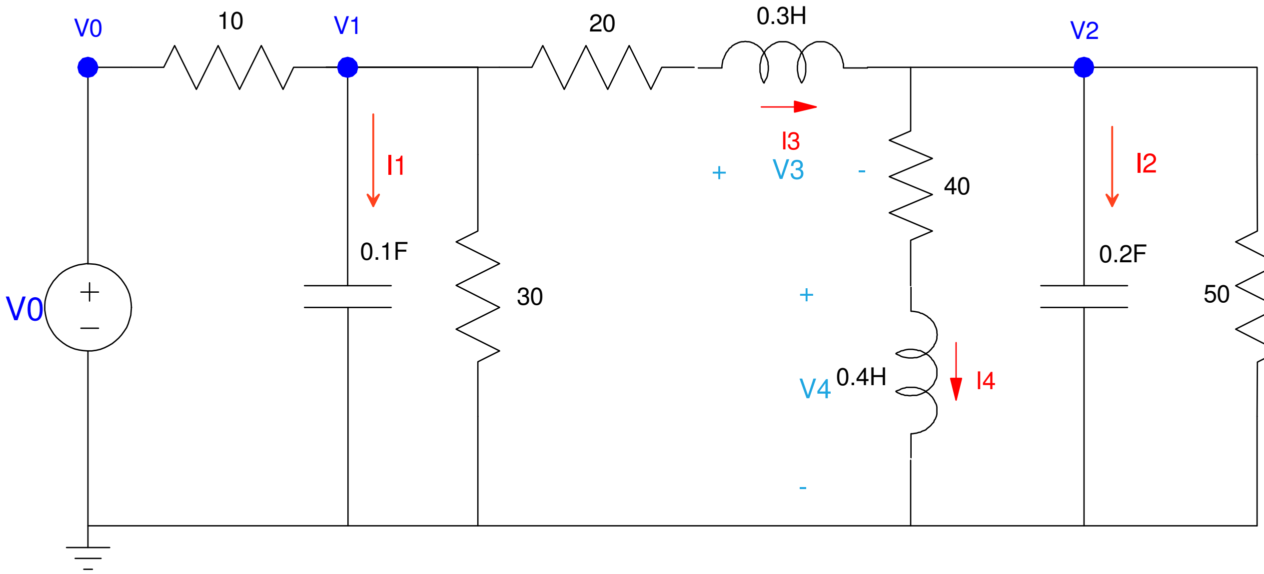
$$sX_3 = 7X_1 + 9X_2 - 4X_3$$

$$sX_4 = 11 \cdot 9X_2 - 5X_4$$

Place in matrix form

$$\begin{bmatrix} sX_1 \\ sX_2 \\ sX_3 \\ sX_4 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 & 10 \\ 8 & -3 & 0 & 0 \\ 7 & 9 & -4 & 0 \\ 0 & 99 & 0 & -5 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \end{bmatrix} U$$

4) Write four coupled differential equations to describe the following circuit. Assume the states are {V1, V2, I3, I4}. Note: For capacitors: $I = C\frac{dV}{dt}$, For inductors: $V = L\frac{dI}{dt}$



$$I_1 = 0.1sV_1 = \left(\frac{V_0 - V_1}{10}\right) - \left(\frac{V_1}{30}\right) - I_3$$

$$I_2 = 0.2sV_2 = I_3 - I_4 - \left(\frac{V_2}{50}\right)$$

$$V_3 = 0.3sI_3 = V_1 - 20I_3 - V_2$$

$$V_4 = 0.4sI_4 = V_2 - 40I_4$$

5) Assume the LaGrangian is:

$$L = 3\sin(x)\dot{x} + 4x^2\cos(x) + 7\dot{x}^2\dot{\theta}^3$$

Determine

$$F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right)$$

Take the partial derivatives

$$F = \frac{d}{dt}\left(3\sin(x) + 14\dot{x}\dot{\theta}^3\right) - \left(3\cos(x)\dot{x} + 8x\cos(x) - 4x^2\sin(x)\right)$$

Take the full derivative

$$F = \left(3\cos(x)\dot{x} + 14\ddot{x}\dot{\theta}^3 + 42\dot{x}\dot{\theta}^2\ddot{\theta}\right) - \left(3\cos(x)\dot{x} + 8x\cos(x) - 4x^2\sin(x)\right)$$