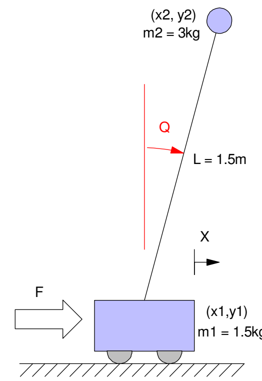
ECE 463/663 - Homework #13

VSS & Saturating Control. Due Monday, May 5th

VSS Control

1) For the cart and pendulum system of homework set #4:



$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.667 \\ -0.444 \end{bmatrix}F$$

Design a VSS control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{4}{(s+2)\left(s^2+s+2\right)}\right)R$$

```
A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0];
B = [0;0;0.6667;-0.4444];
C = [1,0,0,0];

N = length(A);

T1 = [];
for i=1:N
   T1 = [T1,  (A^(i-1))*B];
end


P = poly(eig(A));
T2 = [];
for i=1:N
    T2 = [T2; zeros(1,i-1), P(1:N-i+1)];
end

T3 = zeros(N,N);
for i=1:N
    T3(i, N+1-i) = 1;
end

T = T1*T2*T3;



>> conv([2,1],[2,1,1])

ans =      4      4      3      1

>> Kx = [4,4,3,1] * inv(T)

Kx =    -0.9180    -8.1280    -0.9180    -3.6275

>> eig(A - B*Kx*100)
```

```
ans =

 -97.1559
  -0.4995 + 1.4078i
  -0.4995 - 1.4078i
  -1.8451
```

Another way to see this, the zeros of the transfer funciton from F to the sliding surface should be the desired pole locations

```
>> Kx = num * inv(T)

Kx =    -0.9180   -8.1280   -0.9180   -3.6275

>> G = ss(A, B, Kx, 0);
>> zpk(G)

  (s+2) (s^2 + s + 2)
-----------------------
s^2 (s-4.427) (s+4.427)
```

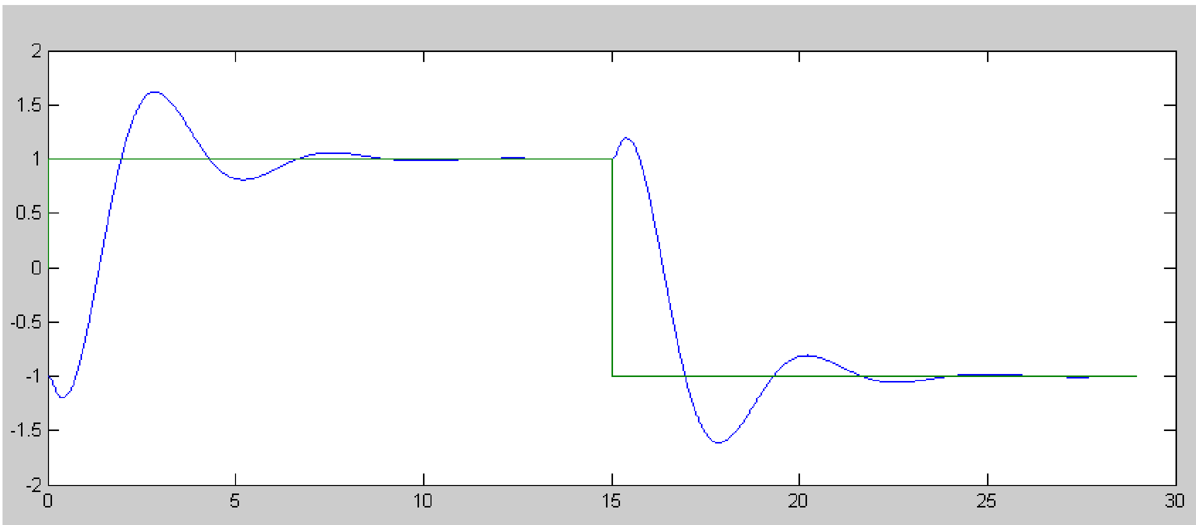
Check: the zeros are the desired closed-loop poles.

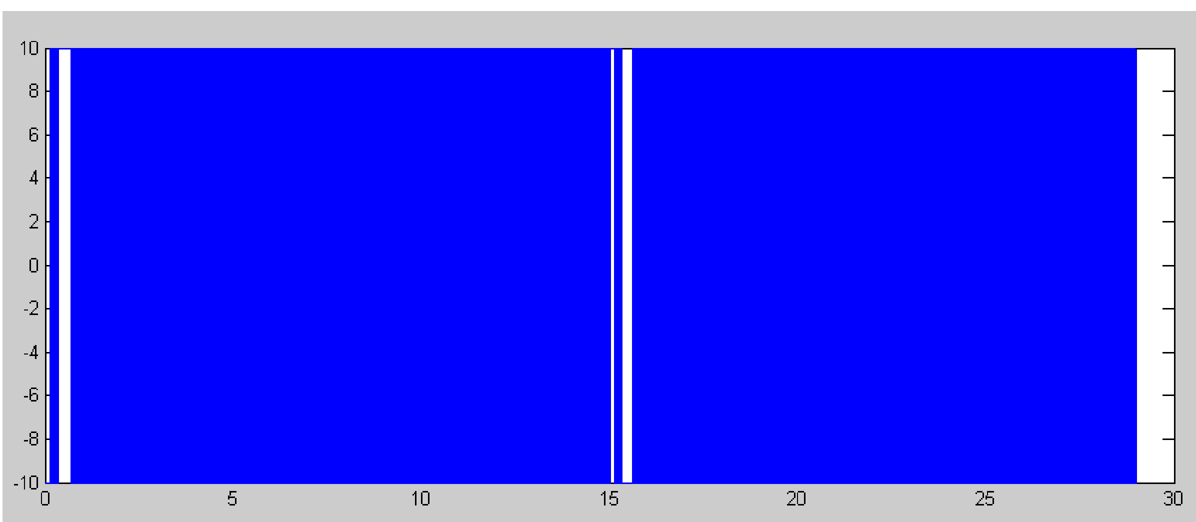
Note that the closed-loop poles go to the zeros as the feedback gain goes to infinity:

```
>> eig(A - B*Kx)

   -4.6859                  gain of one doesn't work:  A - B*Kx*1 is unstable
    3.4162
   -0.3829
    0.6526

>> eig(A - B*Kx*10)

  -9.2943                   still unstable with a gain of 10
   0.1625 + 2.0370i
   0.1625 - 2.0370i
  -1.0307

>> eig(A - B*Kx*100)

 -97.1559                   close - feedback gain of 100 drives poles to zeros
  -0.4995 + 1.4078i
  -0.4995 - 1.4078i
  -1.8451

>> eig(A - B*Kx*1000)

  -997.01                   closer - feedback gain of 1000 drives closer
  -0.5005 + 1.3309i
  -0.5005 - 1.3309i
  -1.9844
```

2) Find the step response of your control law on the linear model

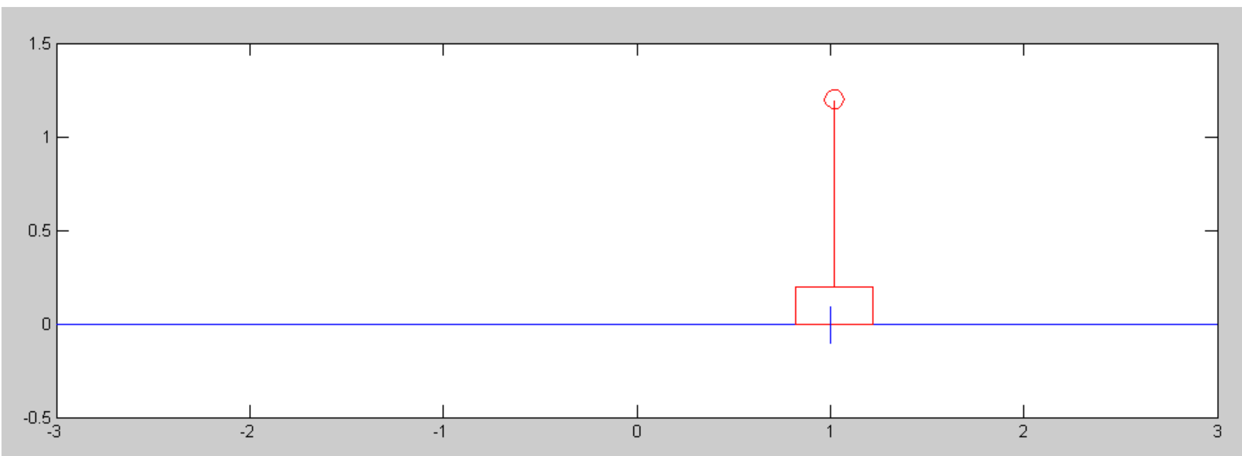
```
Kx = [-0.9180   -8.1280   -0.9180   -3.6275];
while(t < 29)
    Ref = sign(sin(t*2*pi/30);
    U = -10 * sign(Kx * (X - [Ref;0;0;0]));
    dX = A*X + B*U;
    X = X + dX * dt;
    t = t + dt;
    y = [y ; X(1), Ref, U];
    end
```



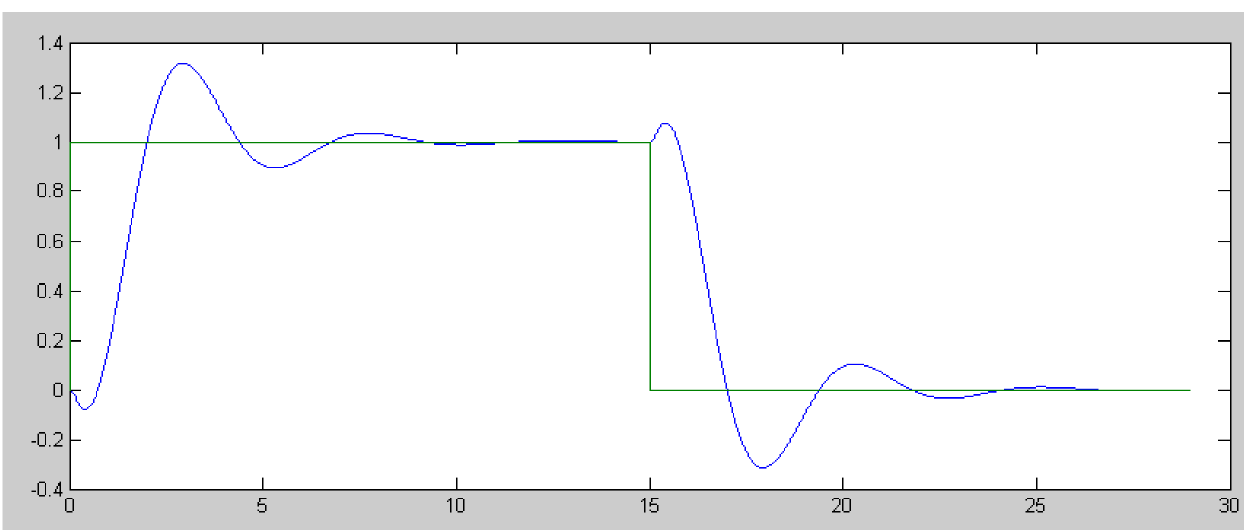


3) Find the step respone of your control law on the nonlinear simulation

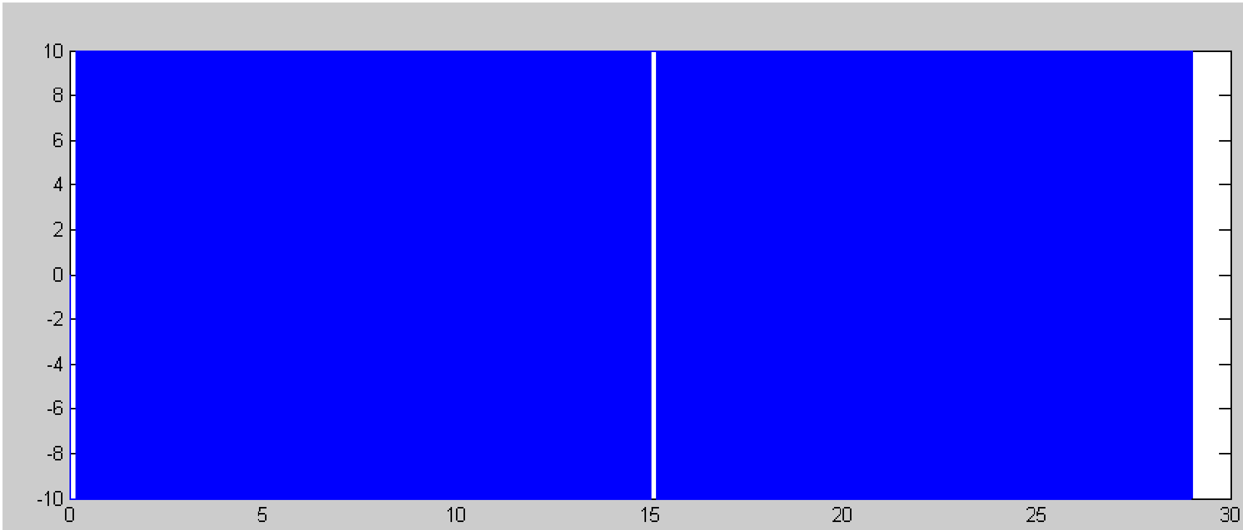
```
Kx = [-0.9180   -8.1280   -0.9180   -3.6275];
while(t < 29)
    Ref = 1*(sin(t*2*pi/30) > 0);
    U = -10 * sign(Kx * (X - [Ref;0;0;0]));

    dX = CartDynamics(X, U);
    X = X + dX * dt;

    t = t + dt;
    n = mod(n+1, 50);
    if(n == 0)
       CartDisplay(X, X, Ref);
       end
    y = [y ; X(1), Ref, U];
    end
```



Nonlinear Simulation for a VSS controller



Position (x) for a VSS controller



Input (U) for a VSS controller

Saturating Control:

4) Design a saturating control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{4}{(s+2)\left(s^2+s+2\right)} \right) R$$

Same as before but change U

$$U = -10 \cdot sign(CX)$$

to

$$U = -10 \cdot \text{limit}(CX)$$

where *limit()* is a saturation funciton

$$\text{limit(x)} = \begin{cases} +1 & x > 1 \\ x & -1 < x < 1 \\ -1 & x < -1 \end{cases}$$

and C places the zeros

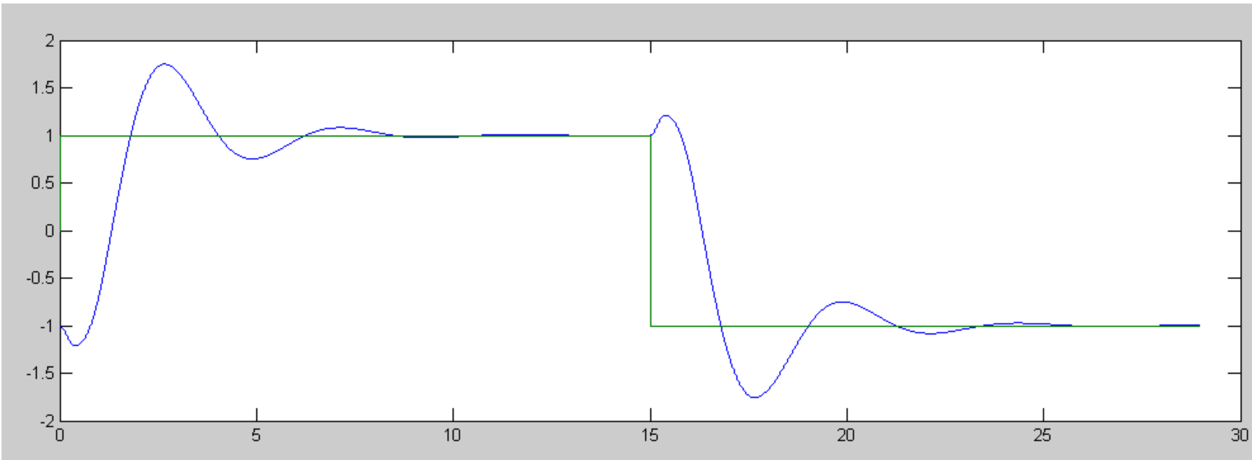
C = Kx = [-0.9180 -8.1280 -0.9180 -3.6275]

(same as before)

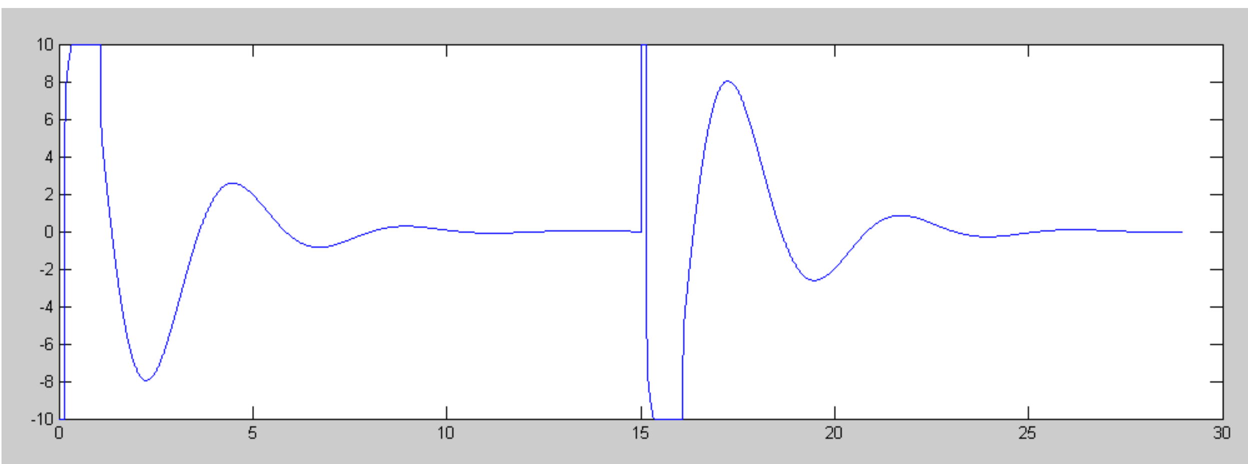
5) Find the step response of your control law on the linear model

Change one line of code:

```
% VSS
%     U = -10 * sign(Kx * (X - [Ref;0;0;0]));
% Saturating
    U = -100 * (Kx * (X - [Ref;0;0;0]));
    U = max(-10, min(10, U));
```

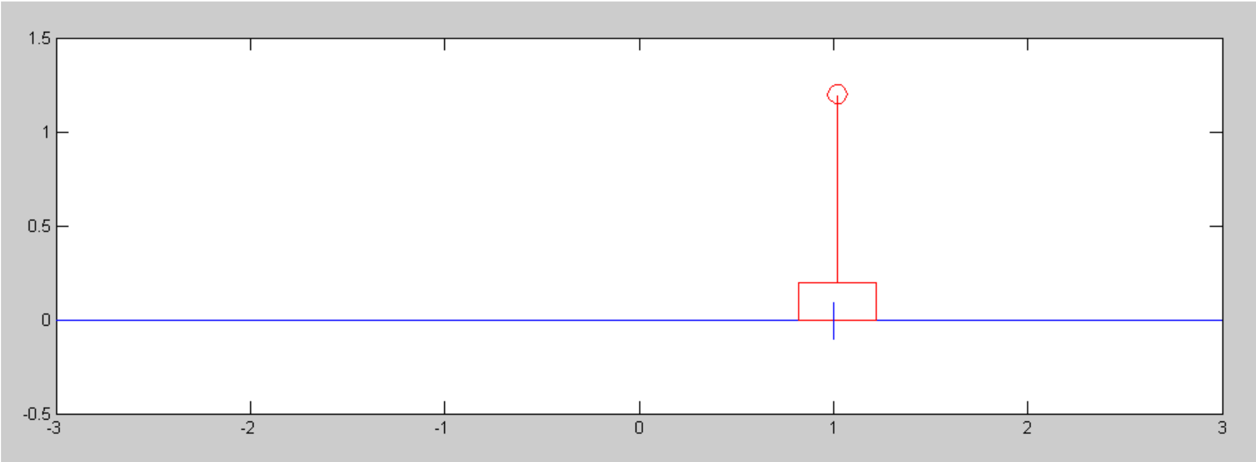


Position for a saturating controller

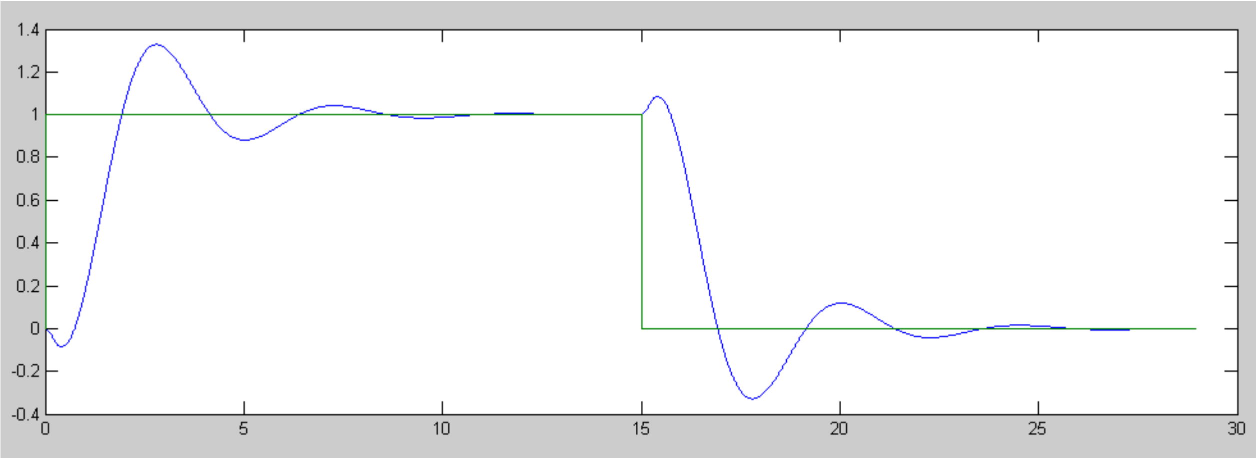


Input (U) for a saturating controller

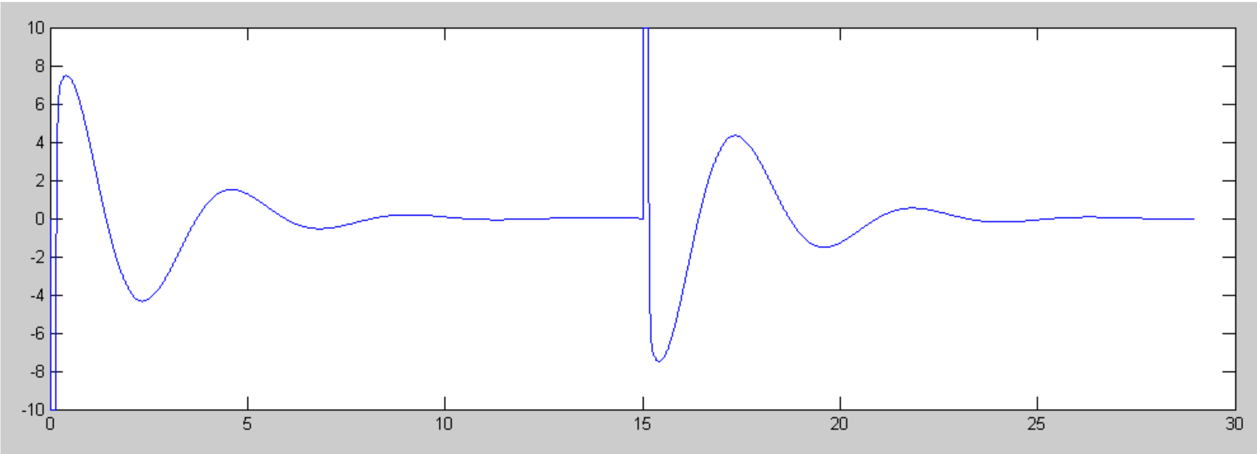
6) Find the step respone of your control law on the nonlinear simulation



Output for a Saturating Controller & Nonlinear Simulation



Position (x) for a Saturating Controller
(almost identical to a VSS controller)



Input (U) for a Saturating Controller