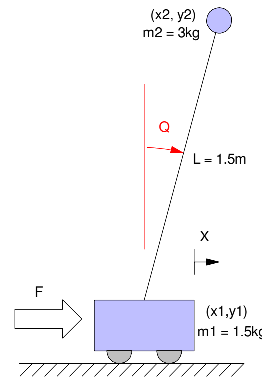
ECE 463/663 - Homework #12

LQG/LTR. Due Monday, April 28th

LQG / LTR

For the cart and pendulum system of homework set #4:

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.667 \\ -0.444 \end{bmatrix}F$$

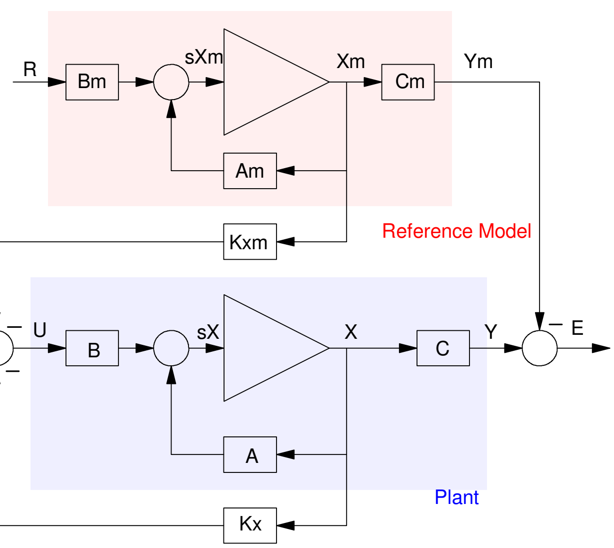


Design a control law so that the cart and pendulum system behaves like the following reference model:

$$y_m = \left(\frac{2}{s^2+s+2}\right)R$$

LQG/LTR without a Servo Compensator:

1) Give a block diagram for your controller



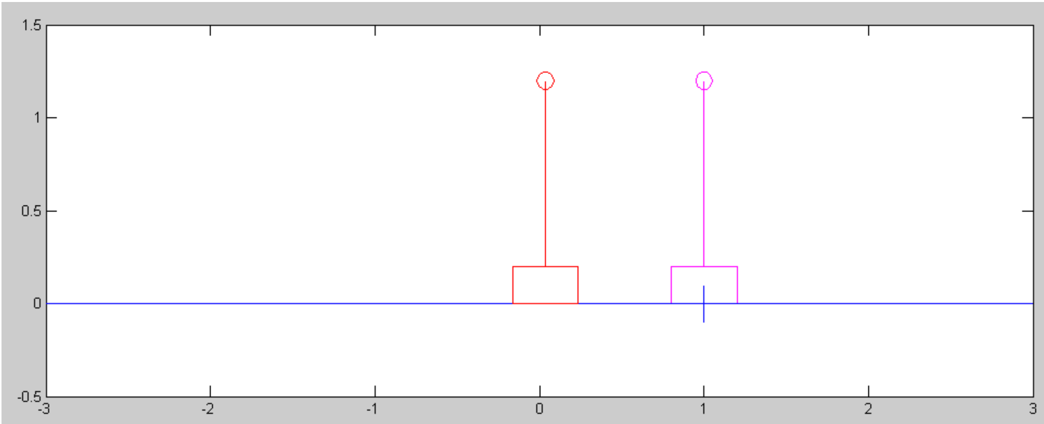
$$s\begin{bmatrix} X \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}U + \begin{bmatrix} 0 \\ B_m \end{bmatrix}R$$

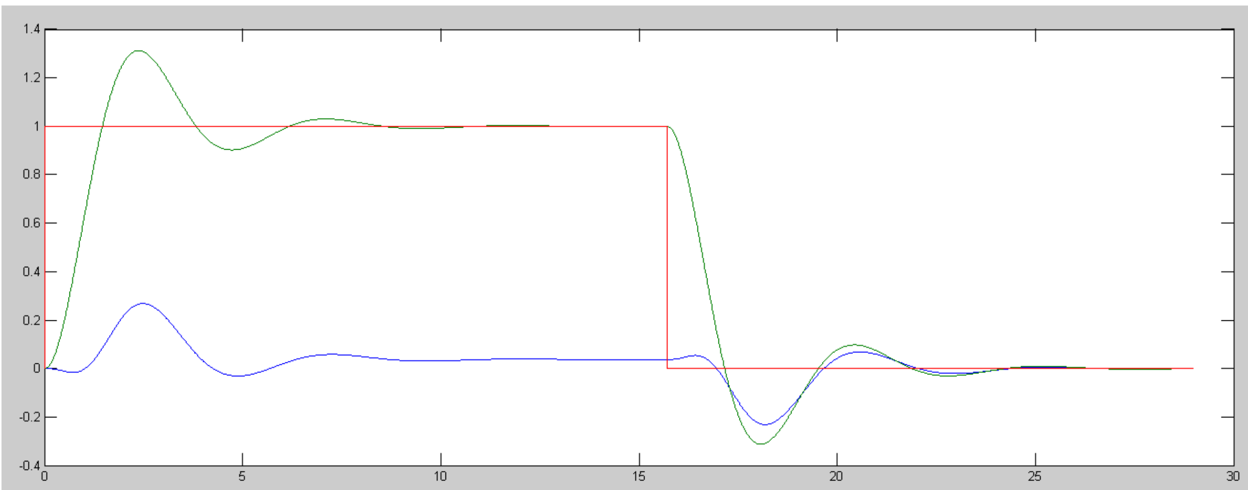
$$E = \begin{bmatrix} C & -C_m \end{bmatrix}\begin{bmatrix} X \\ X_m \end{bmatrix}$$

2) (20pt) Plot the step response of the model and the linearlized plant for yor control law for

- Q = 100 e^2
- Q = 1,000 e^2
- Q = 10,000 e^2

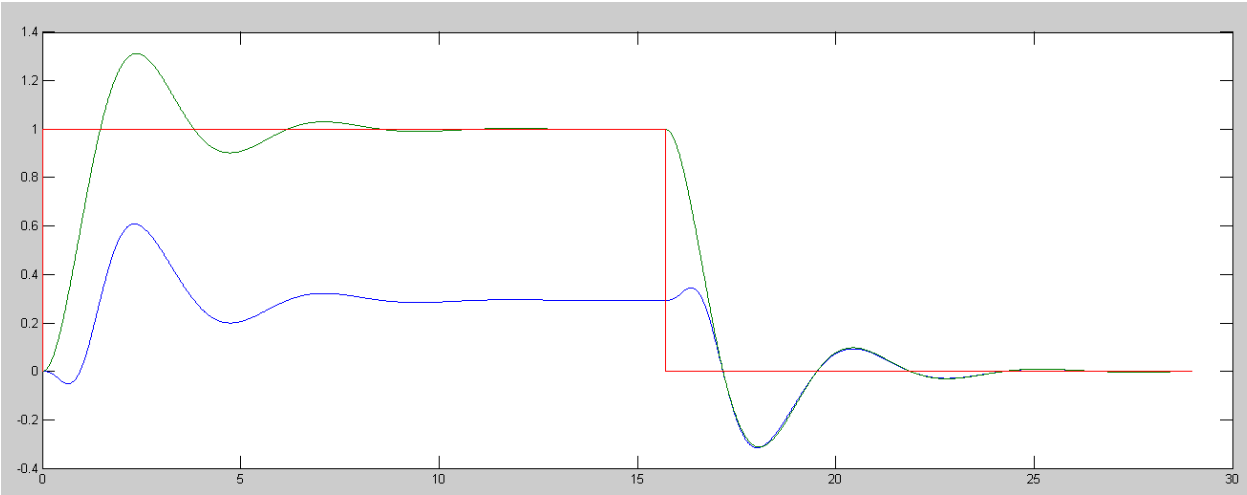
```
Q = 100 e^2
Kx =   -10.0000 -154.5162   -15.0076   -47.5723
Km =      3.1606     0.3732
```





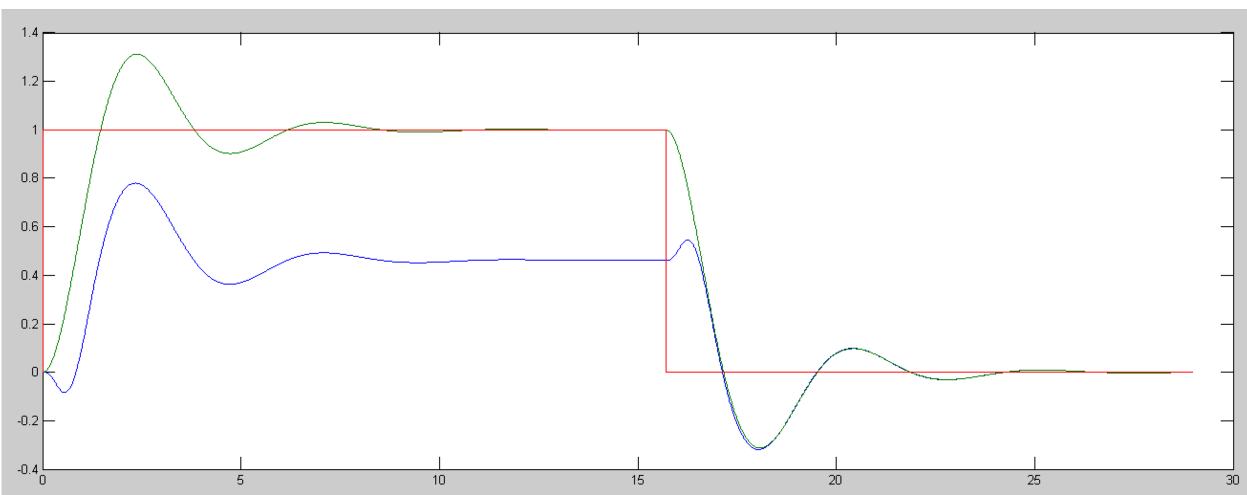
Model (green) & Plant (blue)

```
Q = 1000 e^2
Kx =  -31.6228 -246.4828  -36.1315  -84.1349
Km =    14.0914     9.2704
```



Model (green) & Plant (blue)

```
Q = 10,000 e^2
Kx = -100.0000 -503.5900  -96.8145 -185.1335
Km =    47.3710   46.1804
```



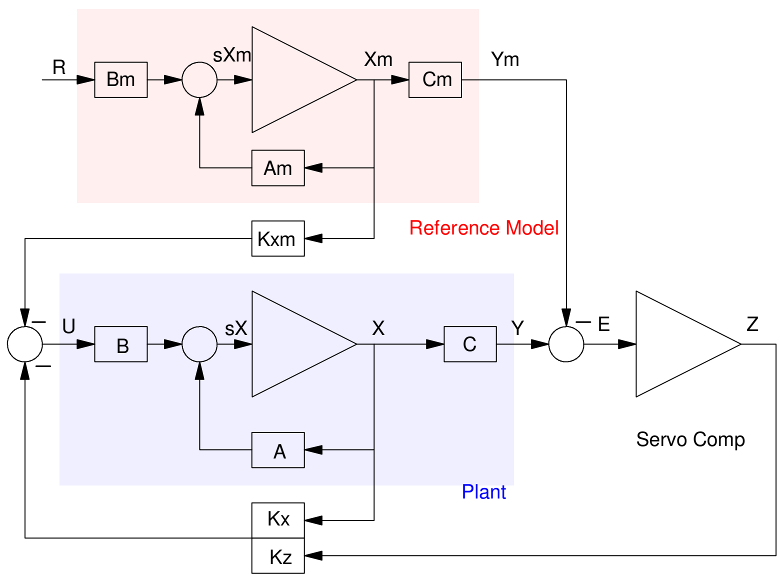
Model (green) & Plant (blue)

Code

```
A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0];
B = [0;0;0.6667;-0.4444];
C = [1,0,0,0];

Ref = 1;
dt = 0.01;
t = 0;

%Reference Model
Gm = tf(2,[1,1,2]);
Gss = ss(Gm);
Am = Gss.A;
Bm = Gss.B;
Cm = Gss.C;
Dm = Gss.D;

[n,m] = size(Am);

A6 = [ A, zeros(4,n) ;
       zeros(n,4), Am];
B6 = [B; zeros(n,1)];
B6r = [zeros(4,1); Bm];

C6 = [C, -Cm];
Q = C6' * C6;
R = 1;

K6 = lqr(A6, B6, Q*1e4, 1);

Kx = K6(1:4);
Km = K6(5:4+n);

X = zeros(4,1);
Xm = zeros(n,1);

n = 0;
y = [];
while(t < 29)
    Ref = 1*(sin(0.2*t) > 0);
    U = -Km*Xm - Kx*X;

    dX = CartDynamics(X, U);
    dXm = Am*Xm + Bm*Ref;

    X = X + dX * dt;
    Xm = Xm + dXm * dt;

    t = t + dt;
    n = mod(n+1, 5);
    if(n == 0)
       CartDisplay(X, [Cm*Xm;0;0;0], Ref);
       end
    y = [y ; X(1), Cm*Xm, Ref];
    end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```

LQG/LTR with a Servo Compensator:

3) Give a block diagram for your controller plus servo compensator



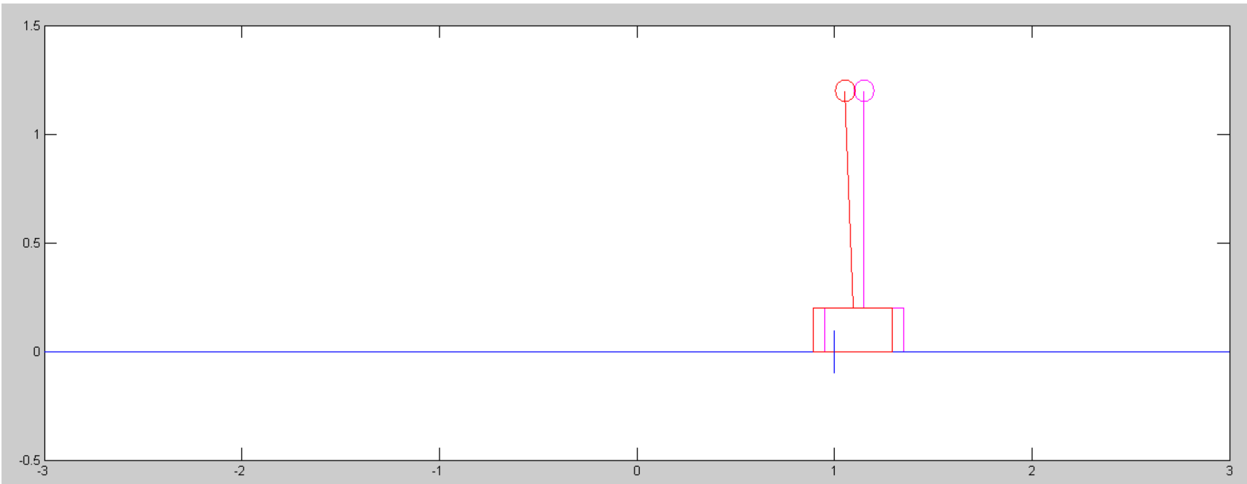
$$s\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & -C_m \\ 0 & 0 & A_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix} + \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix} U + \begin{bmatrix} 0 \\ 0 \\ B_m \end{bmatrix} R$$

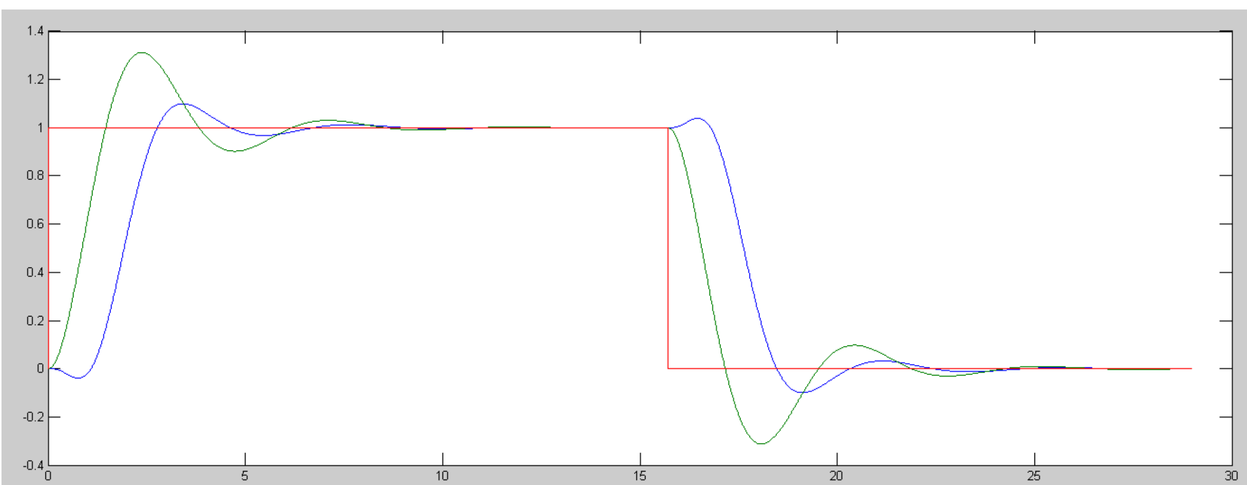
$$U = \begin{bmatrix} -K_x & -K_z & -K_m \end{bmatrix}\begin{bmatrix} X \\ Z \\ X_m \end{bmatrix}$$

4) (20pt) Plot the step response of the model and the linearlized plant for yor control law for

- Q = 100 z^2
- Q = 1,000 z^2
- Q = 10,000 z^2

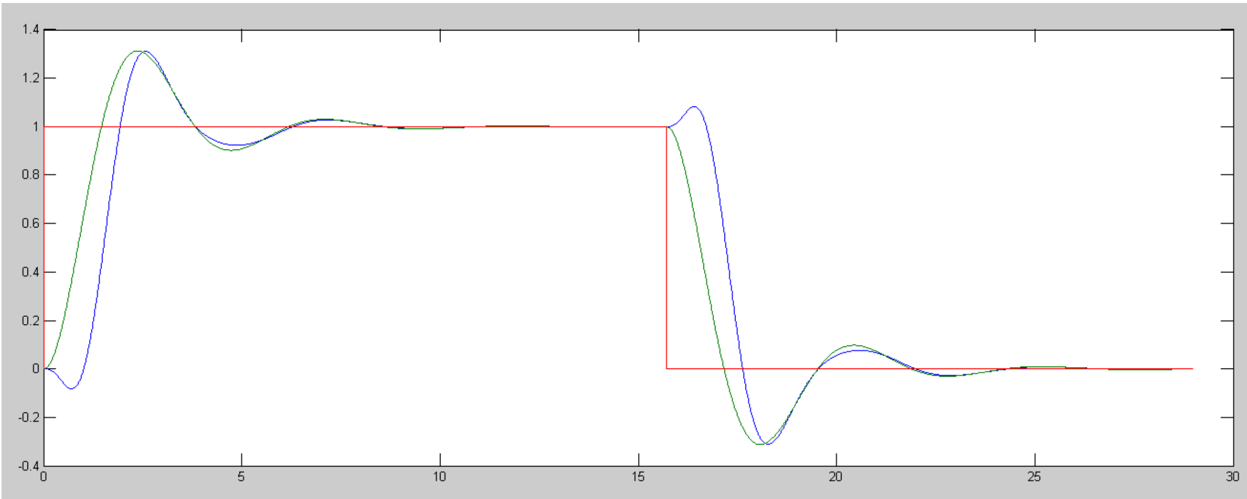
```
Q = 100 z^2
Kx =  -20.6909 -180.7282  -21.4056  -58.0681
Kz =  -10.0000
Km =    5.9343    7.0263
```





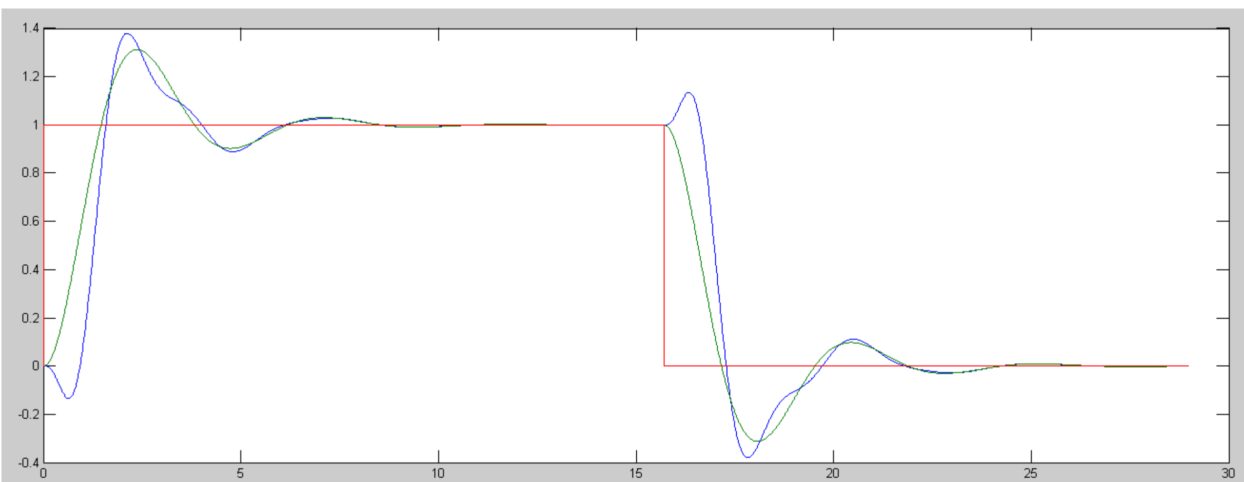
Model (green) & Plant (blue)

```
Q = 1000 z^2
Kx =  -51.0518 -263.4277  -41.2091  -90.8204
Kz =  -31.6228
Km =   16.9527   26.0676
```



Model (green) & Plant (blue)

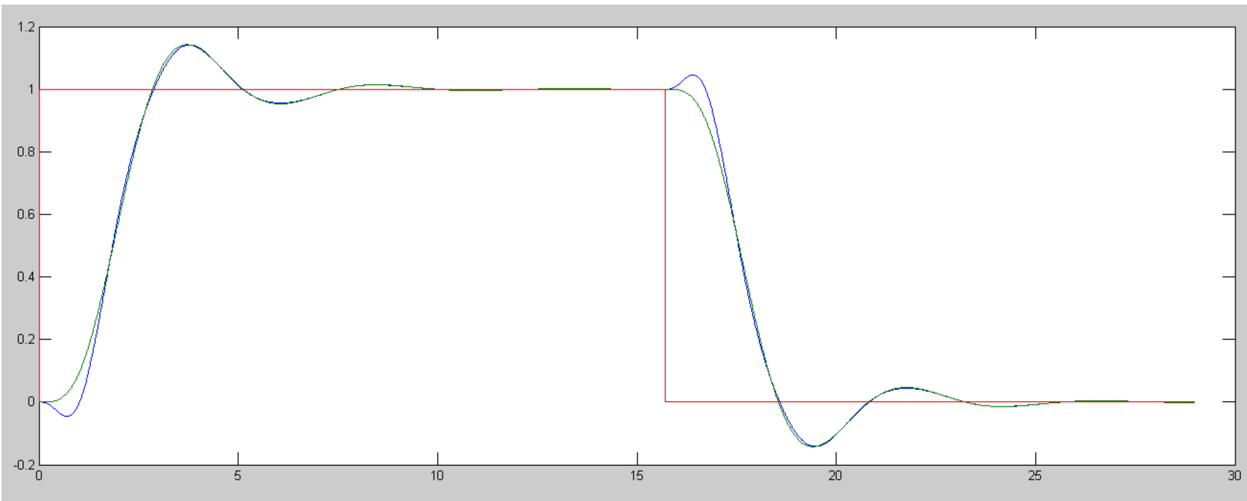
```
Q = 10,000 z^2
Kx = -132.1594 -451.6872  -87.3305 -164.7866
Kz = -100.0000
Km =   43.2913   84.2529
```



Model (green) & Plant (blue)

Sidelight: Part of the problem is you're trying to make a 4th-order system behave like a 2nd-order system. It works a little better if you make the reference model 4th-order as well:

$$G_m = \left(\frac{6}{(s^2+s+2)(s+1.5)(s+2)}\right)$$



Model (green) & Plant (blue)

```
>> K7

Kx = -132.1594 -451.6872  -87.3305 -164.7866
Km = -100.0000  162.3067  144.8585   41.9802   10.5377
```

It works a little better if you add a right-half plane zero to create undershoot

- A reference model that takes into account how the plant *wants* to behave tends to work better than a reference model that ignores the plant's dynamics.

$$G_m = \left(\frac{-2(s-3)}{(s+1.5)(s+2)(s^2+s+2)}\right)$$