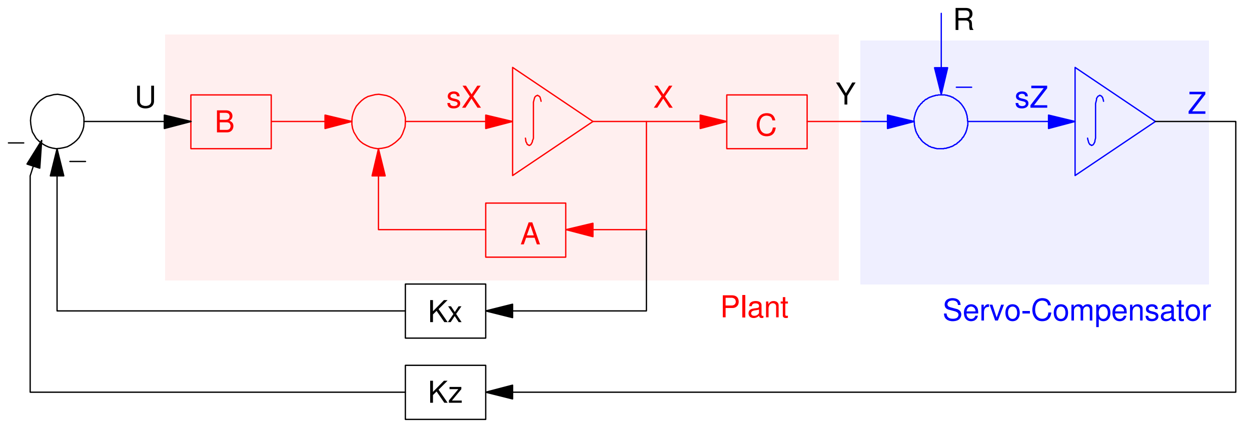
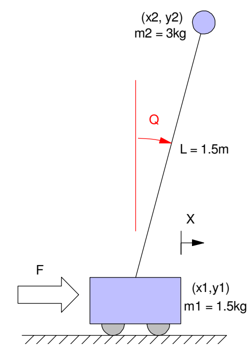
ECE 463/663 - Homework #10

LQG Control with Servo Compensators. Due Monday, April 14th



Cart and Pendulum (HW #4): For the cart and pendulum system of homework #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.667 \\ -0.444 \end{bmatrix}F$$



Use LQG methods to design a full-state feedback control law of the form

$$F = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the cart and pendulum system from homework #4 using LQG control so that

- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

1) Give the control law (Kx and Kz) and explain how you chose Q and R

Define how the system *should* behave

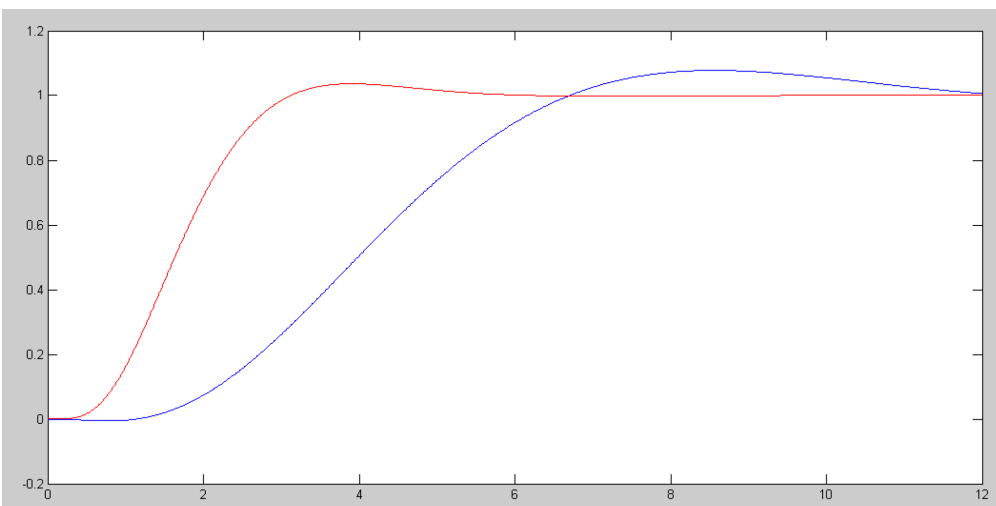
```
Gd = zpk([], [-1+j, -1-j, -4, -5, -6], 1);
DC = evalfr(Gd,0);
Gd = Gd/DC;
t = [0:0.01:12]';
Yd = step(Gd, t);
```

Create an augmented system: plant plus a servo compensator with poles at s=0

```
A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0];
B = [0;0;0.667;-0.444];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5u = [B;0];
B5r = [0*B;-1];
C5 = [1,0,0,0,0];
D5 = 0;
```

Use LQR methods to find full-state feedback gains. Start with Q weighting the servo compensator alone:

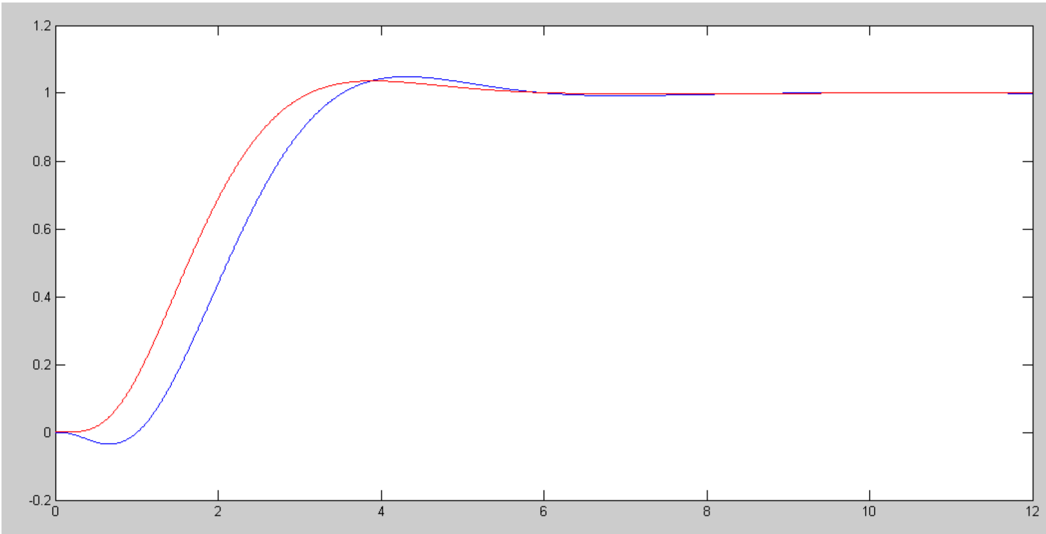
```
K5 = lqr(A5, B5u, diag([0,0,0,0,1]), 1);
Gcl = ss(A5-B5u*K5, B5r, C5, D5);
Y = step(Gcl, t);

plot(t,Y,'b',t,Yd,'r');
```



Red = desired response (Yd), Blue = actual response (Y = x = position)

Adjust the weights on Q to speed up the system (weight on z) and redice the overshoot (weight on x) until the response looks OK

```
K5 = lqr(A5, B5u, diag([10,0,0,0,100]), 1);
Gcl = ss(A5-B5u*K5, B5r, C5, D5);
Y = step(Gcl, t);

plot(t,Y,'b',t,Yd,'r');
```



Desired reponse (red) & actual response (blue)

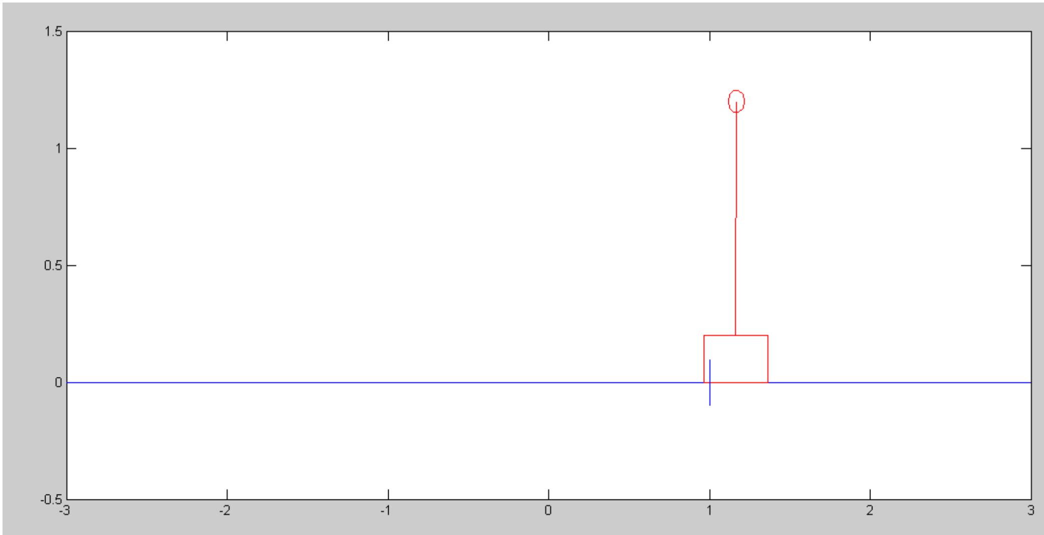
The resulting feedback gains and closed-loop pole locations

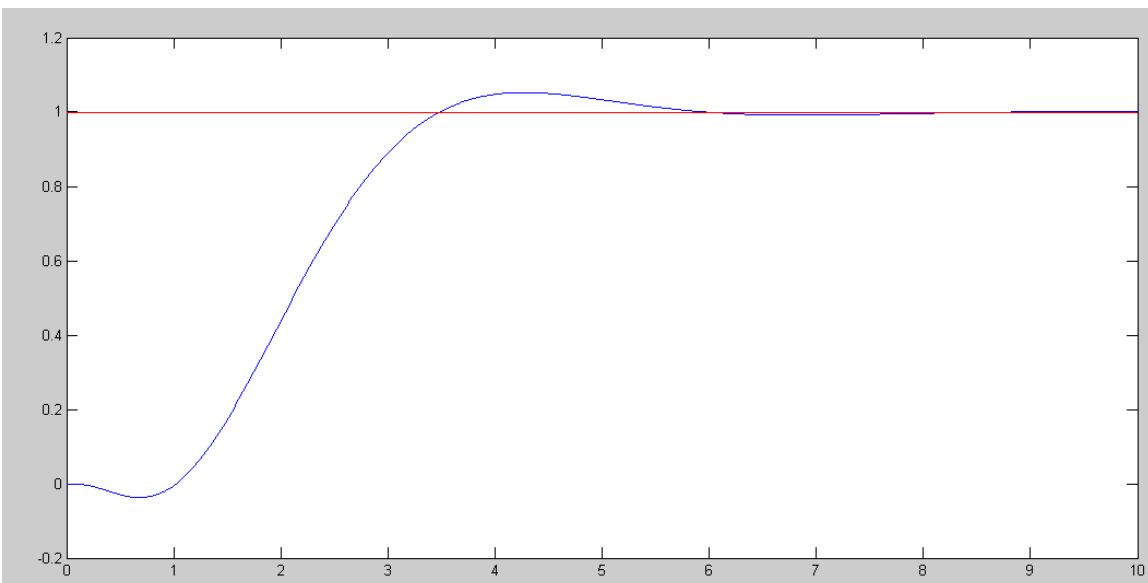
```
>> K5

K5 =   -21.1300 -182.9577  -21.8239  -58.8838  -10.0000

>> eig(A5-B5u*K5)

  -4.4229 + 0.1110i
  -4.4229 - 0.1110i
  -0.7701 + 1.1246i
  -0.7701 - 1.1246i
  -1.2018

>>
```

2) Plot the step response of the linear system

```
shown above
```

3) Check your design with the nonlinear simulation of the cart and pendulum system.





Ball and Beam (HW #4): For the ball and beam system of homework #4

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.434 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} T$$

Use LQG methods to design a full-state feedback control law of the form

$$T = U = -K_z Z - K_x X$$

$$\dot{Z} = (x - R)$$

for the ball and beam system from homework #6 using LQG control so that

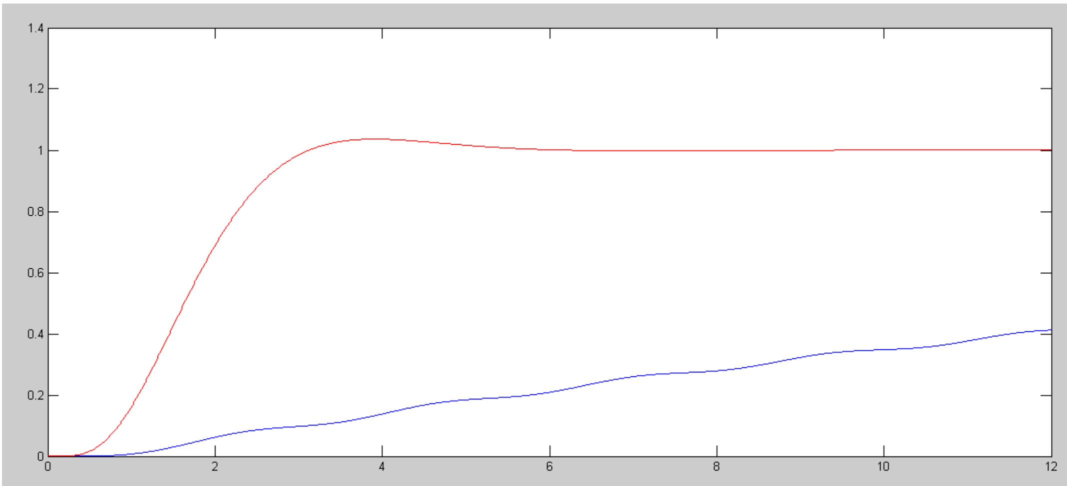
- You track constant setpoints,
- You reject constant disturbances,
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

4) Give the control law (Kx and Kx) and explain how you chose Q and R

Same procedure as before.

- Start with adding a servo compensator with poles at s = 0
- Create the augmented system (plant & servo)
- Find full-state feedback gains using LQR methods
- Start with weighting the servo state

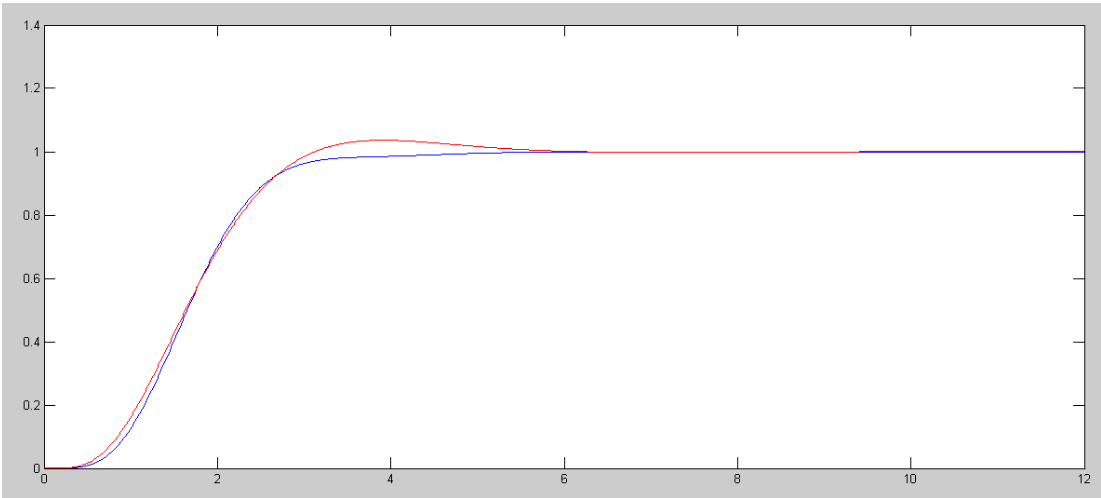
```
A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.424,0,0,0];
B = [0;0;0;0.345];
C = [1,0,0,0];

A5 = [A, zeros(4,1) ; C, 0];
B5u = [B;0];
B5r = [0*B;-1];
C5 = [1,0,0,0,0];
D5 = 0;

K5 = lqr(A5, B5u, diag([0,0,0,0,1]), 1);
Gcl = ss(A5-B5u*K5, B5r, C5, D5);
Y = step(Gcl, t);
```



Desired Response (red) & Actual Response

Increase the weightings in Q until the two are close

```
K5 = lqr(A5, B5u, diag([0,1600,0,0,900]), 1);
Gcl = ss(A5-B5u*K5, B5r, C5, D5);
Y = step(Gcl, t);
```



Red = Desired Step Response: Blue = Actual Step Response after tuning Q

The results are

```
>> K5

K5 =    -73.4849  108.1086  -37.2902   25.0343  -30.0000

>> eig(A5-B5u*K5)

  -1.4516 + 2.2014i
  -1.4516 - 2.2014i
  -2.2031 + 1.7309i
  -2.2031 - 1.7309i
  -1.3274
```

5) Plot the step response of the linear system

shown above

6) Check your design with the nonlinear simulation of the cart and pendulum system.