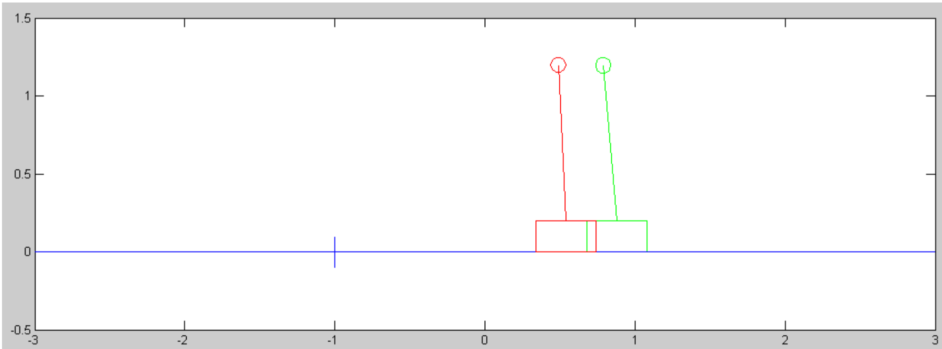
ECE 463: Homework #8

Linear Observers. Due Monday, March 31st



Cart and Pendulum from homework #4 with a state estimator (green)

Use the dynamics for the cart and pendulum from homework set #4

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.6667 \\ -0.4444 \end{bmatrix}F$$

1) Design a full-state feedback control law of the form

$$U = F = K_r R - K_x X$$

so that the closed-loop system has

- A 2% settling time of 6 seconds, and
- 10% overshoot for a step input.

Plot the step response of the linarized system in Matlab.

Translation: Place the closed-loop dominant poles at -0.667 + j0.910

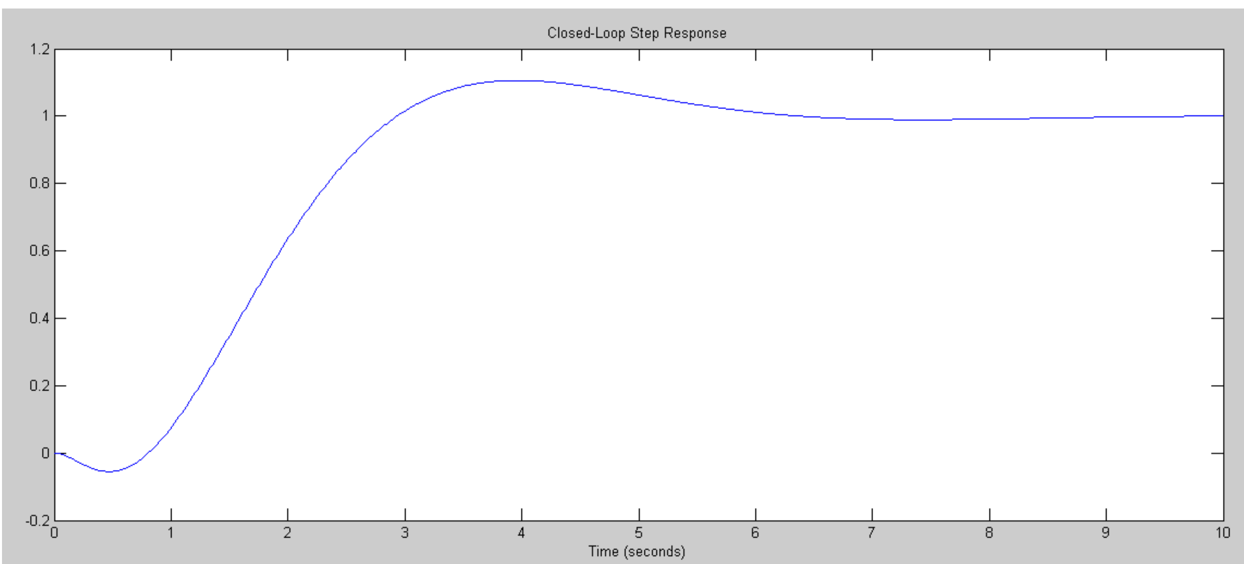
In Matlab:

```
>> A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -19.6000         0         0
         0   19.6000         0         0

>> B = [0;0;0.6667;-0.4444];
>> Kx = ppl(A, B, [-0.667+0.91i, -0.667-0.91i, -3, -4])

Kx =    -3.5060 -100.2440   -5.7192  -27.3334
```

```
>> eig(A-B*Kx)

  -4.0000
  -3.0000
  -0.6670 + 0.9100i
  -0.6670 - 0.9100i

>> C = [1,0,0,0];
>> D = 0;
>> DC = -C*inv(A-B*Kx)*B

DC =   -0.2852

>> Kr = 1/DC
```

Kr =   -3.5060

```
>> G = ss(A-B*Kx, B*Kr, C, D);
>> t = [0:0.01:10]';
>> y = step(G,t);
>> plot(t,y);
>> title('Closed-Loop Step Response');
>> xlabel('Time (seconds)')
>>
```



2) Design a full-order observer to estimate all four states so that the observer is 2-5 times faster than the plant. You may use either cart position or beam angle (or both) as measurements.

$$sX_e = AX_e + BU + H(Y - Y_e)$$

Let the observer poles be {-2+j2, -2-j2, -3, -4}

```
>> H = ppl(A', C', [-2+2i, -2-2i, -3, -4])'

    11.0000
   -16.3061
    67.6000
   -72.4980

>> eig(A-H*C)

   -2.0000 + 2.0000i
   -2.0000 - 2.0000i
   -3.0000
   -4.0000
```

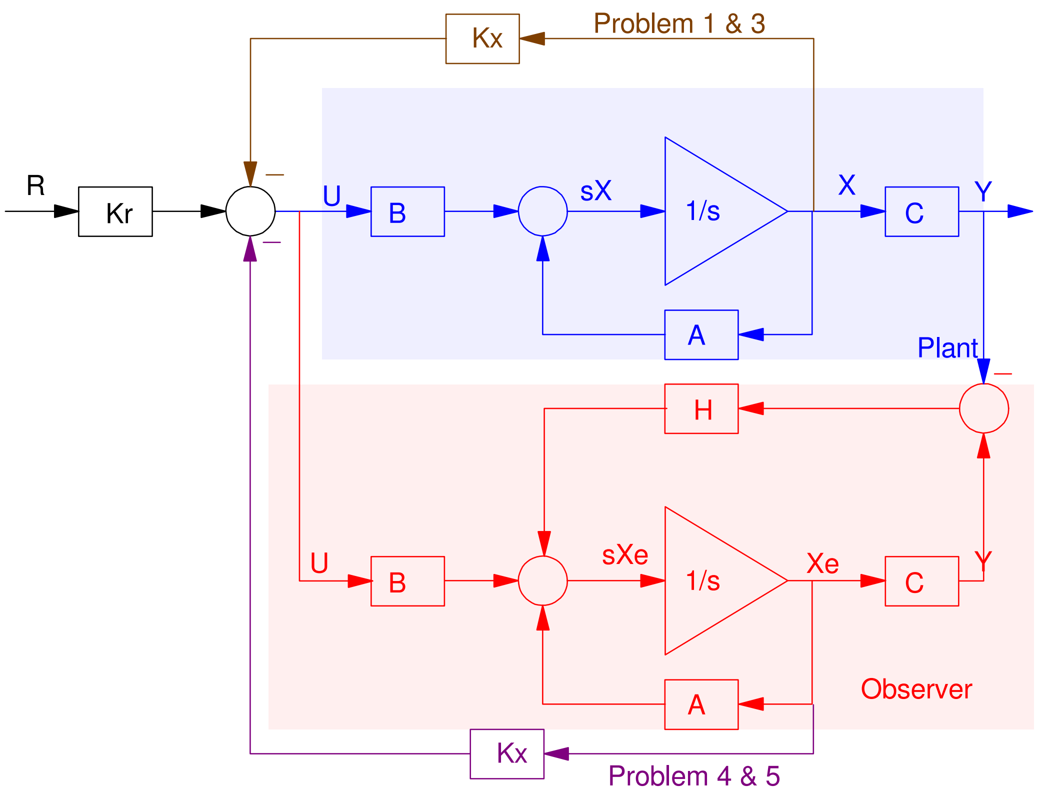
3) Give the state-space model of the closed-loop system using the actual states:

$$U = F = K_r R - K_x X$$

and plot the step response with initial conditions of

$X(0) = [0, 0, 0, 0]'$ $X_e(0) = [0, 0.1, 0, 0]'$

(note: use the function step3)



In State-Space

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A & 0 \\ HC & A-HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} B \\ B \end{bmatrix} U$$

Using this U results in

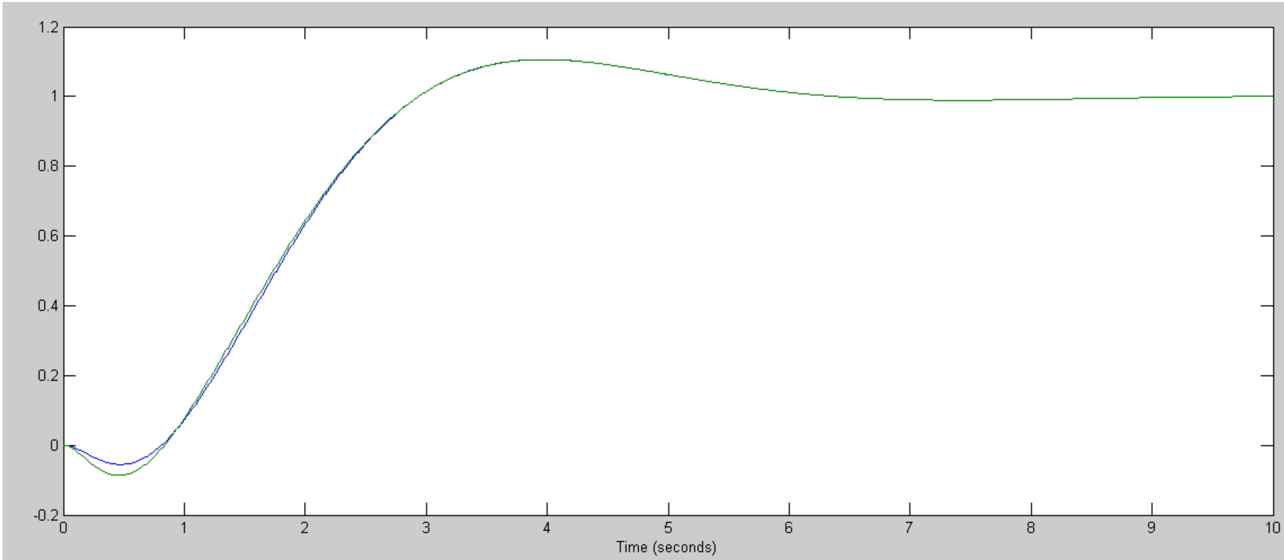
$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A-BKx & 0 \\ HC-BK_x & A-HC \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

Plotting the step response

```
>> A8 = [A-B*Kx, zeros(4,4) ; H*C-B*Kx, A-H*C]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
    2.3374   47.2327    3.8130   18.2232         0         0         0         0
   -1.5581  -24.9484   -2.5416  -12.1470         0         0         0         0
   11.0000         0         0         0  -11.0000         0    1.0000         0
  -16.3061         0         0         0   16.3061         0         0    1.0000
   69.9374   66.8327    3.8130   18.2232  -67.6000  -19.6000         0         0
  -74.0560  -44.5484   -2.5416  -12.1470   72.4980   19.6000         0         0

>> eig(A8)

  -2.0000 + 2.0000i
  -2.0000 - 2.0000i
  -0.6670 + 0.9100i
  -0.6670 - 0.9100i
  -4.0000 + 0.0000i
  -4.0000 - 0.0000i
  -3.0000 + 0.0000i
  -3.0000 - 0.0000i
```

The eigenvalues are where they should be - so the system might be correct.

```
>> B8 = [B*Kr ; B*Kr]

         0
         0
   -2.3374
    1.5581
         0
         0
   -2.3374
    1.5581

>> C8 = [C,0*C ; 0*C, C]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = [0;0];
>> t = [0:0.01:10]';
>> X0 = [0;0;0;0 ; 0;0.1;0;0];
>> R = 0*t+1;
>> y = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y)
>> xlabel('Time (seconds)')
>>
```

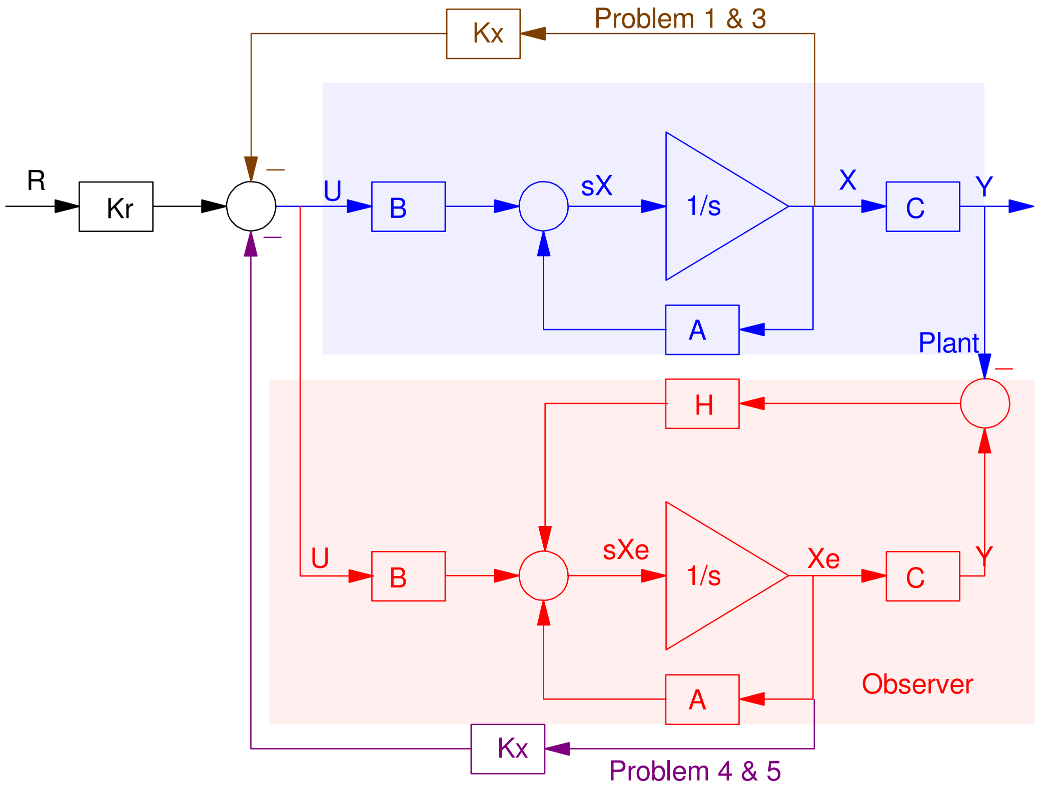

1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
1
2
3
4
5
6
7
8
9
10
Time (seconds)

4) Give the state-space model of the closed loop system using the state estimates:

$$U = K_r R - K_x X_e$$

and plot the step response with initial conditions of

$X(0) = [0, 0, 0, 0]'$ $X_{observer}(0) = [0, 0.1, 0, 0]'$

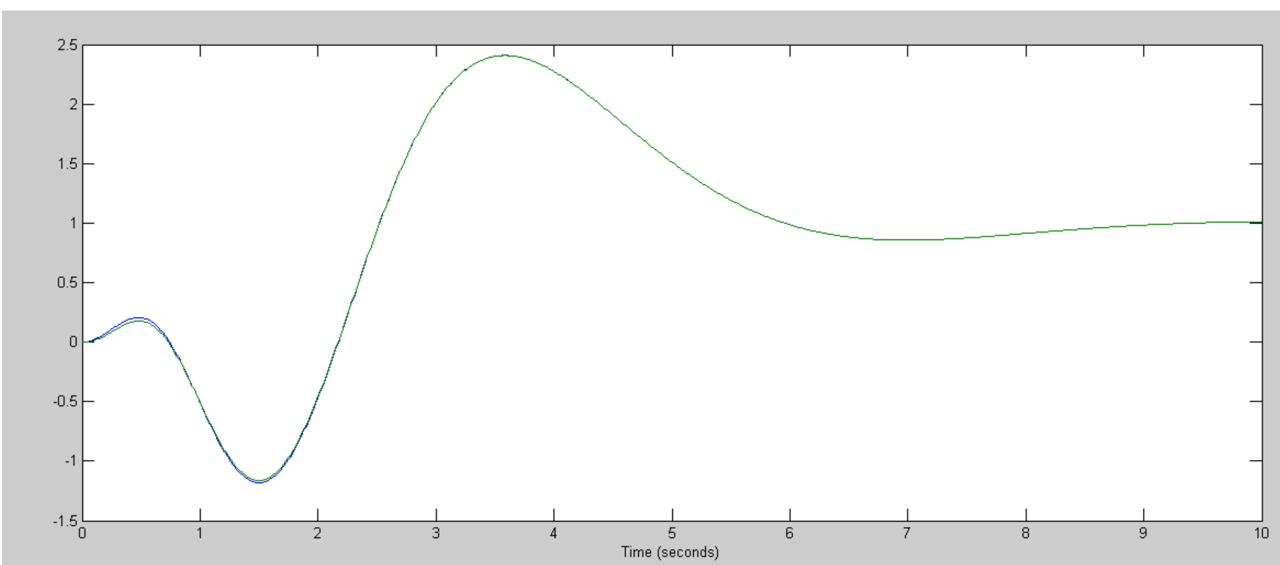


Now the net system is

$$\begin{bmatrix} sX \\ sX_e \end{bmatrix} = \begin{bmatrix} A - BKx & 0 \\ HC & A - HC - BK_x \end{bmatrix} \begin{bmatrix} X \\ X_e \end{bmatrix} + \begin{bmatrix} BK_r \\ BK_r \end{bmatrix} R$$

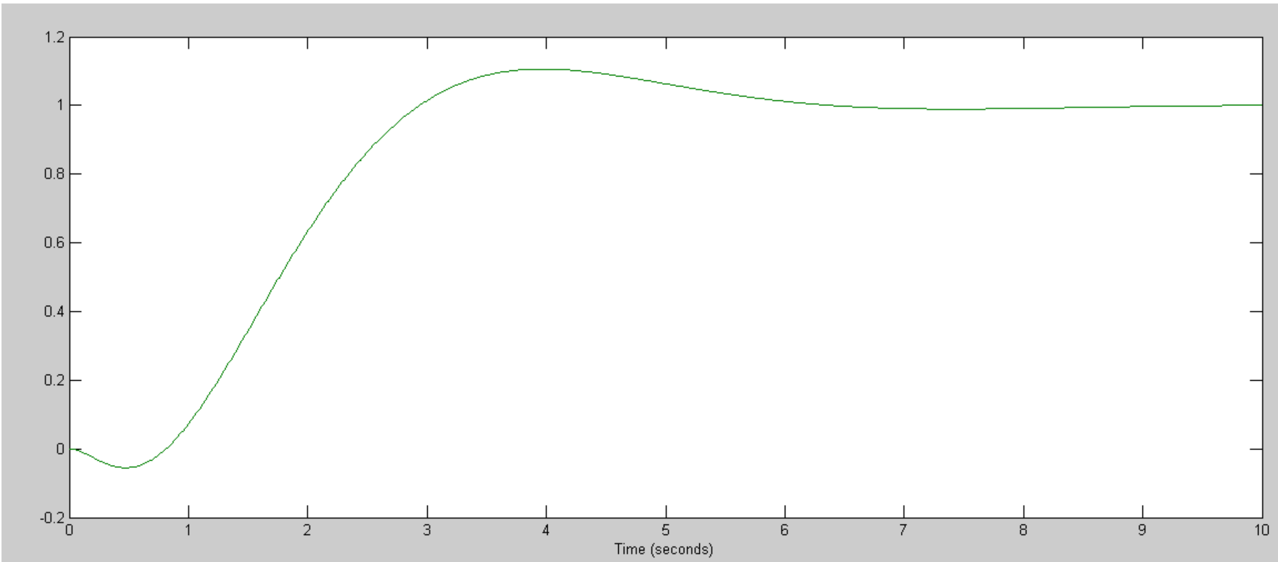
```
>> A8 = [A, -B*Kx ; H*C, A-H*C-B*Kx]

         0         0    1.0000         0         0         0         0         0
         0         0         0    1.0000         0         0         0         0
         0  -19.6000         0         0    2.3374   66.8327    3.8130   18.2232
         0   19.6000         0         0   -1.5581  -44.5484   -2.5416  -12.1470
   11.0000         0         0         0  -11.0000         0    1.0000         0
  -16.3061         0         0         0   16.3061         0         0    1.0000
   67.6000         0         0         0  -65.2626   47.2327    3.8130   18.2232
  -72.4980         0         0         0   70.9399  -24.9484   -2.5416  -12.1470
```

```
>> B8 = [B*Kr ; B*Kr]

         0
         0
   -2.3374
    1.5581
         0
         0
   -2.3374
    1.5581

>> C8 = [C,0*C ; 0*C, C]

     1     0     0     0     0     0     0     0
     0     0     0     0     1     0     0     0

>> D8 = [0;0];
>> X0 = [0;0;0;0 ; 0;0.1;0;0]

         0
         0
         0
         0
         0
    0.1000
         0

>> t = [0:0.01:10]';
>> R = 0*t+1;
>> y = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y)
>> xlabel('Time (seconds)')
>>
```



A little wonky - but that's due to the observer states being slightly off at t=0. Assuming it's locked on:

```
>> X0 = zeros(8,1);
>> y = step3(A8, B8, C8, D8, t, X0, R);
>> plot(t,y)
>> xlabel('Time (seconds)')
>>
```

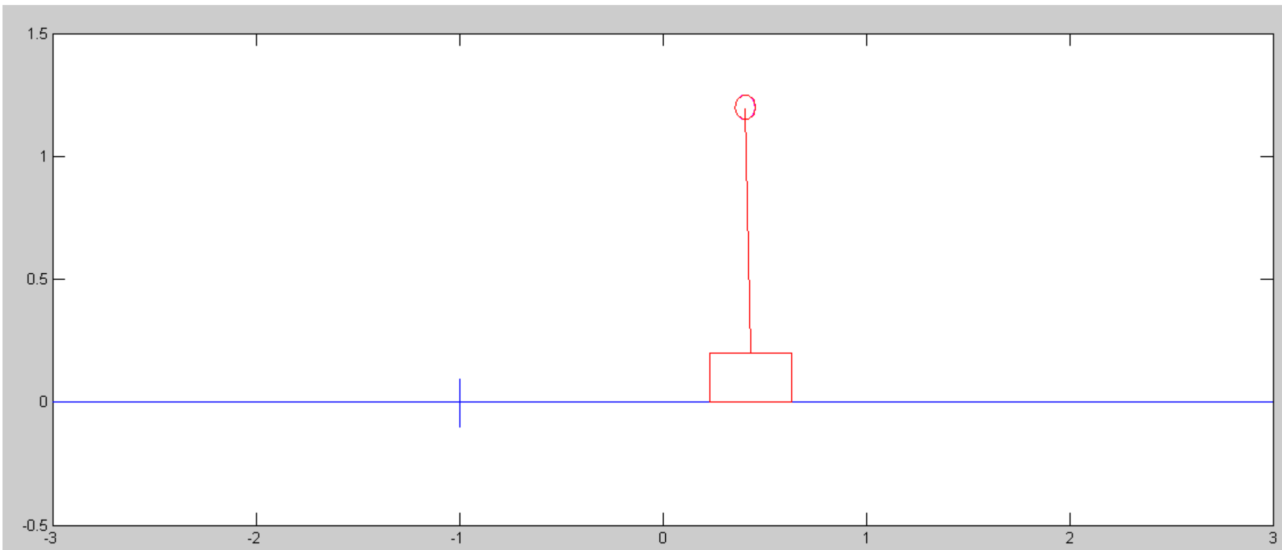


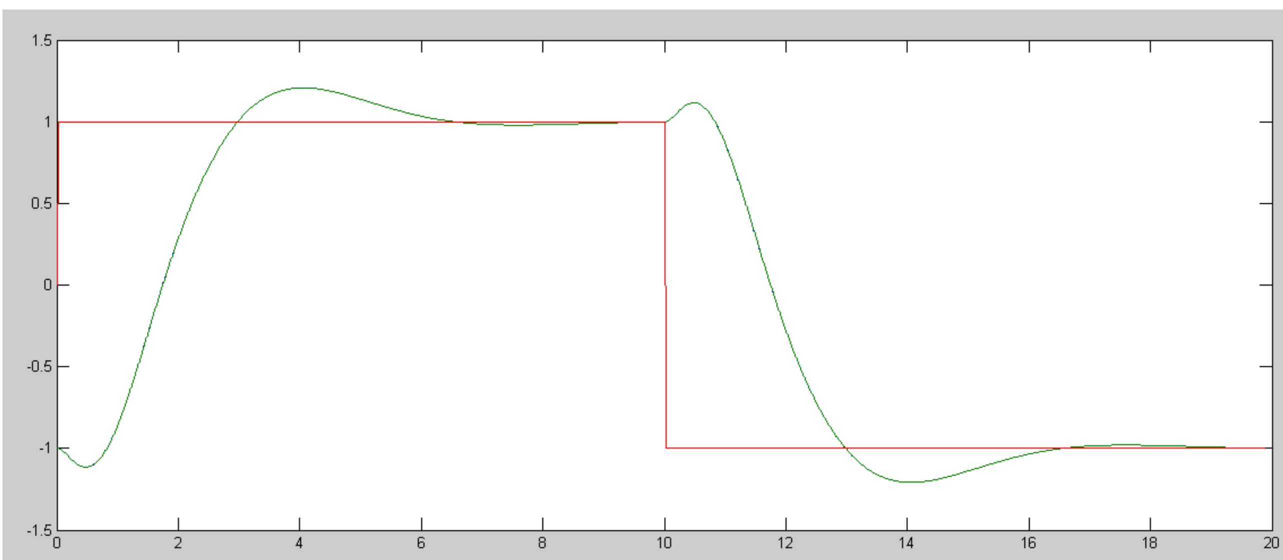
5) (20pt) Modify the cart and pendulum system to include

- your control law, and
- A full-order observer

Plot the step response of the nonlinear system + observer when

- Xe = $[0, 0, 0, 0]^T$
- Xe = $[0.1, 0.1, 0.1, 0.1]^T$





Code:

```
% Cart and Pendulum

X = [-1,0,0,0]';
Ref = 1;
dt = 0.01;
t = 0;
% Control Law
Kx = [-3.5060 -100.2440   -5.7192  -27.3334];
Kr =  -3.5060;
% Full-Order Observer
Ae = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0];
Be = [0;0;0.6667;-0.4444];
Ce = [1,0,0,0];
H = ppl(Ae', Ce', [-2+2i, -2-2i, -3, -4])';
Xe = X;

n = 0;
y = [];
while((t < 19.9) & (abs(X(1)) < 3))
 Ref = sign(sin(2*pi*t/20));
 U = Kr*Ref - Kx*Xe;
 dX = CartDynamics(X, U);
 dXe = Ae*Xe + Be*U + H*(Ce*X - Ce*Xe);

 X = X + dX * dt;
 Xe = Xe + dXe * dt;

 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    CartDisplay(X, Xe, Ref);
 end
 y = [y ; X(1), Xe(1), Ref];
end

hold off;
t = [1:length(y)]' * dt;
plot(t,y);
```