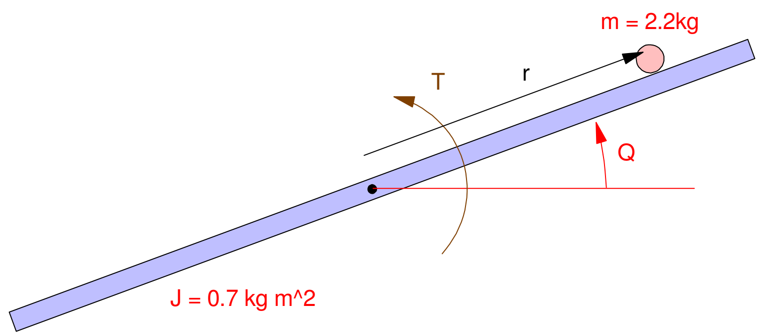
ECE 463/663 - Homework #7

Servo Compensators. Due Monday, March 17th



The dynamics of a Ball and Beam System (homework set #4) with a disturbance are

$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.434 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} d$$

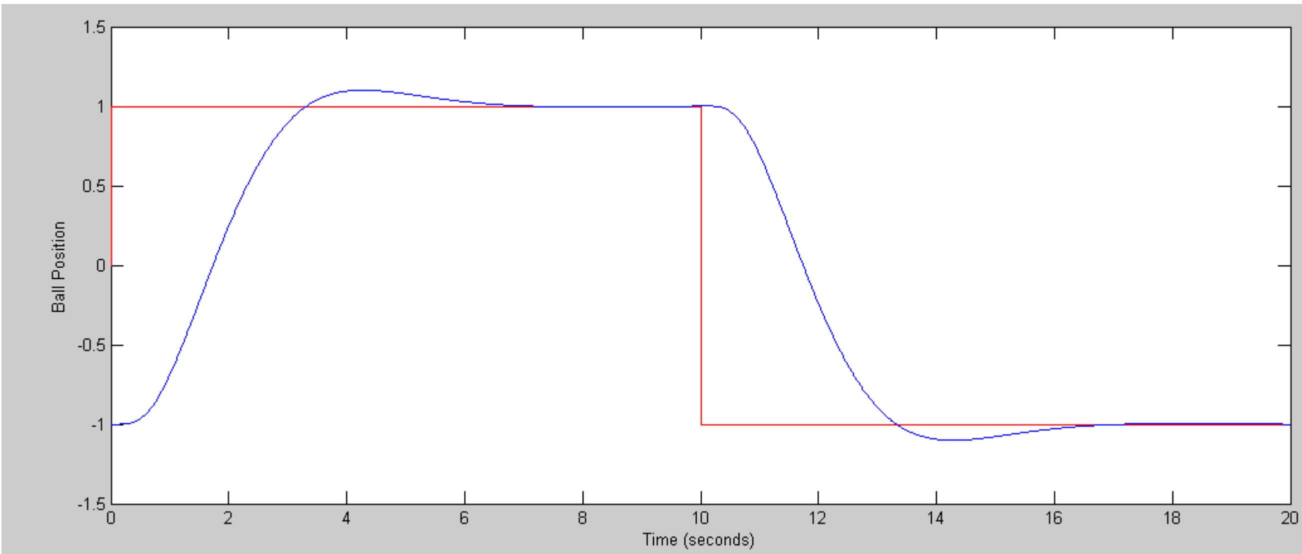
Full-State Feedback with Constant Disturbances

1) For the nonlinear simulation, use the feedback control law you computed in homework #6

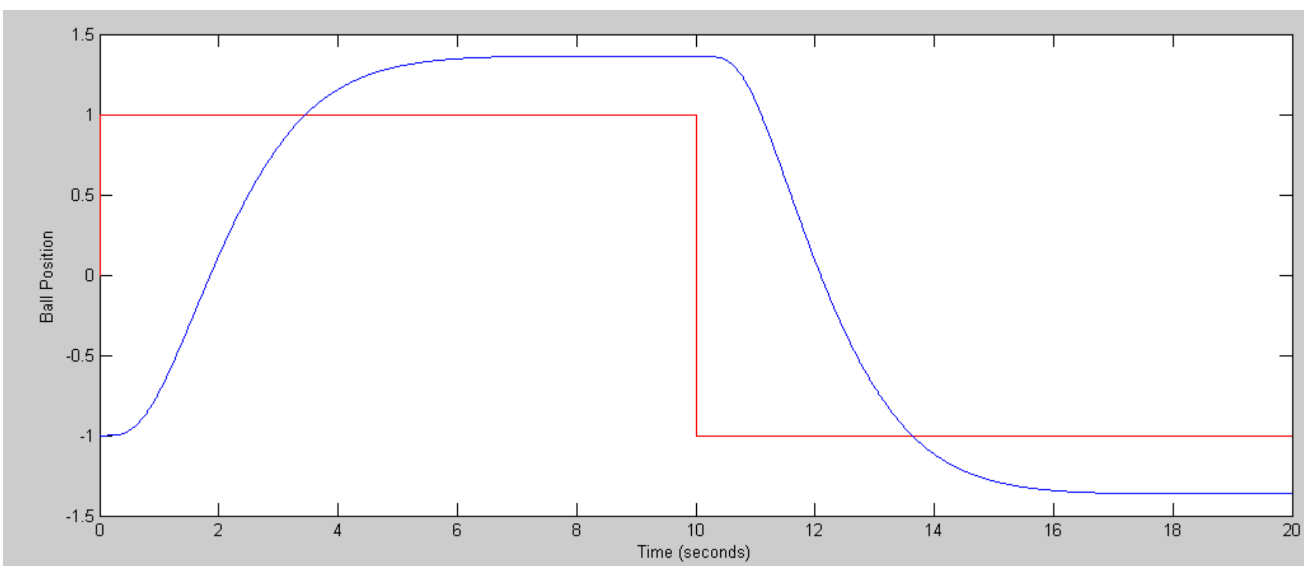
- With R = 1 and the mass of the ball = 2.2kg (same result you got for homework #6), and
- With R = 1 and the mass of the ball decreased to 2.5kg

(i.e. a constant disturbance on the system due to a different mass of the ball)

```
Kx =  -32.6762  103.6051  -18.2583   30.7246
Kr =  -11.1284
```

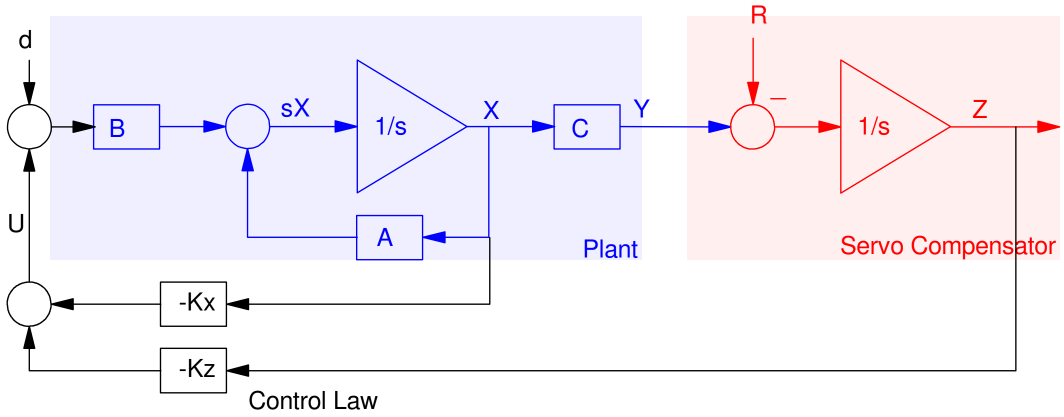


Step Response when m = 2.2kg
nonlinear simulation



Step Response when m = 2.5kg
nonlinear simulation

Servo Compensators with Constant Set-Points



2) Assume a constant disturbance and/or a constant set point. Design a feedback control law that results in

- The ability to track a constant set point (R = constant)
- The ability to reject a constant disturbance (d = constant),
- A 2% settling time of 6 seconds, and
- No overshoot for a step input.

```
>> A5 = [A, zeros(4,1) ; C, 0]

         0         0    1.0000         0         0
         0         0         0    1.0000         0
         0   -7.0000         0         0         0
   -7.4370         0         0         0         0
    1.0000         0         0         0         0

>> B5u = [B ; 0]

         0
         0
         0
    0.3450
         0

>> B5r = [0*B ; -1]

     0
     0
     0
     0
    -1

>> C5 = [1,0,0,0,0];
>> D5 = 0;
>> K5 = ppl(A5, B5u, [-4/6, -4/6, -3, -4, -5])

K5 =  -63.3325  183.8969  -53.0021   38.6473  -11.0421

>> Kx = K5(1:4)
```

`Kx =  -63.3325  183.8969  -53.0021   38.6473`

```
>> Kz = K5(5)
```

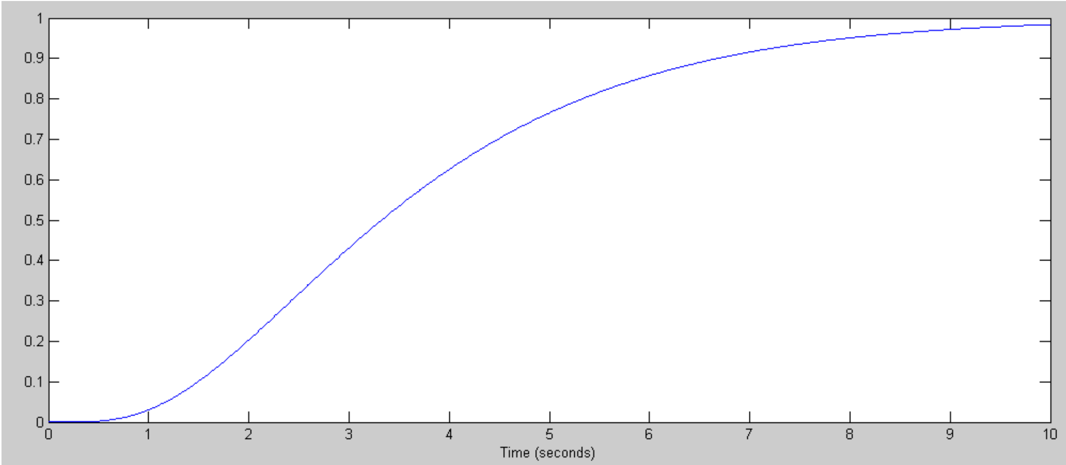
`Kz =  -11.0421`

3) For the linear system, plot the step response

- With respect to a step change in R, and
- With respect to a step change in d

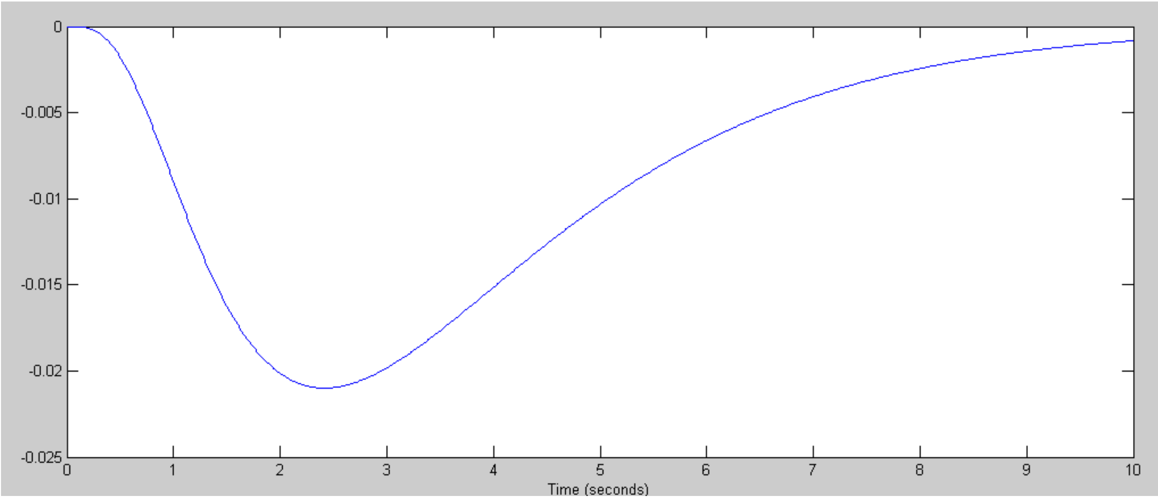
Form the closed-loop system

```
>> G5 = ss(A5 - B5u*K5, B5r, C5, D5);
>> t = [0:0.01:10]';
>> y = step(G5,t);
>> plot(t,y);
>> xlabel('Time (seconds)');
>>
```



Response to a step input in Ref

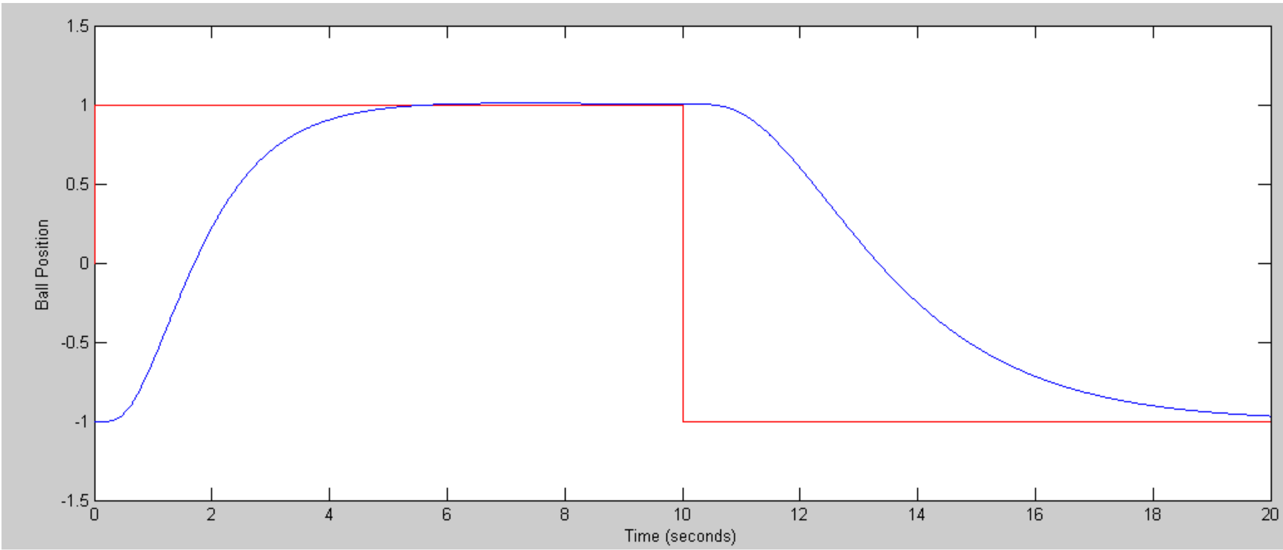
```
>> B5d = [B ; 0];
>> G5 = ss(A5 - B5u*K5, B5d, C5, D5);
>> y = step(G5,t);
>> plot(t,y);
>> xlabel('Time (seconds)');
```



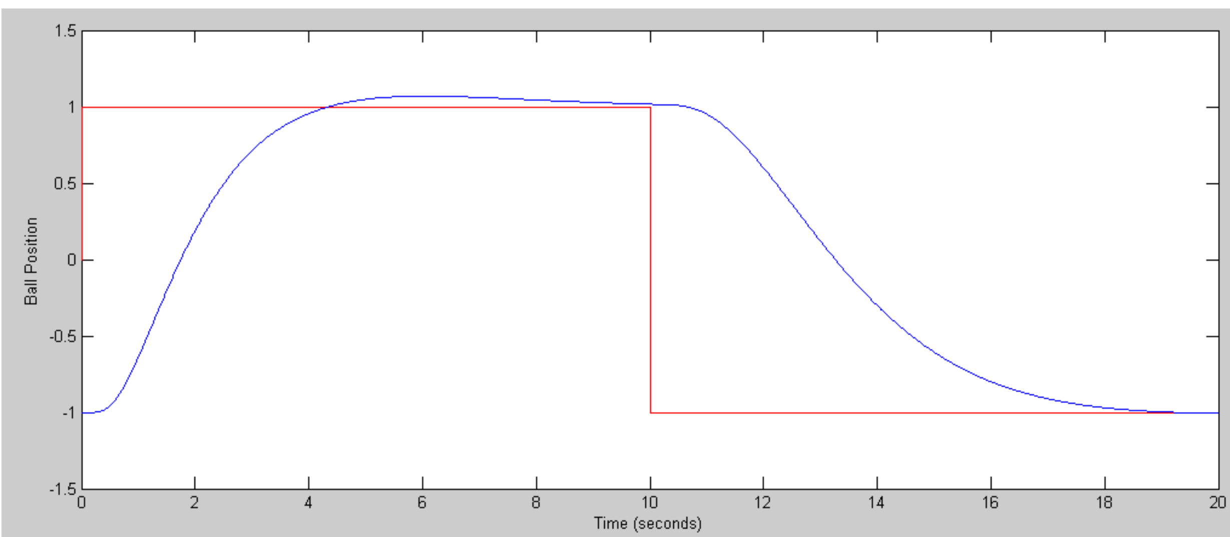
Response in a step change in the disturbance

4) Implement your control law on the nonlinear ball and beam system

- With R = 1 and the mass of the ball being 2.2kg, and
- With R = 1 and the mass of the ball being 2.5kg



Step response with m = 2.2kg

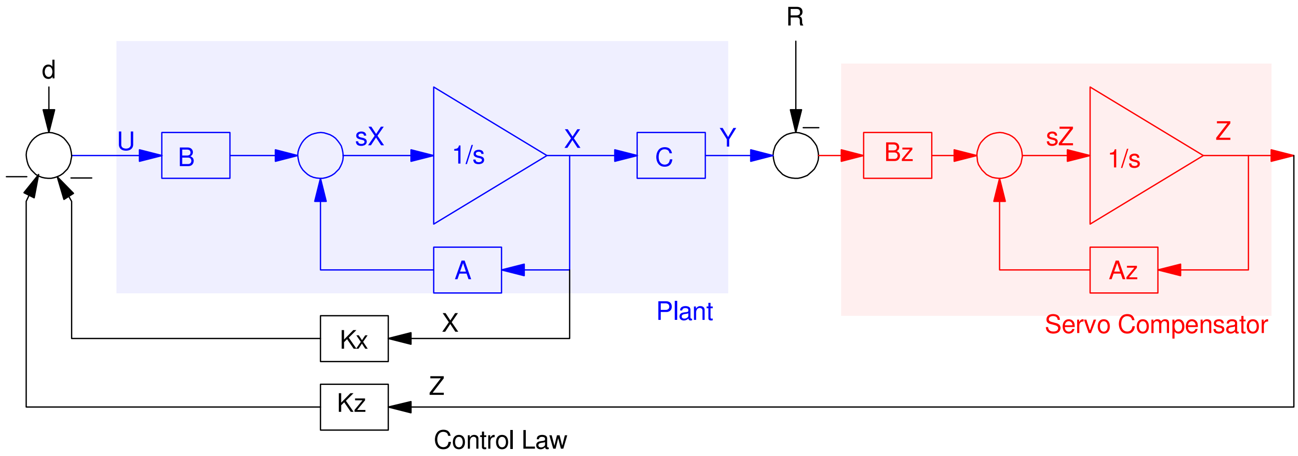


Step Response when m = 2.5kg

Code:

```
% Ball & Beam System

X = [-1, 0, 0, 0]';
dt = 0.01;
t = 0;

% Servo Compensator
Kx = [-63.3325  183.8969  -53.0021   38.6473];
Kz = -11.0421;
Z = 0;

n = 0;
y = [];

while(t < 20)
    Ref = sign(sin(t*pi/10));
    U = -Kx*X - Kz*Z;

    dX = BeamDynamics(X, U);
    dZ = X(1) - Ref;

    X = X + dX * dt;
    Z = Z + dZ * dt;

    t = t + dt;

    y = [y ; Ref, X(1)];
    n = mod(n+1,5);
    if(n == 0)
        BeamDisplay(X, Ref);
        end
    end

t = [1:length(y)]' * dt;

plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```

Servo Compensators with Sinulsoidal Set-Points



5) Assume a 0.6 rad/sec disturbance and/or set point (R). Design a feedback control law that results in

- The ability to track a constant set point (R = sin(0.6t))
- The ability to reject a constant disturbance (d = sin(0.6t)),
- A 2% settling time of 12 seconds, and

First, define a servo compensator with poles at +/- j0.6

```
>> Az = [0,0.6;-0.6,0]

         0    0.6000
   -0.6000         0

>> eig(Az)

        0 + 0.6000i
        0 - 0.6000i

>> Bz = [1;1];
```

Next, form the augmented system

```
>> A6 = [A, zeros(4,2) ; Bz*C, Az]

         0         0    1.0000         0         0         0
         0         0         0    1.0000         0         0
         0   -7.0000         0         0         0         0
   -7.4370         0         0         0         0         0
    1.0000         0         0         0         0    0.6000
    1.0000         0         0         0   -0.6000         0

>> B6u = [B ; 0;0]

         0
         0
         0
    0.3450
         0
         0
```

```
>> B6r = [0*B ; -Bz]

      0
      0
      0
      0
     -1
     -1

>> C6 = [C, 0, 0];
>> D6 = 0;
```

Find the feedback gains using pole-placement. Since the open-loop poles are at {2.68, -2.68, 2.68i, -2.68i}, place them here (somewhat arbitrary):

```
>> eig(A)

   -2.6861
    0.0000 + 2.6861i
    0.0000 - 2.6861i
    2.6861

>> K6 = ppl(A6, B6u, [-0.5, -0.5, -0.5+3i, -0.5-3i, -2.7, -2.8])

K6 =  -72.7410   83.1884  -32.9896   21.7391   -2.2420  -20.8874

>> Kx = K6(1:4)
```

Kx = -72.7410   83.1884 -32.9896   21.7391

```
>> Kz = K6(5:6)
```

Kz =   -2.2420 -20.8874

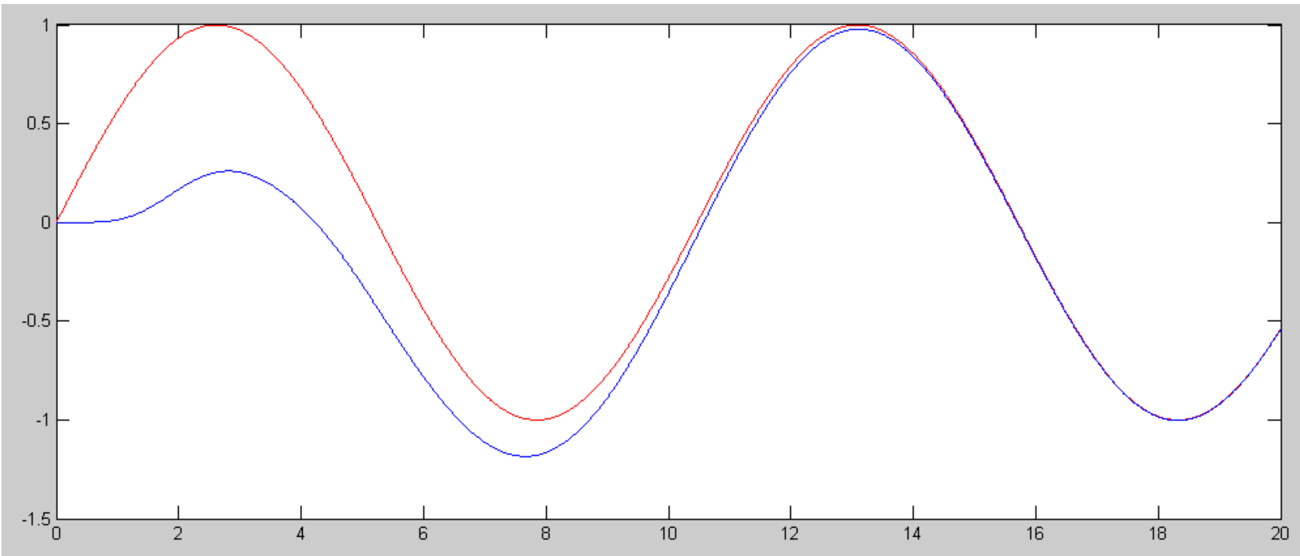
```
>>
```

6) For the linear system, plot the response

- With R(t) = sin(0.6t), and
- With d(t) = sin(0.6t)

Response to R(t)

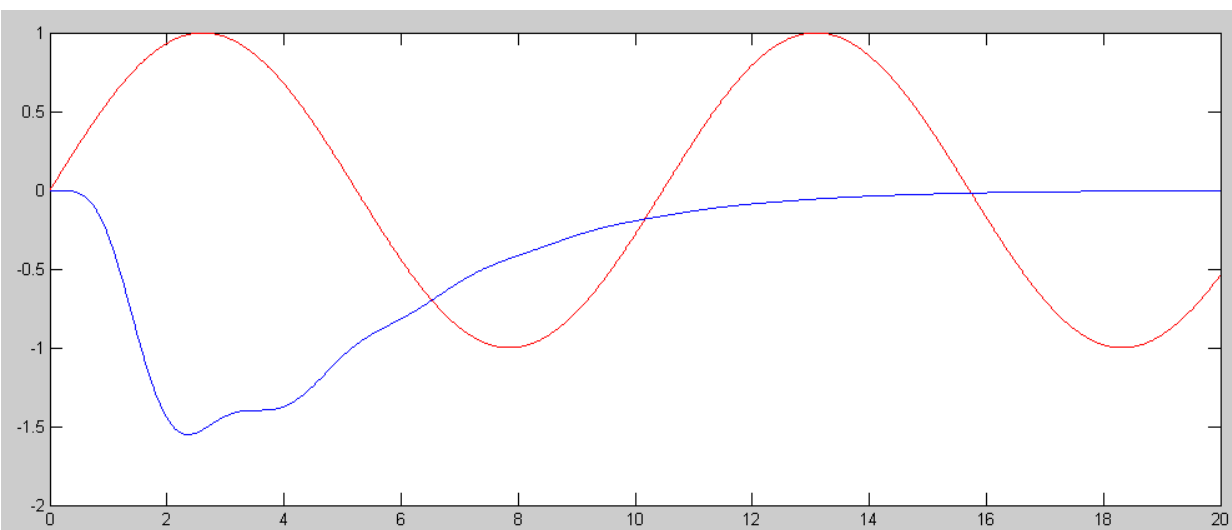
```
>> G6 = ss(A6 - B6u*K6, B6r, C6, D6);
>> t = [0:0.01:20]';
>> R = sin(0.6*t);
>> X0 = zeros(6,1);
>> y = step3(A6-B6u*K6, B6r, C6, D6, t, X0, R);
>> plot(t,R,'r',t,y,'b')
```



Response to R(t) = sin(0.6t). red = R(t), blue = y(t)

Response to d(t)

```
>> B6d = [B ; 0*Bz];
>> y = step3(A6-B6u*K6, B6d, C6, D6, t, X0, R);
>> plot(t,R,'r',t,y*100,'b')
```

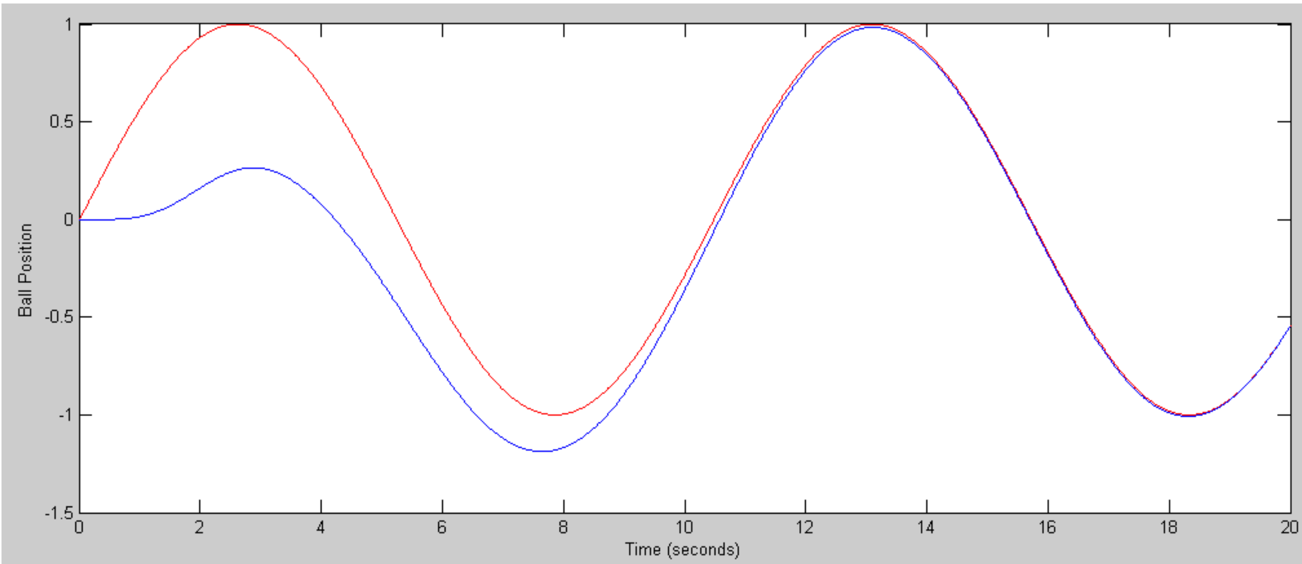


Response to d(t) = sin(0.6t). red = d(t), blue = 100*y(t)

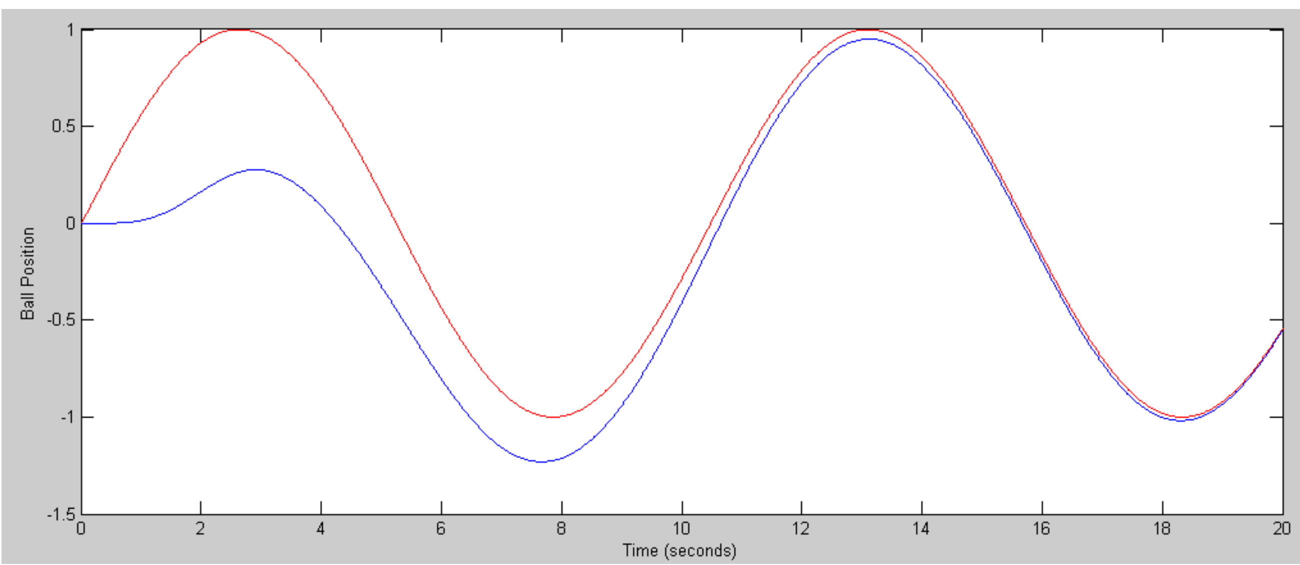
7) Implement your control law on the nonlinear ball and beam system

- With R = sin(0.6t) and the mass of the ball being 2.2kg, and
- With R = sin(0.6t) and the mass of the ball being 2.5kg

Nominal Case: m = 2.2kg



m = 2.5kg



Code:

```
% Ball & Beam System

X = [0, 0, 0, 0]';
dt = 0.01;
t = 0;

% Servo Compensator at 0.6 rad/sec
Az = [0,0.6 ; -0.6,0];
Bz = [1;1];
Kx = [-72.7410   83.1884  -32.9896   21.7391];
Kz = [-2.2420  -20.8874];
Z = zeros(2,1);

n = 0;
y = [];

while(t < 20)
    Ref = sin(0.6*t);
    U = -Kx*X - Kz*Z;

    dX = BeamDynamics(X, U);
    dZ = Az*Z + Bz*(X(1) - Ref);

    X = X + dX * dt;
    Z = Z + dZ * dt;

    t = t + dt;

    y = [y ; Ref, X(1)];
    n = mod(n+1,5);
    if(n == 0)
        BeamDisplay(X, Ref);
        end
    end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```