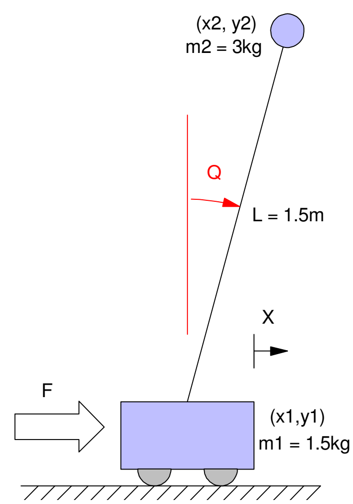
ECE 463/663 - Homework #6

Pole Placement. Due Monday, March 3rd

Problem 1) (30pt) Use the dynamics of a Cart and Pendulum System from homework set #4:

$$s\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -19.6 & 0 & 0 \\ 0 & 19.6 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0.6667 \\ -0.4444 \end{bmatrix}F$$



(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 5 seconds, and
- 5% overshoot for a step input

(10pt) Check the step response of the linear system in Matlab

(10pt) Check the step response of the nonlinear system

Start with where the closed-loop poles belong

5 second settling time means real(s) = -0.8

5% overshoot means the damping ratio is 0.6901

s = -0.8 + j0.8389

Place the closed-loop poles at

s = { -0.8 + j0.8389, -0.8 - j0.8389, -4, -5}

In Matlab

```
>> A = [0,0,1,0;0,0,0,1;0,-19.6,0,0;0,19.6,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0  -19.6000         0         0
         0   19.6000         0         0

>> B = [0;0;0.6667;-0.4444]

         0
         0
    0.6667
   -0.4444

>> P = [-0.8 + 0.8389i, -0.8 - 0.8389i, -4, -5];
>> Kx = ppl(A, B, P)

Kx =   -6.1681 -133.7895  -10.1200  -39.0347
```

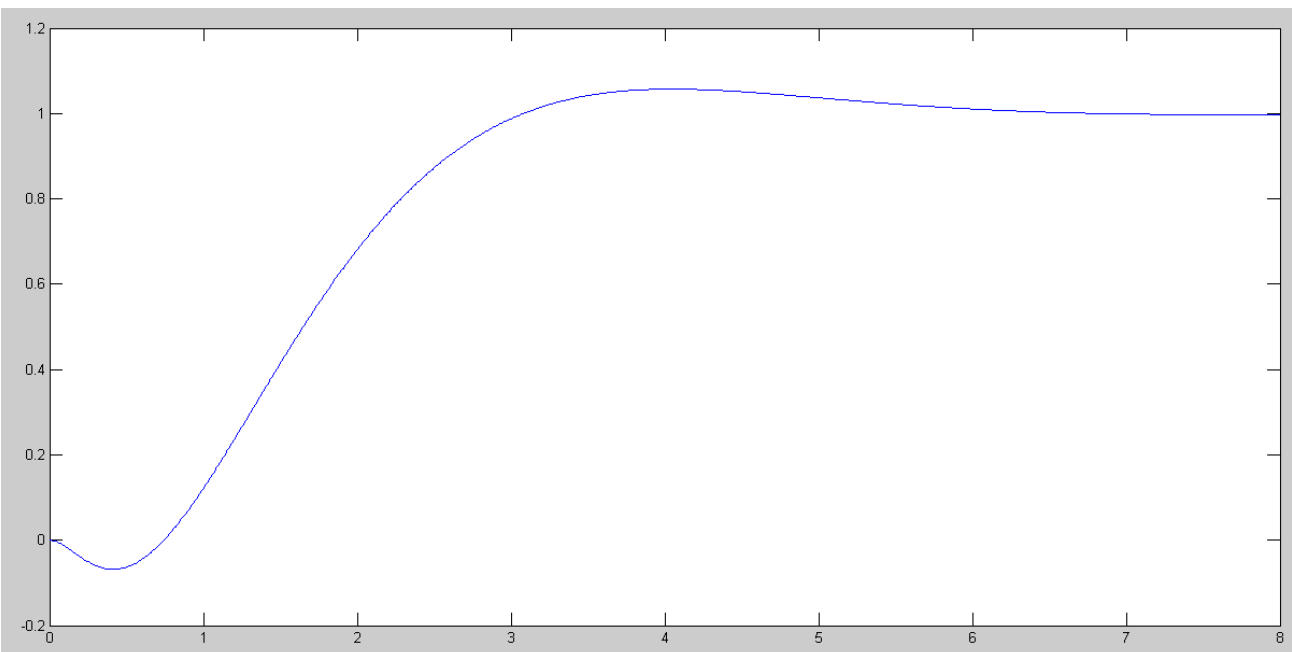
```
>> eig(A - B*Kx)

  -5.0000
  -4.0000
  -0.8000 + 0.8389i
  -0.8000 - 0.8389i

>> C = [1,0,0,0];
>> DC = -C*inv(A-B*Kx)*B

DC =   -0.1621

>> Kr = 1/DC
```

Kr =   -6.1681

```
>>
```

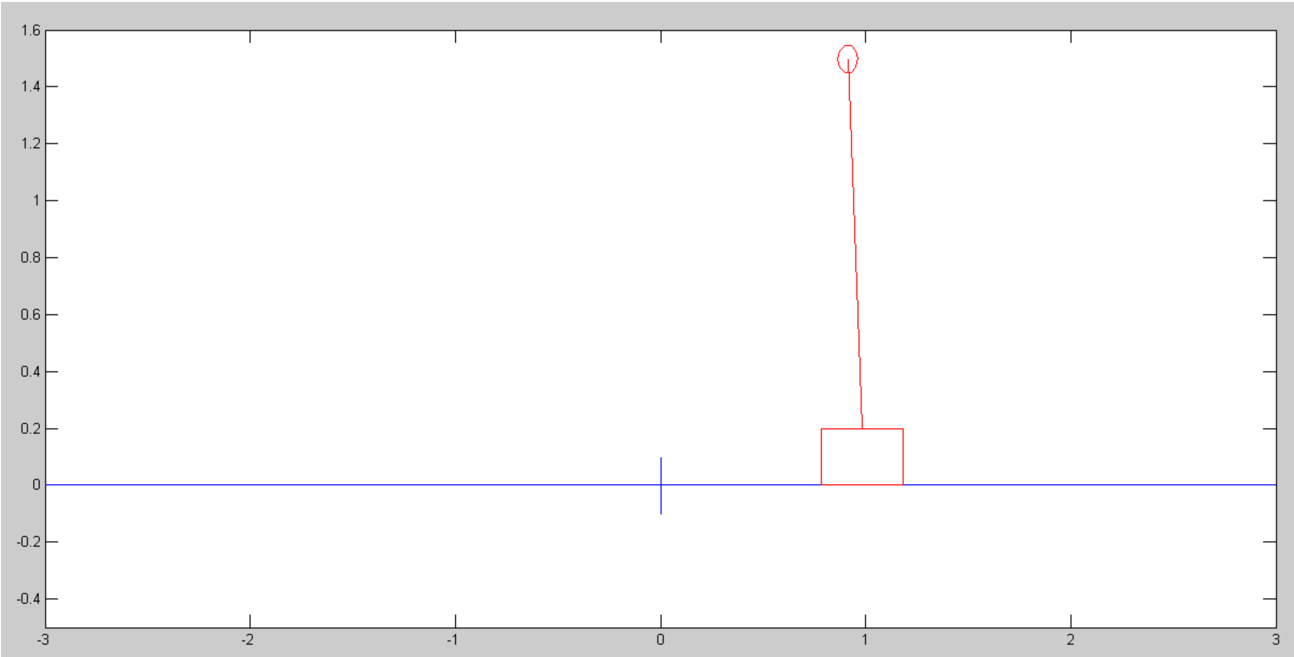
Step Response of the linearized system:

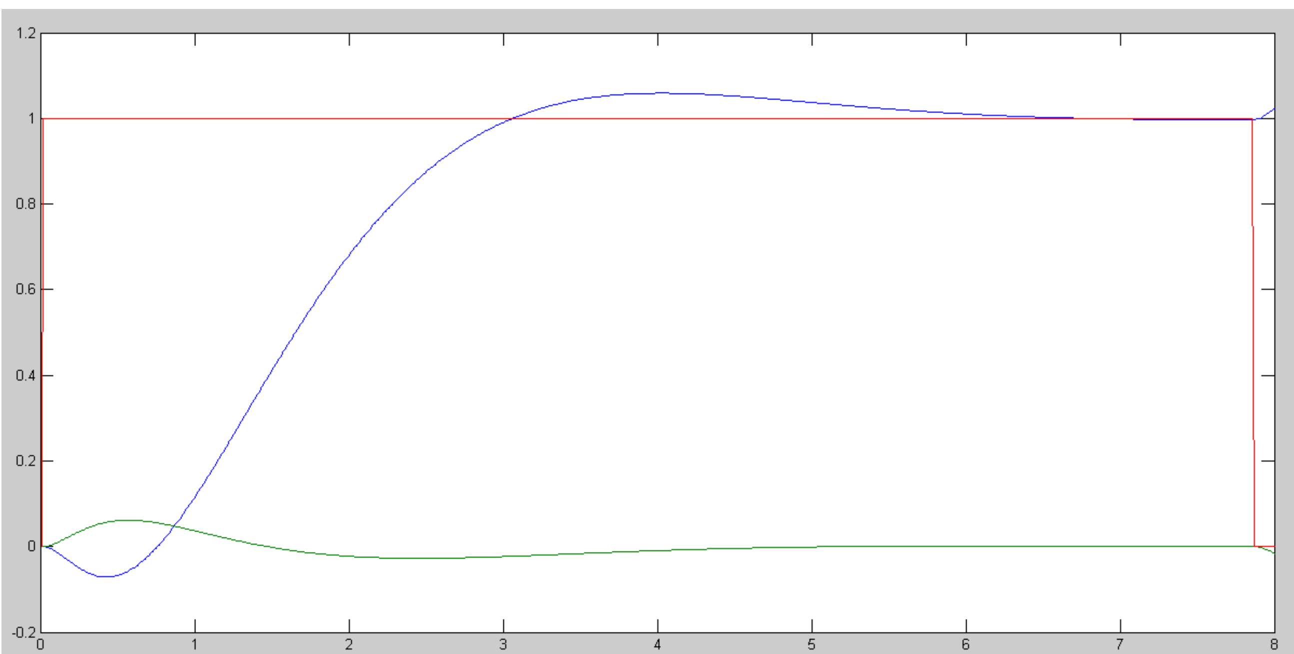
```
>> Gcl = ss(A-B*Kx, B*Kr, C, 0);
>> t = [0:0.01:8]';
>> y = step(Gcl, t);
>> plot(t,y)
```



Step Response of the nonlinear model

- Looks about the same as the linear model
- The nonlinearities don't matter that much for this choice of feedback gains
- You could push this system and make it faster





Code:

```
% Cart and Pendulum
% Homework #6: Pole Placememt

X = [0,0,0,0]';
Ref = 1;
dt = 0.01;
t = 0;
n = 0;
y = [];

Kx = [-6.1681 -133.7895  -10.1200  -39.0347];
Kr = -6.1681;

while(t < 15)
 Ref = 1*(sin(0.4*t) > 0);
 U = Kr*Ref - Kx*X;
 dX = CartDynamics(X, U);
 X = X + dX * dt;
 t = t + dt;
 n = mod(n+1, 5);
 if(n == 0)
    CartDisplay(X, Ref);
 end
 y = [y ; X(1), X(2), Ref];
end

t = [1:length(y)]' * dt;
plot(t,y);
```

Problem 2) (30pt) Use the dynamics for the Ball and Beam system from homework set #4.

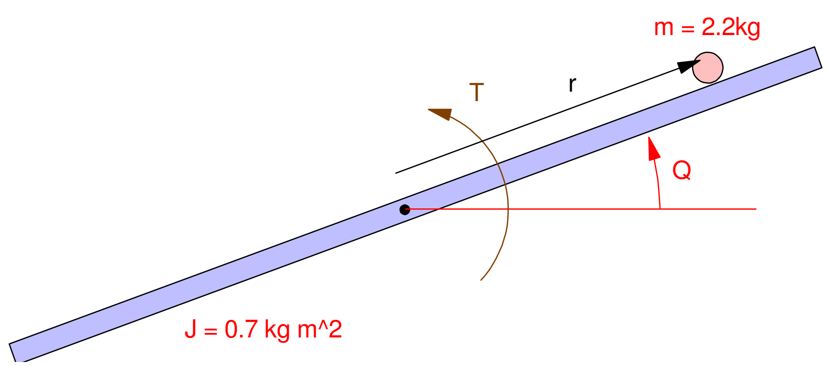
$$s\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -7 & 0 & 0 \\ -7.434 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.345 \end{bmatrix} T$$

(10pt) Design a feedback control law so that the closed-loop system has

- A 2% settling time of 5 seconds, and
- 5% overshoot for a step input

(10pt) Check the step response of the linear system in Matlab

(10pt) Check the step response of the nonlinear system




```
>> A = [0,0,1,0;0,0,0,1;0,-7,0,0;-7.434,0,0,0]

         0         0    1.0000         0
         0         0         0    1.0000
         0   -7.0000         0         0
   -7.4340         0         0         0

>> B = [0;0;0;0.345]

         0
         0
         0
    0.3450

>> C = [1,0,0,0];
>> D = 0;
>> P = [-0.8 + 0.8389i, -0.8 - 0.8389i, -4, -5];
>> Kx = ppl(A, B, P)
```

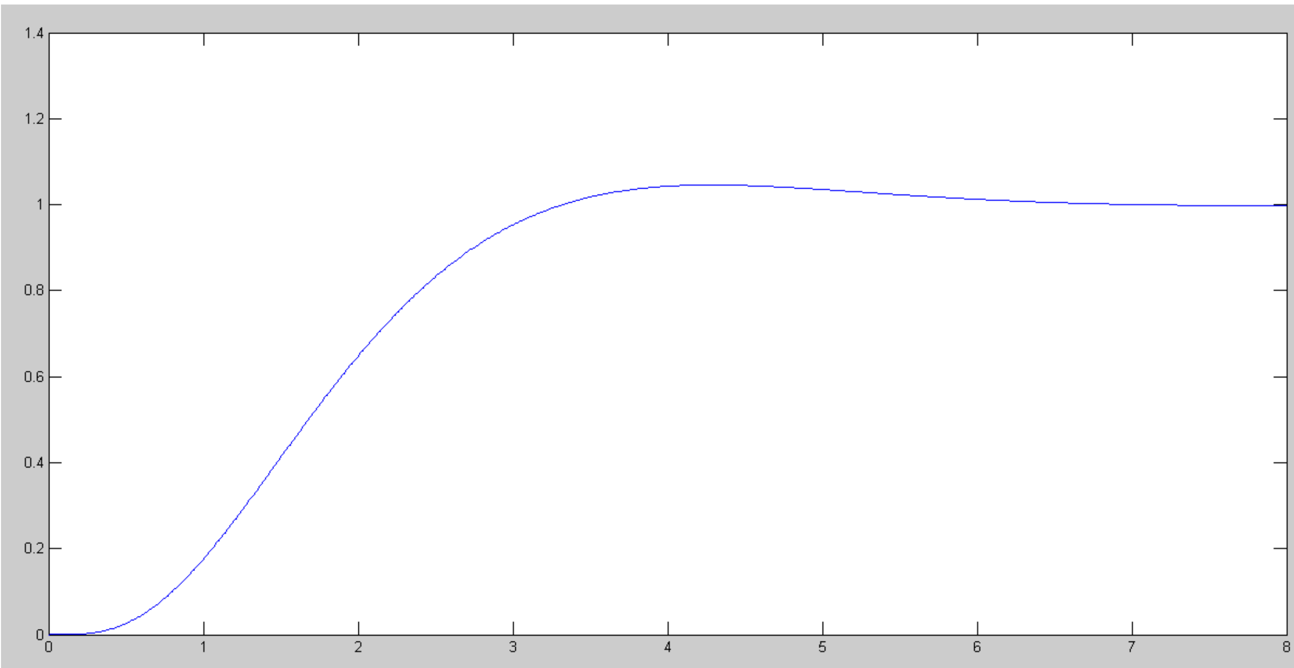
Kx = -32.6762 103.6051 -18.2583 30.7246

```
>> eig(A - B*Kx)

  -5.0000
  -4.0000
  -0.8000 + 0.8389i
  -0.8000 - 0.8389i

>> DC = -C*inv(A-B*Kx)*B
```

```
DC =   -0.0899

>> Kr = 1/DC
```

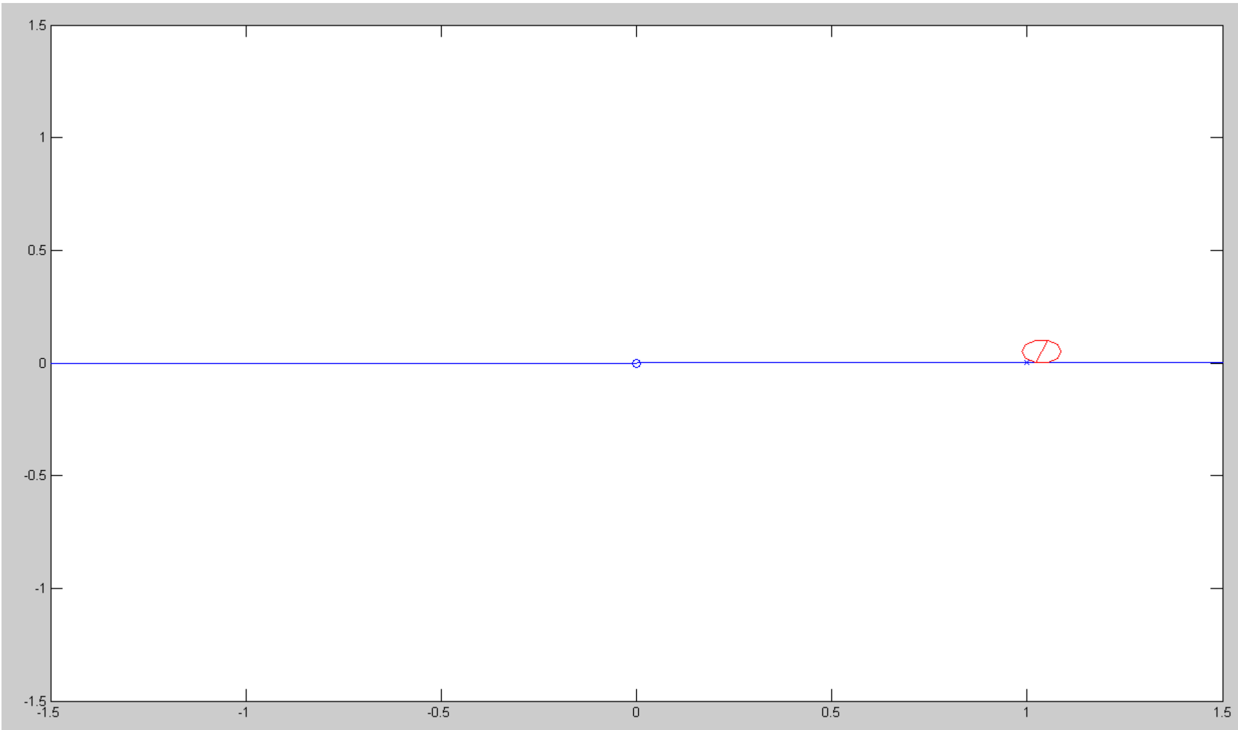
Kr = -11.1284

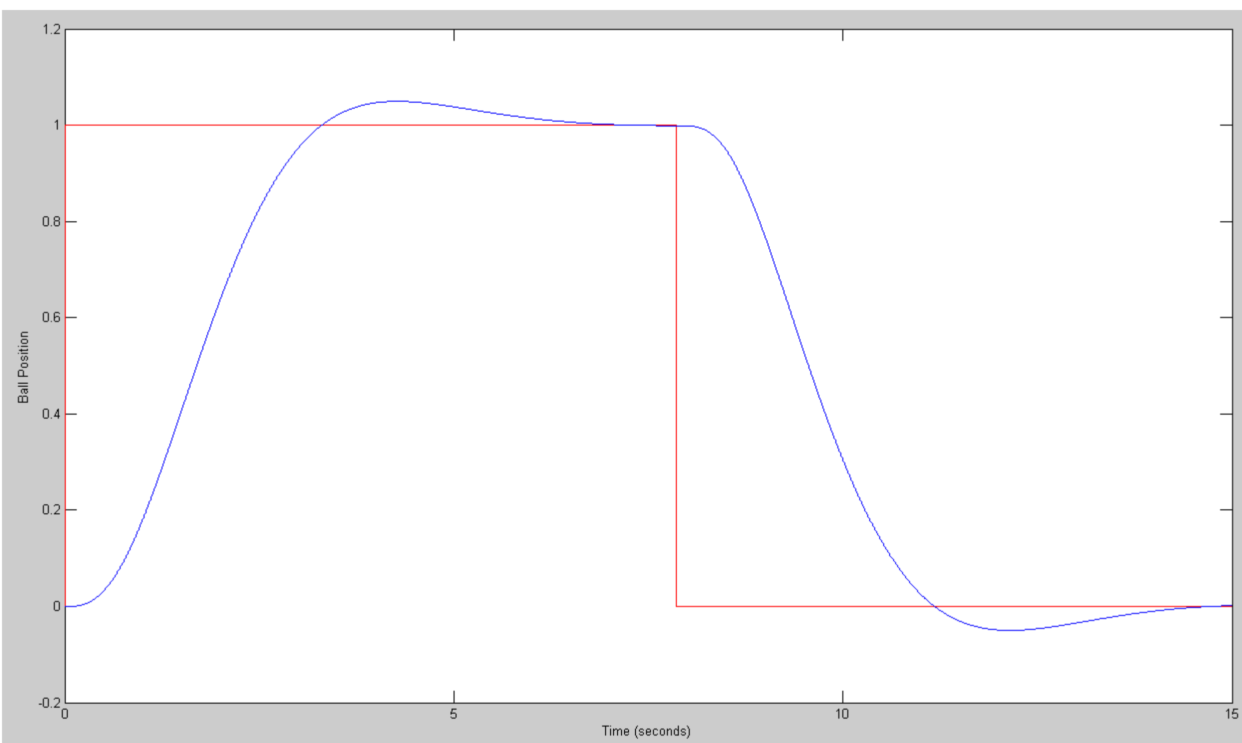
```
>> Gcl = ss(A-B*Kx, B*Kr, C, 0);
>> t = [0:0.01:8]';
>> y = step(Gcl, t);
>> plot(t,y)
>>
```



Nonlinear Simulation

- About the same step response
- The linearized model is a good approximation for the plant with these feedback gains
- You could push this system and make it faster

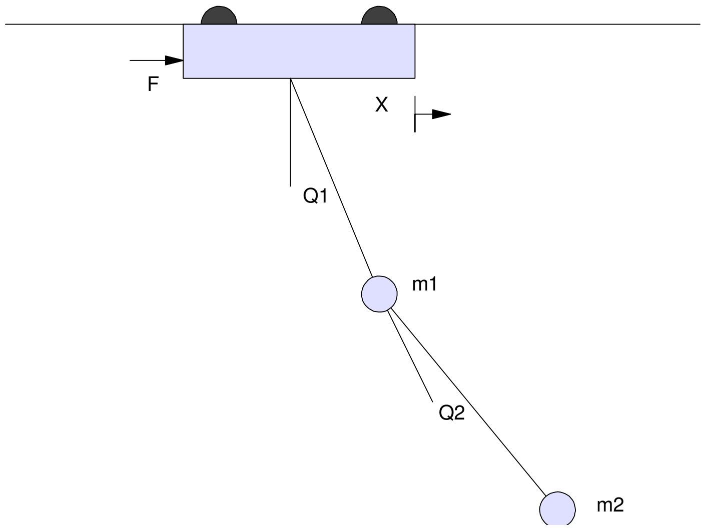




Code

```
% Ball & Beam System
% Spring 2025
% Homework #6

X = [0, 0, 0, 0]';
dt = 0.002;
t = 0;
n = 0;
y = [];

Kx = [  -32.6762  103.6051  -18.2583   30.7246];
Kr = [-11.1284];

while(t < 15)
 Ref = 1*(sin(0.4*t) > 0);
 U = Kr*Ref - Kx*X;
 dX = BeamDynamics(X, U);

 X = X + dX * dt;
 t = t + dt;

 y = [y ; Ref, X(1)];
 n = mod(n+1,5);
 if(n == 0)
    BeamDisplay(X, Ref);
 end
 end

t = [1:length(y)]' * dt;


plot(t,y(:,1),'r',t,y(:,2),'b');
xlabel('Time (seconds)');
ylabel('Ball Position');
```

Problem #3 (30pt): The dynamics of a double gantry (Gantry2) are



$$s\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2g & 0 & 0 & 0 & 0 \\ 0 & -3g & g & 0 & 0 & 0 \\ 0 & 3g & -3g & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \theta_1 \\ \theta_2 \\ \dot{x} \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} F$$

(10pt) Design a feedback control law of the form

U = Kr * R - Kx * X

so that the closed-loop system has

- A 2% settling time of 5 seconds, and
- 5% overshoot for a step input

(10pt) Determine the step response of the linear system in Matlab

(10pt) Determine the step response of the nonlinear system

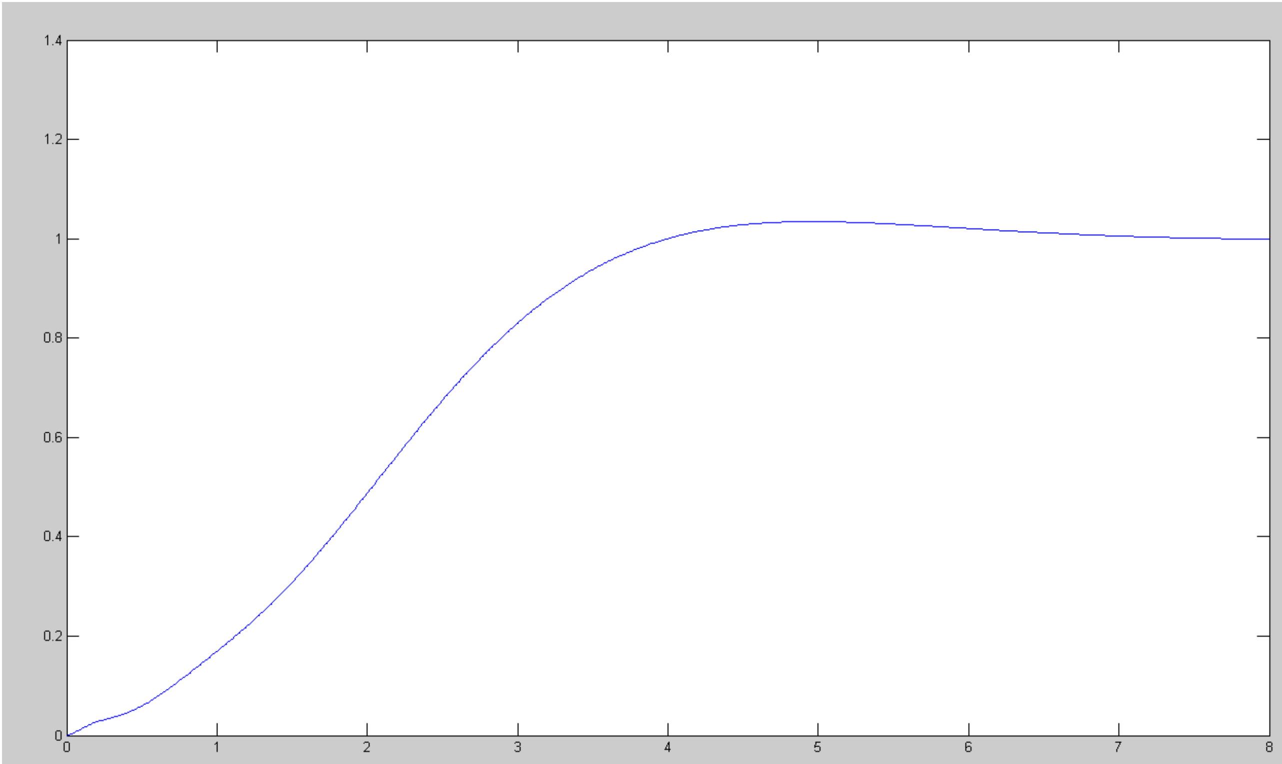
```
>> Z = zeros(3,3);
>> I = eye(3,3);
>> g = 9.8;
>> A21 = [0,2,0;0,-3,1;0,3,-3]*g;
>> A = [Z,I ; A21,Z]

         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
         0   19.6000         0         0         0         0
         0  -29.4000    9.8000         0         0         0
         0   29.4000  -29.4000         0         0         0

>> B = [0;0;0;1;-1;1]

     0
     0
     0
     1
    -1
     1

>> C = [1,0,0,0,0,0];
>> D = 0;
>> P = [-0.8 + 0.8389i, -0.8 - 0.8389i, -4, -5, -6,-7];
>> Kx = ppl(A, B, P)
```

Kx =   5.8765 -66.0579   84.8094   11.4604 -25.7508 -13.6112

```
>> eig(A - B*Kx)

  -7.0000
  -6.0000
  -5.0000
  -4.0000
  -0.8000 + 0.8389i
  -0.8000 - 0.8389i

>> DC = -C*inv(A-B*Kx)*B

DC =    0.1702

>> Kr = 1/DC
```

Kr =   5.8765

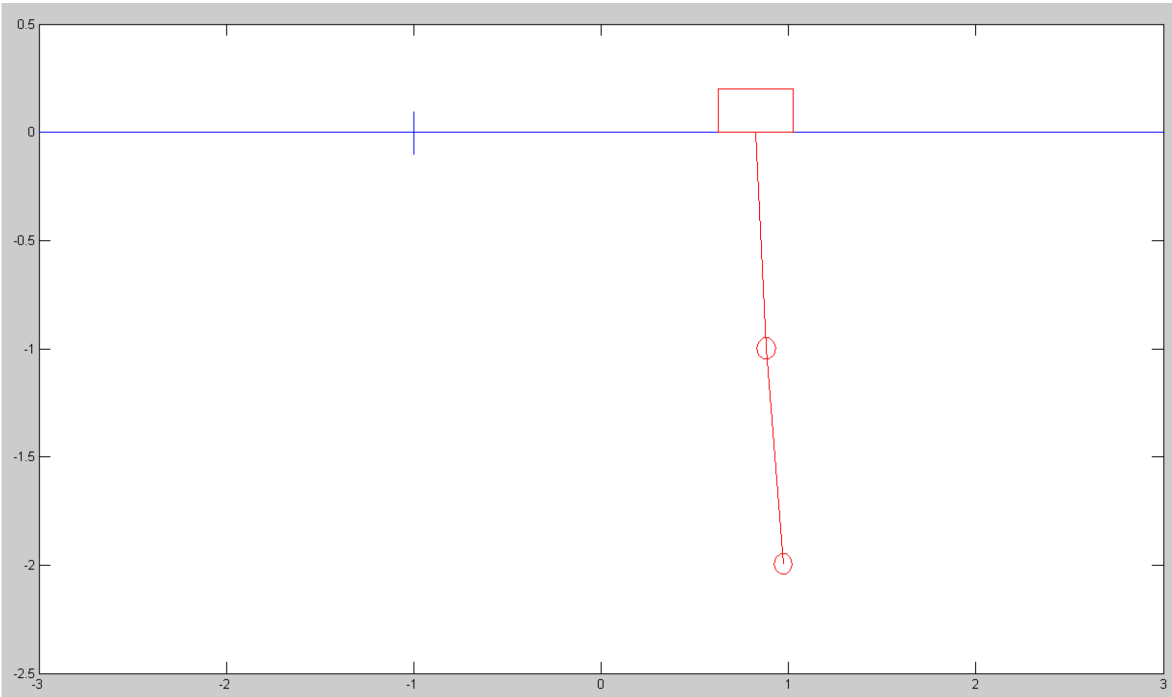
```
>> Gcl = ss(A-B*Kx, B*Kr, C, 0);
>> t = [0:0.01:8]';
>> y = step(Gcl, t);
>> plot(t,y)
>>
```

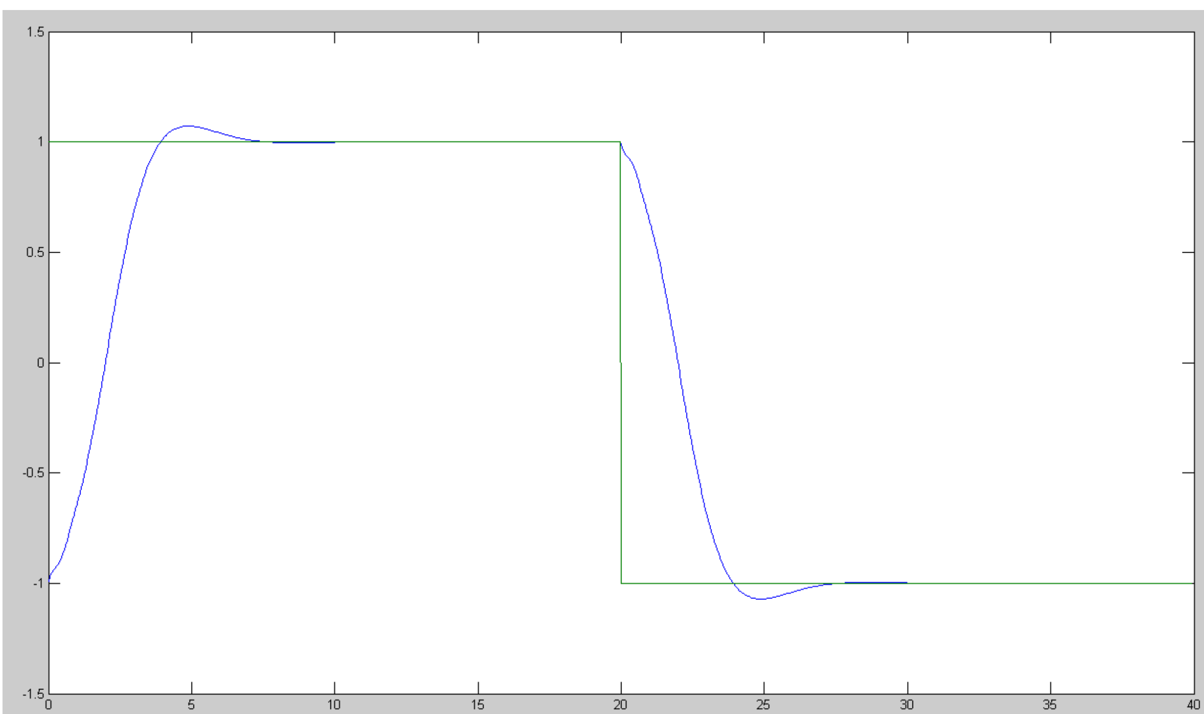


1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8

Nonlinear Simulatin

- About the same step response
- The linearized model is a good approximation for the plant with these feedback gains
- You could push this system and make it faster





Code

```
X = [-1, 0, 0, 0, 0, 0]';
Ref = 1;
dt = 0.01;
U = 0;
t = 0;
Kx = [5.8765  -66.0579   84.8094   11.4604  -25.7508  -13.6112];
Kr = 5.8765;
n = 0;
y = [];
while(t < 40)
 Ref = sign(sin(pi*t/20));
 U = Kr*Ref - Kx*X;
 dX = Gantry2Dynamics(X, U);
 X = X + dX * dt;
 t = t + dt;
 n = mod(n+1,5);
 if(n == 0)
    Gantry2Display(X, Ref);
    plot([Ref, Ref],[-0.1,0.1],'b');
    y = [y ; X(1), Ref];
 end
end

pause(2);
t = [0:length(y)-1]' * dt*5;
plot(t,y);
```