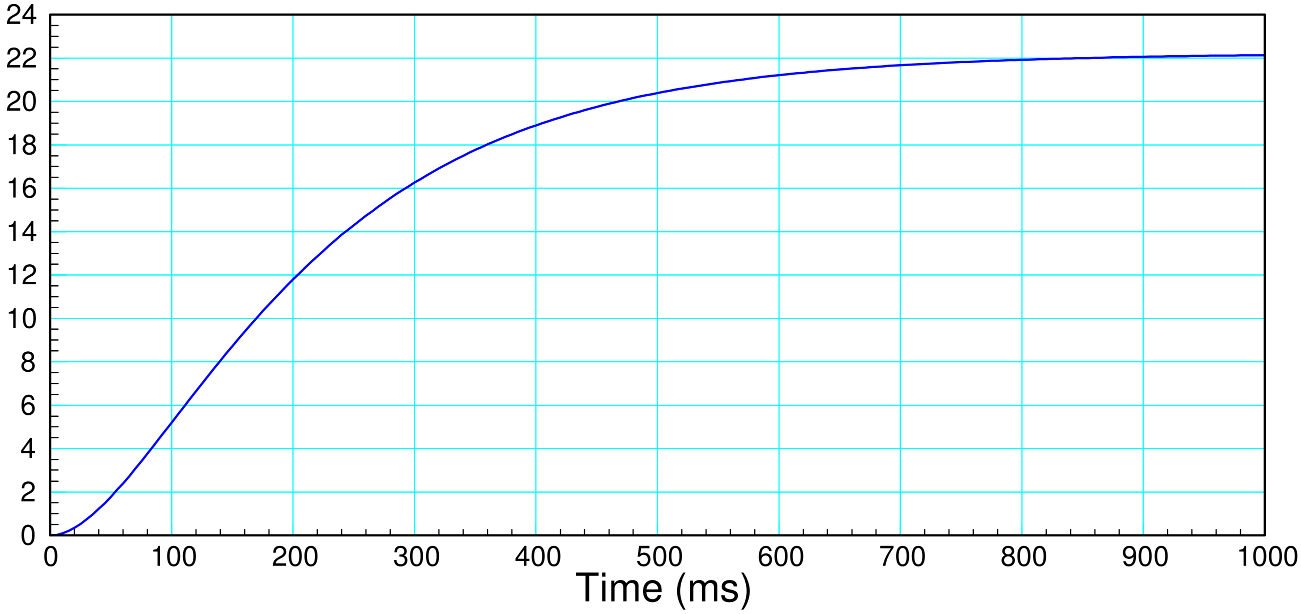
ECE 463/663 - Homework #1

LaPlace Transforms and Dominant Poles. Due Wednesday, Jan 22nd

1) Name That System! Give the transfer function for a system with the following step response.



This is 1st-order system (no oscillations), meaning G(s) is of the form

$$G(s) \approx \left(\frac{a}{s+b}\right)$$

b: The 2% settling time is about 700ms

$$b = \frac{4}{700ms} = 5.71$$

a: The DC gain is about 22

$$\left(\frac{a}{s+b}\right)_{s=0} = 22$$

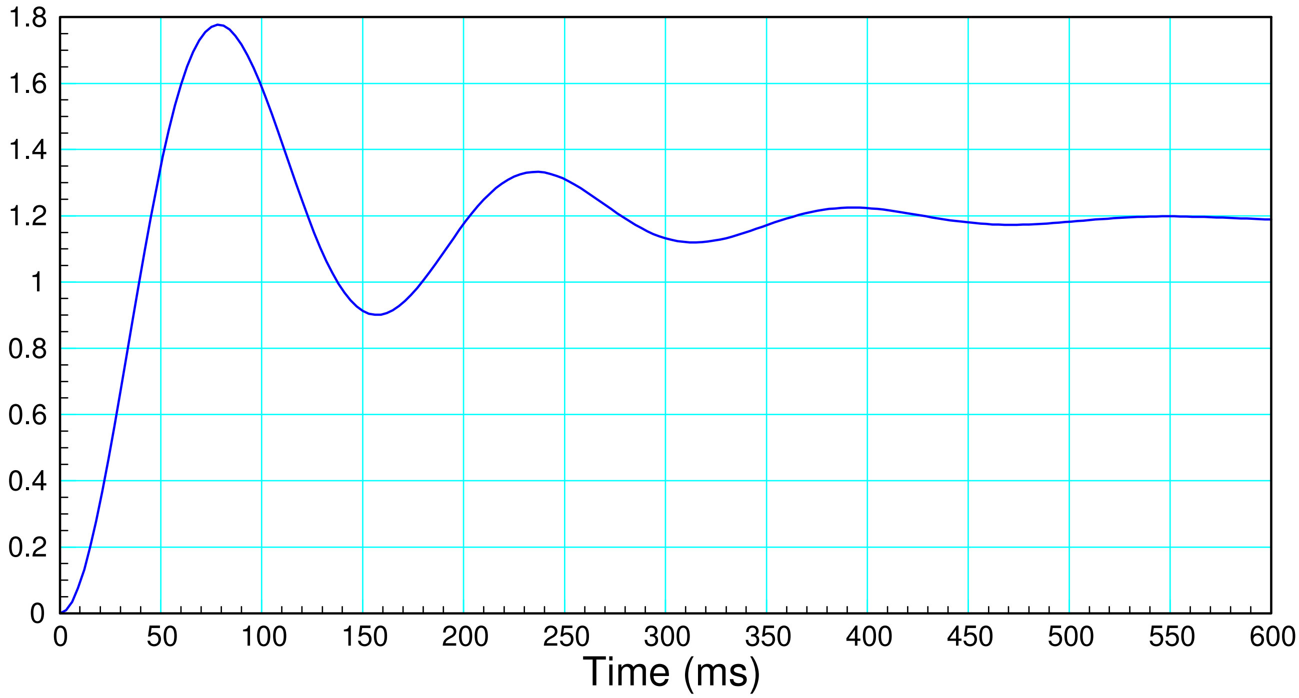
$$a = 22b = 125.71$$

so

$$\boxed{G(s) \approx \left(\frac{125.71}{s+5.71}\right)}$$

answers will vary

2) Name That System! Give the transfer function for a system with the following step response.



This is a 2nd-order system (oscillations mean complex poles). G(s) is of the form

$$G(s) = \left(\frac{a}{s^2+bs+c}\right) = \left(\frac{k}{(s+\sigma+j\omega)(s+\sigma-j\omega)}\right)$$

real: The 2% setttling time is about 450ms

$$\sigma \approx \frac{4}{450ms} = 8.89$$

imag: the frequency of oscillation has two cycles in 315ms

$$\omega \approx \left(\frac{2 \text{ cycles}}{0.315\text{s}}\right) 2\pi = 39.89$$

so G(s) is of the form

$$G(s) \approx \left(\frac{k}{(s+8.89+j39.89)(s+8.89-j39.89)}\right)$$

k: The DC gain is about 1.2

$$\left(\frac{k}{(s+8.89+j39.89)(s+8.89-j39.89)}\right)_{s=0} = 1.2$$

$$k = 2004.29$$

so

$$G(s) \approx \left(\frac{2004.89}{(s+8.89+j39.89)(s+8.89-j39.89)}\right)$$

Problem 3 - 6) Assume

$$Y = \left(\frac{200}{(s+1.5)(s+5)(s+7)}\right)X$$

3) What is the differential equation relating X and Y?

Multiply out the denominator

```
>> poly([-1.5,-5,-7])

ans =    1.0000   13.5000   53.0000   52.5000
```

$$Y = \left(\frac{200}{s^3+13.5s^2+53s+52.5}\right)X$$

Cross multiply

$$(s^3 + 13.5s^2 + 53s + 52.5)Y = (200)X$$

note that 'sY' means 'the derivative of y(t)'

$$y''' + 13.5y'' + 53y' + 52.5y = 200x$$

4) Determine y(t) assuming x(t) is a sinusoidal input:

$$x(t) = 3\cos(2t) + 7\sin(2t)$$

Use phasors

$$s = j2$$

$$X = 3 - j7$$

$$Y = \left(\frac{200}{s^3+13.5s^2+53s+52.5}\right)_{s=j2} \cdot (3 - j7)$$

In Matlab

```
>> s = 2i
s =        0 + 2.0000i

>> X = 3 - 7i
X =   3.0000 - 7.0000i

>> Y = ( 200 / (s^3 + 13.5*s^2 + 53*s + 52.5) ) * X
Y = -14.3761 - 5.9024i
```

Another way to do it

```
>> G = zpk([],[-1.5,-5,-7],200)

         200
-------------------
(s+1.5) (s+5) (s+7)

>> s = 2i
s =        0 + 2.0000i
```

```
>> X = 3 - 7i
X =   3.0000 - 7.0000i

>> Y = evalfr(G,s) * X
Y = -14.3761 - 5.9024i

>>
```

The net result is

$$y(t) = -14.3761\cos(2t) + 5.9024\sin(2t)$$

5) Determine y(t) assuming x(t) is a step input:

$$Y = \left(\frac{200}{(s+1.5)(s+5)(s+7)}\right)X$$

$$x(t) = u(t)$$

Substitute the LaPlace transorm for x(t)

$$Y = \left(\frac{200}{(s+1.5)(s+5)(s+7)}\right)\left(\frac{1}{s}\right)$$

Take the partial fraction expanse

$$Y = \left(\frac{a}{s+1.5}\right) + \left(\frac{b}{s+5}\right) + \left(\frac{c}{s+7}\right) + \left(\frac{d}{s}\right)$$

Using Matlab

```
>> Y = zpk([],[-1.5,-5,-7,0],200)

         200
---------------------
s (s+1.5) (s+5) (s+7)

>> s = -1.5 + 1e-9;
>> a = evalfr(Y,s) * (s+1.5)
a =   -6.9264

>> s = -5 + 1e-9;
>> b = evalfr(Y,s) * (s+5)
b =    5.7143

>> s = -7 + 1e-9;
>> c = evalfr(Y,s) * (s+7)
c =   -2.5974

>> s = 0 + 1e-9;
>> d = evalfr(Y,s) * (s)
d =    3.8095
```

so

$$Y = \left(\frac{-6.9264}{s+1.5}\right) + \left(\frac{5.7143}{s+5}\right) + \left(\frac{-2.5974}{s+7}\right) + \left(\frac{3.8095}{s}\right)$$

Take the inverse LaPlace transform

$$y(t) = (-6.9264e^{-1.5t} + 5.7143e^{-5t} - 2.5974e^{-7t} + 3.8095)u(t)$$

```
>>
```

6a) Determine a 1st-order approximation for this system

$$Y = \left(\frac{200}{(s+1.5)(s+5)(s+7)}\right)X \approx \left(\frac{a}{s+b}\right)X$$

Keep the dominant pole

$$Y \approx \left(\frac{a}{s+1.5}\right)X$$

Match the DC gain

$$\left(\frac{200}{(s+1.5)(s+5)(s+7)}\right)_{s=0} = 3.8095$$

$$\left(\frac{a}{s+1.5}\right)_{s=0} = 3.8095$$

$$a = 5.7142$$

so

$$Y \approx \left(\frac{5.7142}{s+1.5}\right)X$$

6b) Compare the step response of your 1st-order model to the actual 3rd-order system

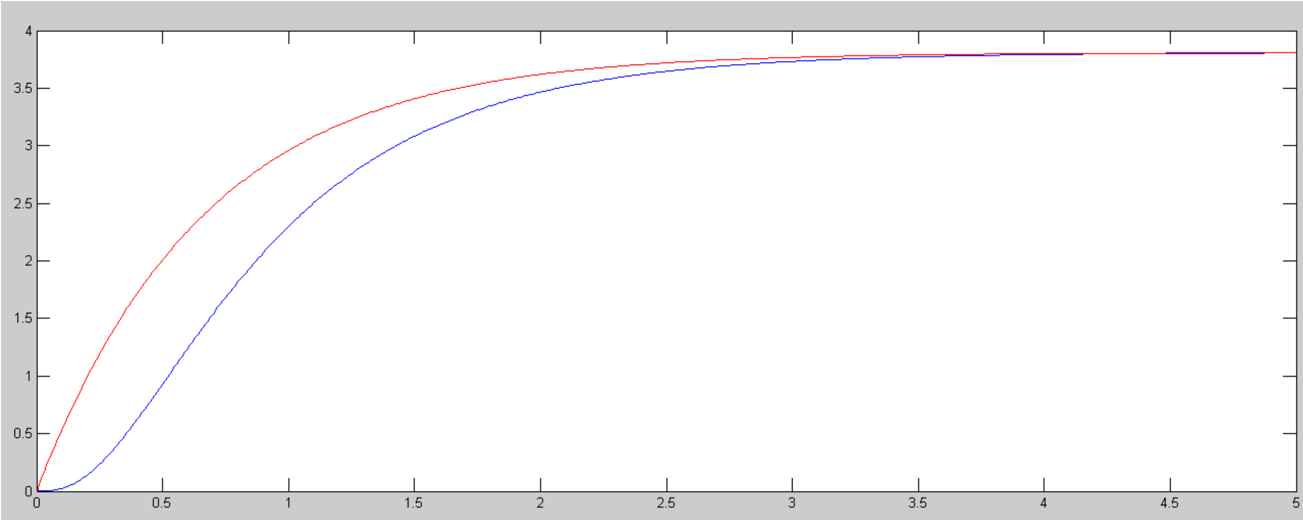
```
>> G3 = zpk([],[-1.5,-5,-7],200)

         200
-------------------
(s+1.5) (s+5) (s+7)

>> G1 = zpk([],-1.5,5.7142)

5.7142
-------
(s+1.5)

>> t = [0:0.01:5]';
>> y3 = step(G3,t);
>> y1 = step(G1,t);
>> plot(t,y3,'b',t,y1,'r')
>>
```



Step response of 3rd-order system (blue) & 1st-order approximation (red)

Comment:

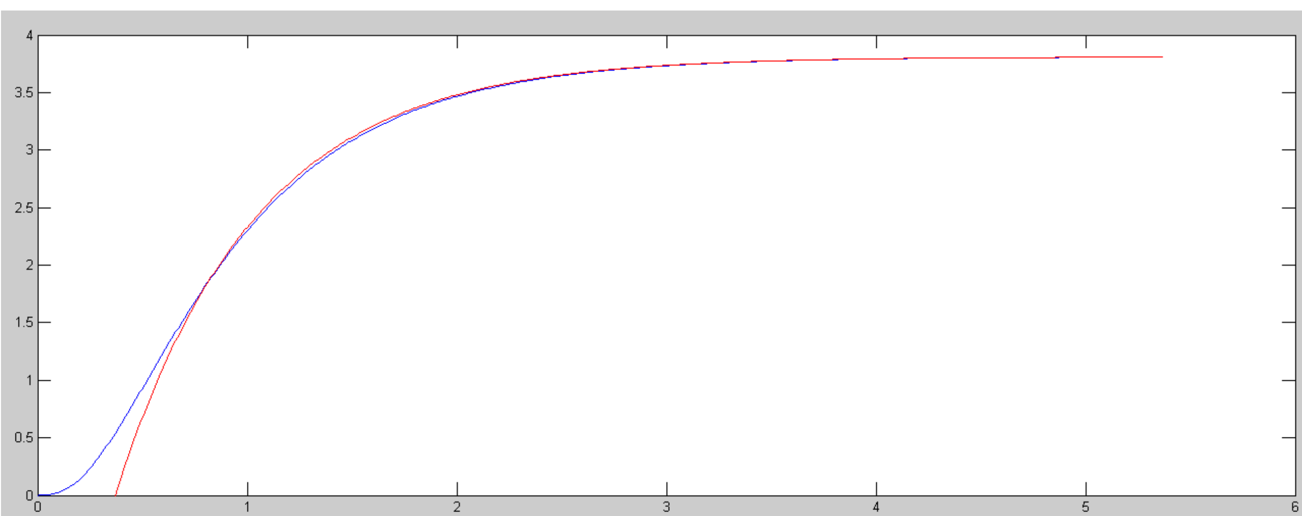
- Same DC gain
- Same shape
- Same settlimg time

Almost the same system.

Comment 2: The model would be better if you add a delay. Using trial and error until the delay matches up

```
>> plot(t,y3,'b',t+0.37,y1,'r')
```

$$Y \approx \left(\frac{5.7142}{s+1.5}\right) \cdot e^{-0.37s}$$



Step response of 3rd-order system (blue) & 1st-order approximation with a delay (red)