

# ECE 463/663 - Homework #11

LQR Observers. Due Wednesday, April 23rd

## Kalman Filters

**Cart and Pendulum (HW #4):** The dynamics for a cart and pendulum system with sensor and input noise is as follows

$$s \begin{bmatrix} \mathbf{x} \\ \theta \\ \dot{\mathbf{x}} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0 & 0 & 0 \\ 0 & -0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \theta \\ \dot{\mathbf{x}} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ - \\ - \end{bmatrix} (F + \eta_u)$$

$$y_1 = \mathbf{x} + n_x$$

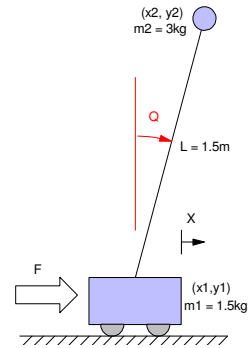
$$y_2 = \theta + n_\theta$$

where there is Gaussian noise at the input and output

$$n_u \sim N(0, 1.5^2) \quad \text{mean zero, standard deviation 1.5}$$

$$n_x \sim N(0, 0.2^2) \quad \text{mean zero, standard deviation 0.2}$$

$$n_\theta \sim N(0, 0.3^2) \quad \text{mean zero, standard deviation 0.3}$$



1) Use a servo-compensator to force the DC gain to one (i.e. use the servo compensator from homework set #10).

Plot the step response

- Without noise (same as homework set #9)
- With noise

2) Design a full-order observer using pole-placement to place the observer poles at  $\{-3, -3, -3, -3\}$

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.

3) Design a Kalman filter (i.e. a full-order observer with a specific Q and R)

- Simulate the response of the cart with noise added at the input and output.
- Plot the states of the plant and the observer with noise,.