

# ECE 463/663 - Homework #9

Calculus of Variations. Ricatti Equation. LQG Control. Due Monday, April 7th

## Soap Film

- 1) Calculate the shape of a soap film connecting two rings around the X axis:
  - $Y(0) = 8$
  - $Y(5) = 7$
- 2) Calculate the shape of a soap film connecting two rings around the X axis:
  - $Y(0) = 8$
  - $Y(3) = \text{free}$

## Hanging Chain

- 3) Calculate the shape of a hanging chain subject to the following constraints
  - Length of chain = 15 meters
  - Left Endpoint: (0,8)
  - Right Endpoint: (10,7)

## Ricatti Equation

- 4) Find the function,  $x(t)$ , which minimizes the following functional

$$J = \int_0^{10} (x^2 + 10\dot{x}^2) dt$$

$$x(0) = 8$$

$$x(10) = 7$$

- 5) Find the function,  $x(t)$ , which minimizes the following functional

$$J = \int_0^{10} (x^2 + 10u^2) dt$$

$$\dot{x} = -0.5x + u$$

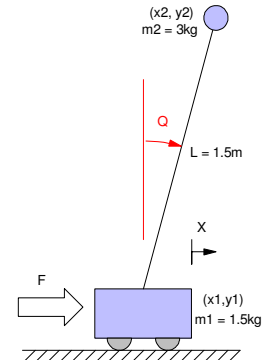
$$x(0) = 8$$

$$x(10) = 7$$

## LQG Control for a Cart & Pendulum

### 6) Cart & Pendulum (HW #4 & HW#6):

$$s \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0 & 0 & 0 \\ 0 & -0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ - \\ - \end{bmatrix} F$$



Design a full-state feedback control law of the form

$$F = U = K_r R - K_x X$$

for the cart and pendulum system from homework #4 using LQG control so that

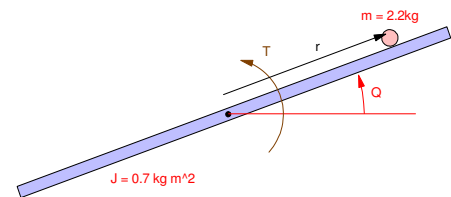
- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?

### 7) Ball and Beam (HW #4 & HW#6):

$$s \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0 & 0 & 0 \\ -0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r \\ \theta \\ \dot{r} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ - \end{bmatrix} T$$



Design a full-state feedback control law of the form

$$T = U = K_r R - K_x X$$

for the ball and beam system from homework #4 using LQG control so that

- The DC gain is 1.00
- The 2% settling time is 6 seconds, and
- There is less than 10% overshoot for a step input.

Compare your results with homework #6

- Where are the closed-loop poles with pole placement and with LQG control?
- Are the feedback gains larger or smaller with LQG control?
- Which one works better?