

ECE 463/663 - Homework #3

Canonical Forms, Similarity Transforms, LaGrangian Dynamics, Block Diagrams
Due Monday, February 3rd

Canonical Forms

Problem 1-3) For the system

$$Y = \left(\frac{20(s^2+9)}{(s+2)(s+4)(s+6)} \right) U$$

- 1) Express this system in controller canonical form. (Give the A, B, C, D matrices)
- 2) Express this system in cascade form
- 3) Express this system in Jordan (diagonal) form
- 4) Assume a system's dynamics are

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \\ sV_4 \end{bmatrix} = \begin{bmatrix} -3 & 1 & 0 & 0 \\ 1 & -3 & 1 & 0 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} V_0$$

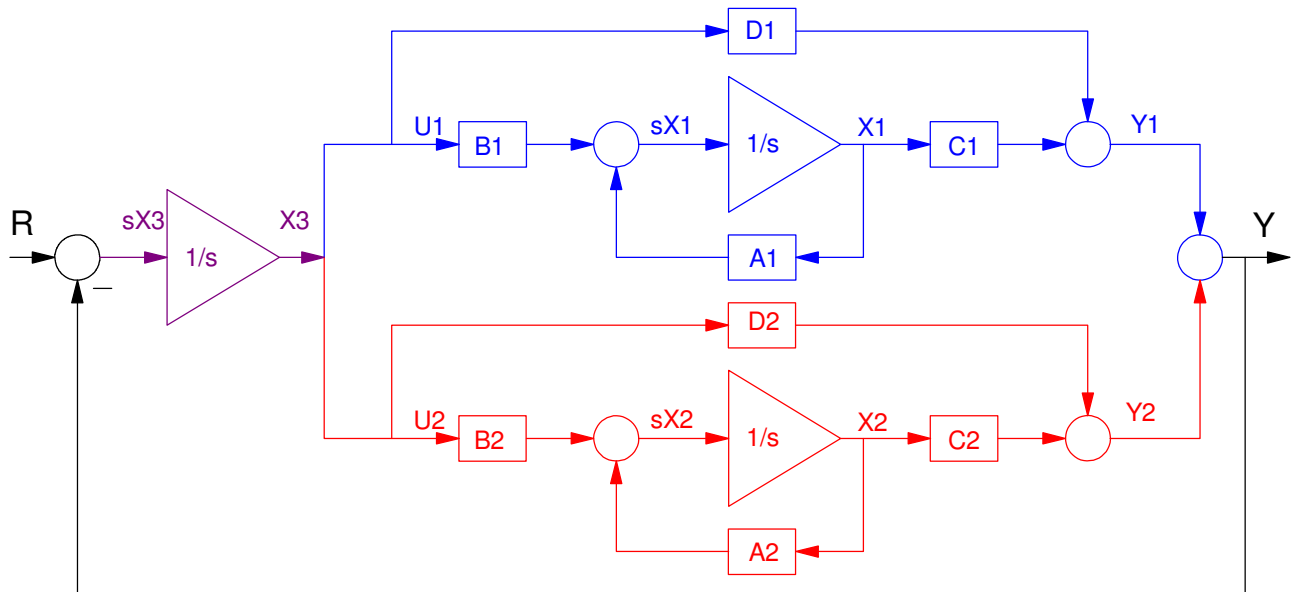
$$Y = V_4$$

Express these dynamic with the change in variable

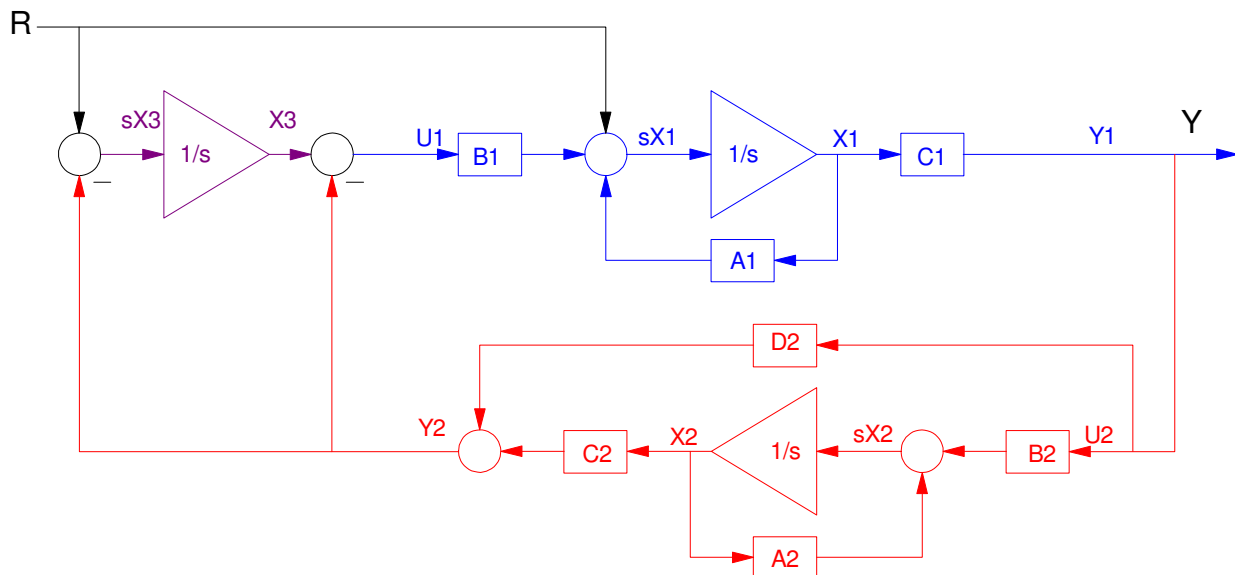
$$\begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} = \begin{bmatrix} V_1 - V_2 \\ V_2 - V_3 \\ V_3 - V_4 \\ V_4 \end{bmatrix}$$

Block Diagrams

5) Determine the state-space model the following system:



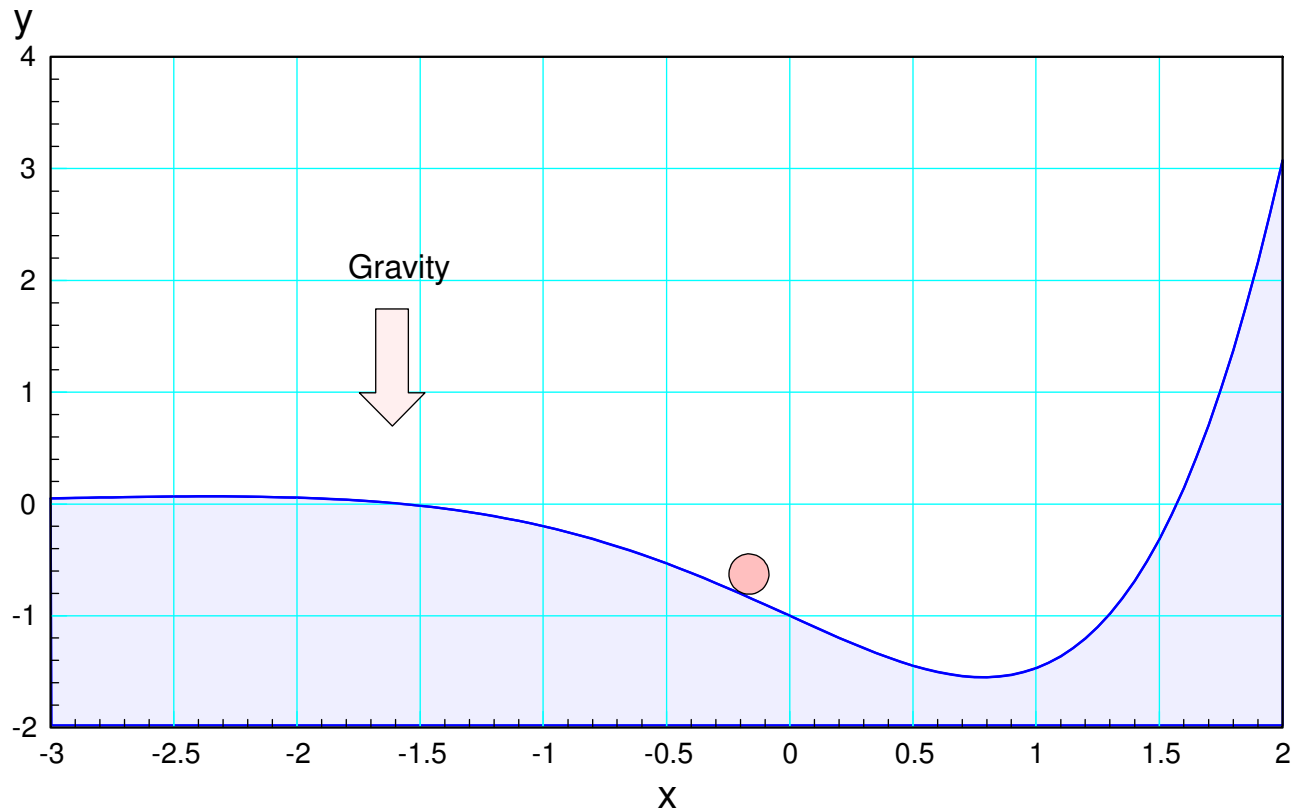
6) Determine the state-space model for the following system:



LaGrangian Dynamics

A 1kg ball is rolling on a surface defined by:

$$y = -\exp(x) \cdot \cos(x)$$



7) Determine the kinetic and potential energy of this ball as a function of x : Gravity is in the $-y$ direction.

Assuming a solid sphere:

8) Determine the dynamics for this ball as it rolls on this surface