ECE 461/661 - Test #3: Name ____________________

Digital Control & Frequemncy Domain techniques - Fall 2024

s to z conversion

1) Determine the discrete-time equivalent for K(s). Assume a sampling rate of T = 0.05 seconds

$$K(s) = \left(\frac{30(s+2)(s+5)}{s(s+15)}\right)$$

Convert from the s-plane to the z-plane as z = exp(sT)

$$s = -2 \qquad z = e^{sT} = 0.9048$$

$$s = -5 \qquad z = e^{sT} = 0.7788$$

$$s = 0 \qquad z = e^{sT} = 1$$

$$s = -15 \qquad z = e^{sT} = 0.4724$$

so K(z) is of the form

$$K(z) = k\left(\frac{(z-0.9048)(z-0.7788)}{(z-1)(z-0.4724)}\right)$$

Pick 'k' to match the gain somewhere. Since the DC gain is infinity, pick some other frequency

$$s = j$$

$$\left(\frac{30(s+2)(s+5)}{s(s+15)}\right)_{s=j} = 22.7530\angle -55.9391^0$$

$$z = e^{sT} = e^{j0.05}$$

$$k\left(\frac{(z-0.9048)(z-0.7788)}{(z-1)(z-0.4724)}\right)_{z=e^{j0.05}} = 0.9081k\angle -56.0428^0$$

Pick 'k' so that the gains match

$$22.7530 = 0.9081k$$

$$k = 25.0565$$

and

$$K(z) = 25.0565\left(\frac{(z-0.9048)(z-0.7788)}{(z-1)(z-0.4724)}\right)$$

Digital Compensators: K(z)

2) Assume a unity feedback system with a sampling rate of T = 0.1 second

$$G(z) = \left(\frac{0.01z^2}{(z-0.95)(z-0.85)(z-0.6)}\right)$$

Design a digital compensator, K(z), which results in

- No error for a step input,
- Closed-Loop Dominant poles at z = 0.8 + j0.2, and
- Is causal (the number of poles in K(z) is equal to or greater than the number of zeros)

Translation:

- Add a pole at z = +1 to make this a type-1 system
- Cancel the slow poles and replace then with fast poles
- So that G(z)*K(z) = -1 at z = 0.8 + j0.2

As a first guess, let K(z) be of the form

$$K(z) = k\left(\frac{(z-0.95)(z-0.85)}{(z-1)(z-a)}\right)$$

Then

$$GK = \left(\frac{0.01k\cdot z^2}{(z-1)(z-0.6)(z-a)}\right)$$

Analyze what we know:

$$\left(\frac{0.01z^2}{(z-1)(z-0.6)}\right)_{z=0.8+j0.2} = 0.0850\angle -151.9275^0$$

The angle should be 180 degrees for this point to be on the root locus. Pick 'a' so that the angle of (z-a) makes the phase equal to 180 degrees

$$\angle(z-a) = 28.0725^0$$

$$a = 0.8 - \frac{0.2}{\tan\left(28.0725^0\right)} = 0.4250$$

so

$$K(z) = k\left(\frac{(z-0.95)(z-0.85)}{(z-1)(z-0.4250)}\right)$$

$$GK = \left(\frac{0.01k\cdot z^2}{(z-1)(z-0.6)(z-0.4250)}\right)$$

Pick 'k' to make the gain one

$$\left(\frac{0.01 \cdot z^2}{(z-1)(z-0.6)(z-0.4250)}\right)_{z=0.8+j0.2} = 0.2000\angle 180^0$$

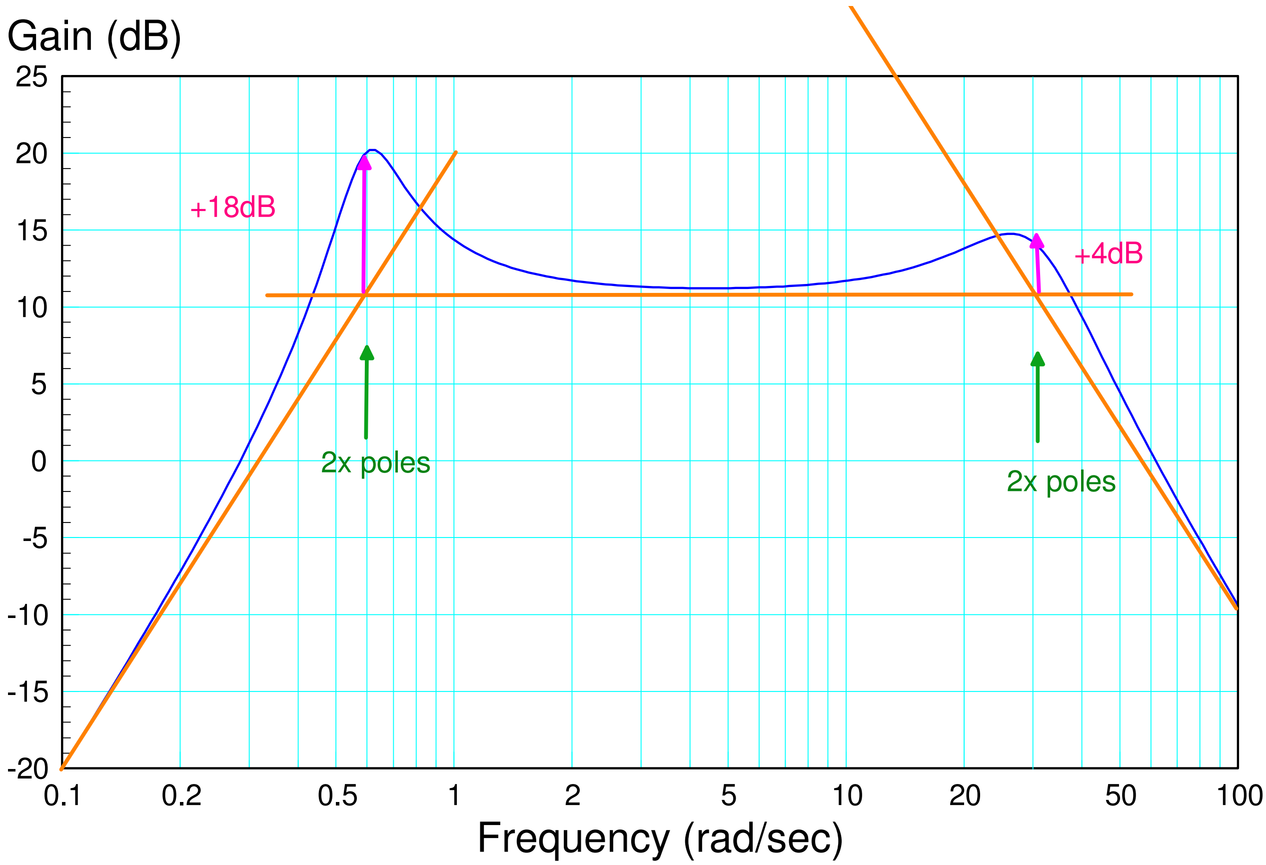
$$k = \frac{1}{0.2} = 5$$

giving

$$K(z) = 5\left(\frac{(z-0.95)(z-0.85)}{(z-1)(z-0.4250)}\right)$$

3) Bode Plots

Determine the system, G(s), which has the following gain vs. frequency



Step 1: Draw the straight line asymptotes (shown in orange). The intersections tell you where the poles and zeros are

$$G(s) = \left(\frac{ks^2}{(s+0.6\angle\pm\theta)(s+31\angle\pm\phi)} \right)$$

To find the angles, check the gain at the corner. At the pole at 0.6

$$gain = +18dB = 10^{18/20} = 7.9433$$

$$\frac{1}{2\zeta} = 7.9433$$

$$\zeta = 0.0629 \;\; \Rightarrow \theta = 86.39^0$$

At the pole at 31

$$\frac{1}{2\zeta} = gain = +4dB = 1.5849$$

$$\zeta = 0.3133 \Rightarrow \phi = 71.61^0$$

$$G(s) = \left(\frac{ks^2}{\left(s+0.6\angle\pm 86.39^0\right)\left(s+31\angle\pm 71.61^0\right)} \right)$$

To find k, match the gain somewhere like s = j5

$$\left(\frac{ks^2}{\left(s+0.6\angle\pm 86.39^0\right)\left(s+31\angle\pm 71.61^0\right)} \right)_{s=j5} = 11dB = 3.548$$

$$k = 3291.3$$

so

$$G(s) \approx \left(\frac{3291.3s^2}{\left(s+0.6\angle\pm 86.39^0\right)\left(s+31\angle\pm 71.61^0\right)} \right)$$

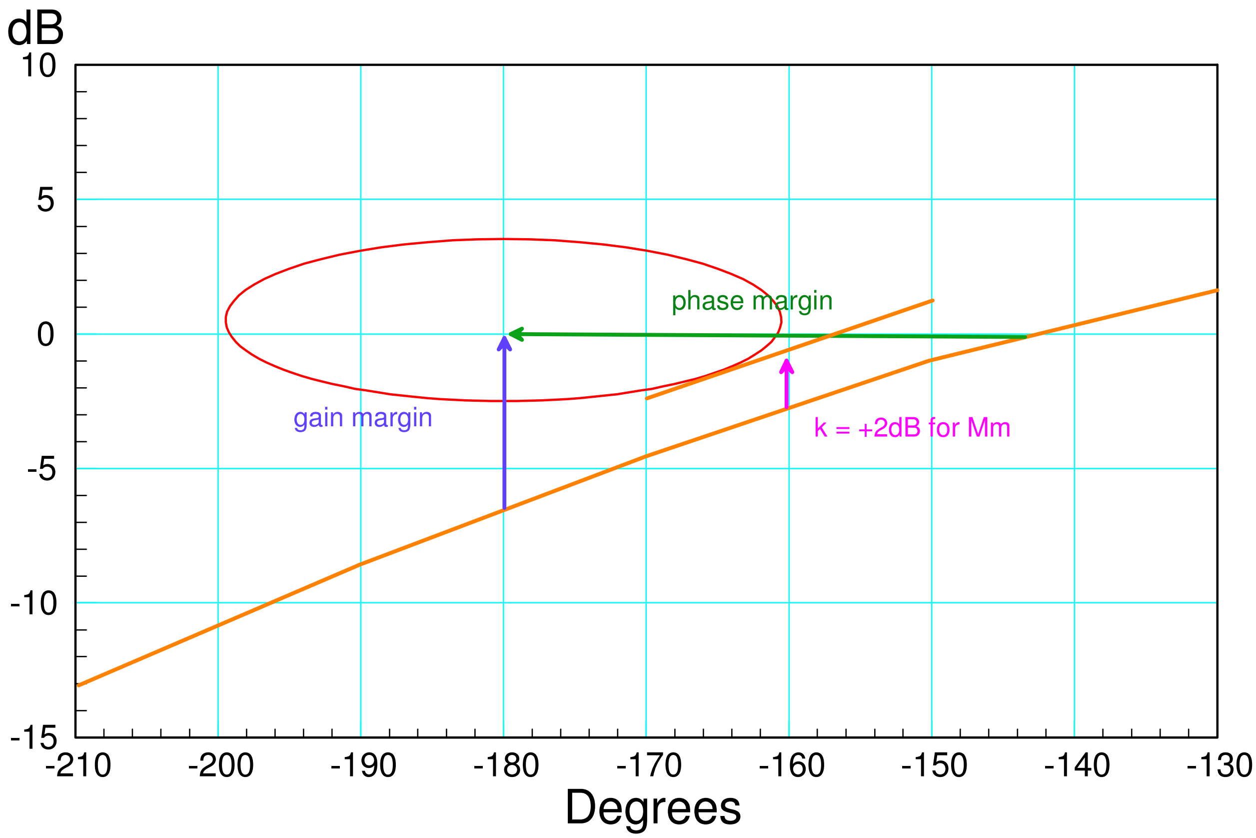
4) Nichols Charts

Assume a unity feedback system where the gain of G(s) is as follows:

freq (rad/sec)	3	5	7	9	15
Gain	2dB	-1dB	-4dB	-8dB	-13dB
Phase	-130 deg	-150 deg	-170 deg	-190 deg	-210 deg

Transfer this data to a Nichols chart (shown below) and determine:

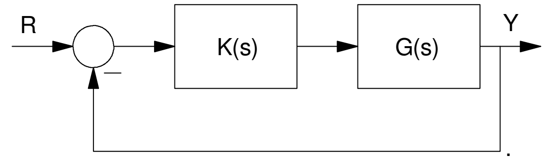
Mm for the M-circle shown	Gain Margin for k=1	Phase Margin for k=1	k for max gain = Mm
3.196	+7dB shown in purple	36 degrees shown in green	+2dB shown in pink



Mm: Pick any point on the m-circle such as $1\angle-162^0$. The closed-loop gain is then

$$M_m = \left(\frac{G}{1+G}\right) = \left(\frac{1\angle-162^0}{1+1\angle-162^0}\right) = 3.196$$

5) Analog Compensator (Bode Plots)



Assume a unity feedback system with

$$G(s) = \left(\frac{50}{(s+1)(s+4)(s+10)}\right)$$

Determine a compensator, K(s), which results in

- No error for a step input
- A phase margin of 20 degrees
- A 0dB gain frequency of 3 rad/sec

Translation:

- Add a pole at s = 0 to make this type-1
- Add poles and zeros so that $GK(j3) = 1\angle -160^0$

Starting out, let K(s) be of the form

$$K(s) = \left(\frac{k(s+1)(s+4)}{s(s+a)}\right)$$

$$GK = \left(\frac{50k}{s(s+10)(s+a)}\right)$$

Analyze what we know

$$\left(\frac{50}{s(s+10)}\right)_{s=j3} = 1.5964\angle -106.6992^0$$

For the phase to add up to -160 degrees

$$\angle(s+a) = 53.3008^0$$

Using some trig

$$a = \frac{3}{\tan\left(53.3008^0\right)} = 2.2361$$

and

$$K(s) = \left(\frac{k(s+1)(s+4)}{s(s+2.2361)}\right)$$

$$GK = \left(\frac{50k}{s(s+2.2361)(s+10)}\right)$$

Pick k to make the gain one at s = j3

$$\left(\frac{50}{s(s+2.2361)(s+10)}\right)_{s=j3} = 0.4266\angle -160^0$$

$$k = \frac{1}{0.4266} = 2.3439$$

and

$$K(s) = \left(\frac{2.3439(s+1)(s+4)}{s(s+2.2361)}\right)$$

Other valid solutions are

$$K(s) = \left(\frac{1.6414(s+1)(s+4)(s+10)}{s(s+4.2844)^2}\right)$$

$$K(s) = \left(\frac{33.2191(s+1)}{s(s+10.1729)}\right)$$