ECE 461/661 - Test #2: Name ____________________

Feedback and Root Locus - Fall 2024

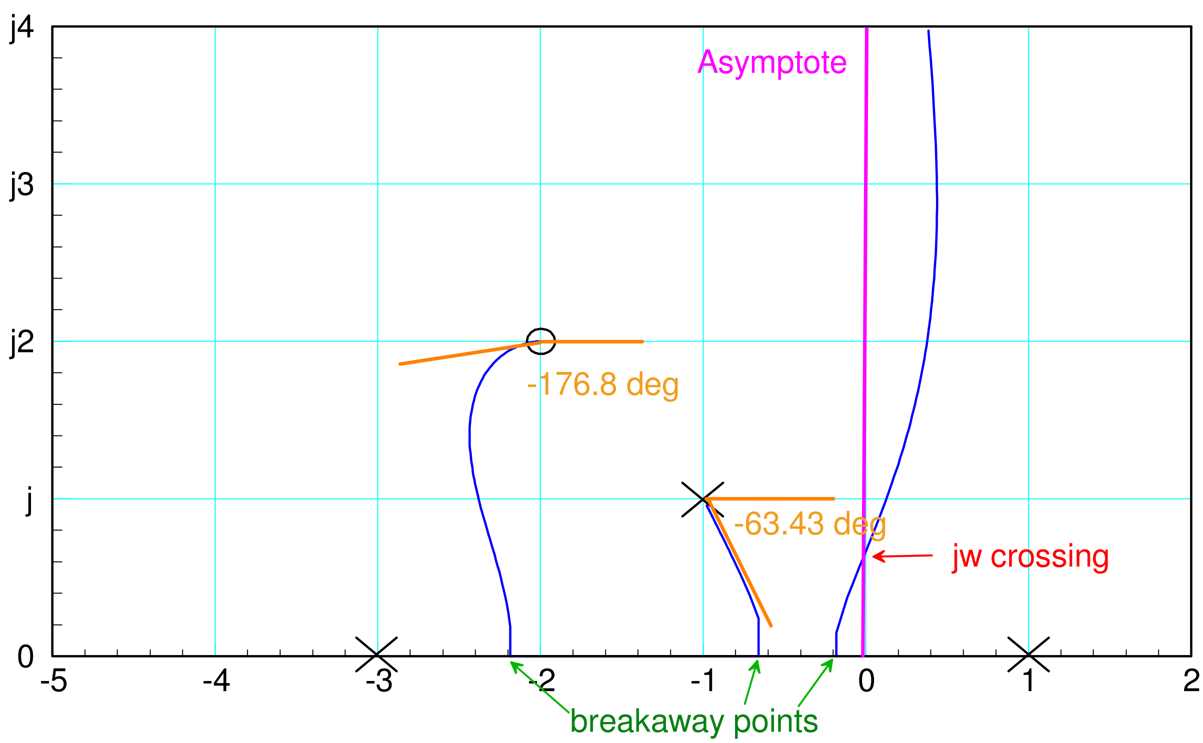
Root Locus

1) The root locus of G(s) is shown below.

$$G(s) = \left(\frac{10(s+2+j2)(s+2-j2)}{(s-1)(s+3)(s+1+j)(s+1-j)}\right)$$

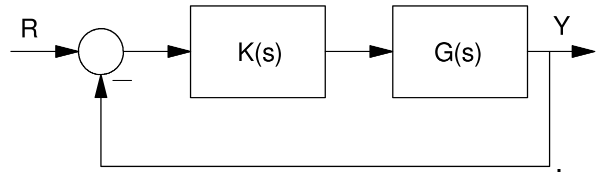
Determine the following

Approach Angle to the zero at -2+j2	Departure Angle from the pole at -1+j1	Real Axis Loci
-176.8 deg	-63.43 deg	(+1, -3)
Breakaway Points (approx)	**Asymptotes**	**jw Crossing(s)**
{-0.1891, -0.6270, -2.1794}	show on graph	j0.6667



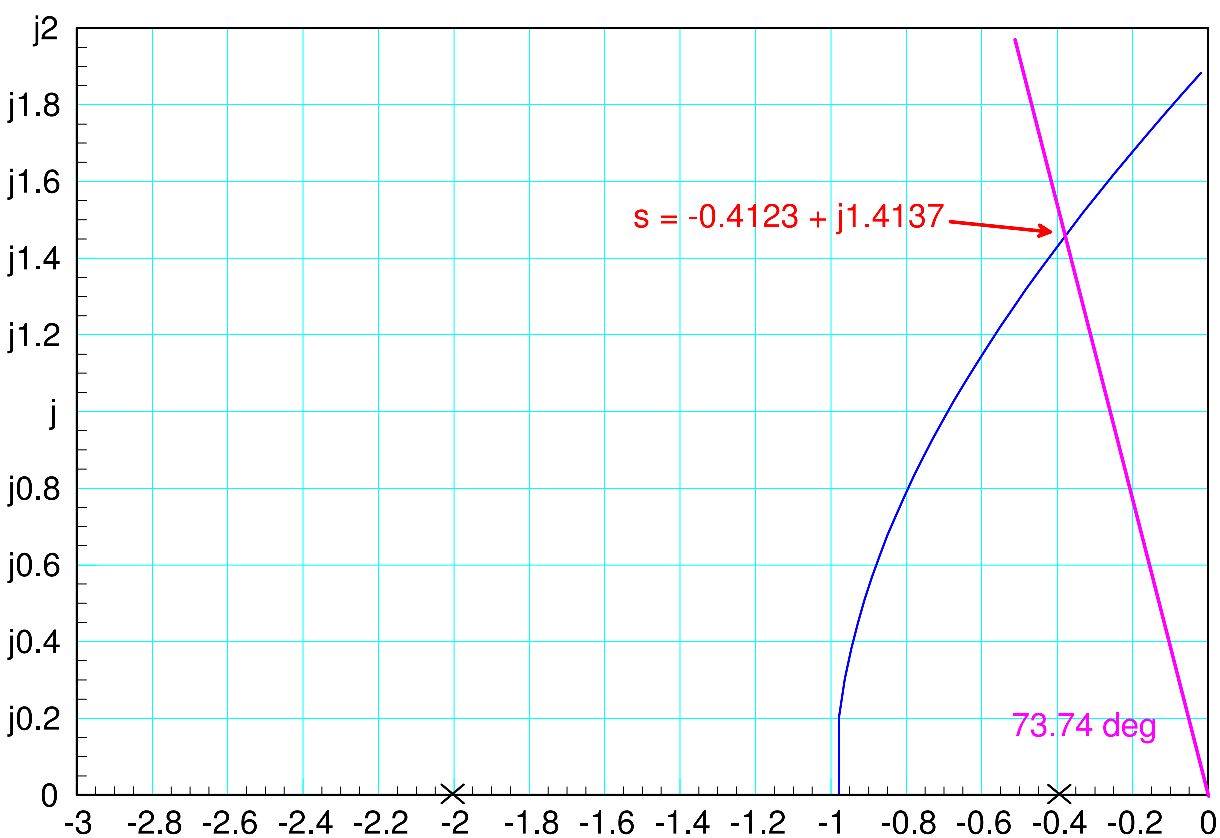
Gain Compensation

2) Determine the gain (K(s) = k) so that the feedback system has 40% overshoot for a step input. Also determine the closed-loop dominant pole(s) and error constant, Kp

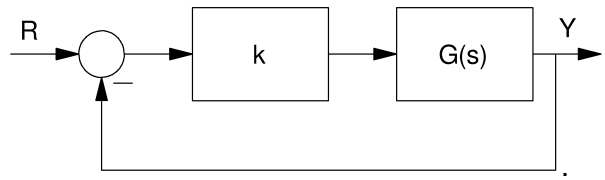


$$G(s) = \left(\frac{100}{(s+0.4)(s+2)(s+4)(s+6)(s+7)}\right)$$

Damping Ratio 40% overshoot	Angle of Pole	k 40% overshoot	Closed Loop Dominant Pole(s)	Kp Error Constant
0.2800	73.74 deg	4.50	-0.4123 + j1.4137	3.3482



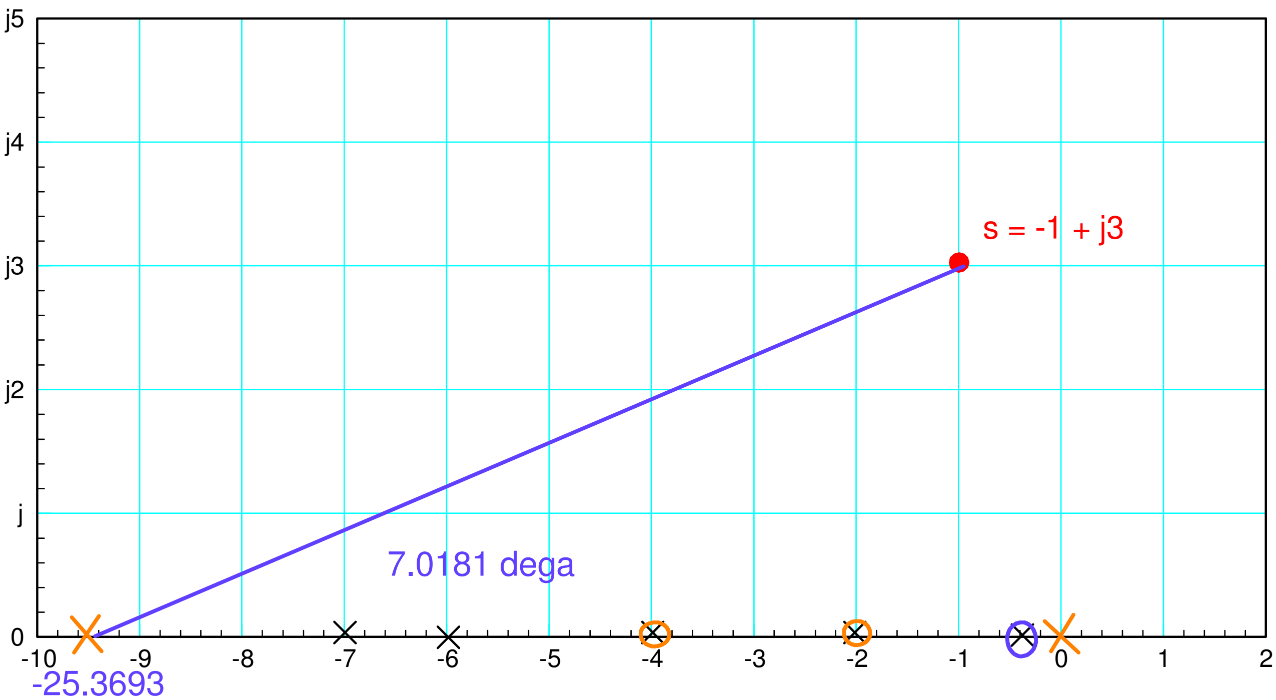
Lead/PI Compensation



3) Design a compensator, K(s), so that the closed-loop system has

- No error for a step input
- Closed-Loop dominant poles at s = -1 + j3, and
- Finite gain as s→ ∞ (i.e. have at least as many poles as zeros)

$$G(s) = \left(\frac{100}{(s+0.4)(s+2)(s+4)(s+6)(s+7)}\right)$$



$$K(s) = k\left(\frac{(s+0.4)(s+2)(s+4)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{100k}{s(s+6)(s+7)(s+a)^2}\right)$$

analyze what you know

$$\left(\frac{100}{s(s+6)(s+7)}\right)_{s=-1+j3} = 0.8085\angle -165.96^0$$

To make the angle -180 degrees

$$\angle(s+a) = 7.0181^0$$

$$a = 1 + \frac{3}{\tan\left(7.0181^0\right)} = 25.369$$

to find k:

$$GK = \left(\frac{100k}{s(s+6)(s+7)(s+25.3693)^2}\right)_{s=-1+j3} = 0.0013k\angle 180^0$$

$$k = \frac{1}{0.0013} = 745.7$$

so

$$\boxed{K(s) = 745.7\left(\frac{(s+0.4)(s+2)(s+4)}{s(s+25.3693)^2}\right)}$$

There are other solutions as well

Compensator Design (hardware)

4) Design a circuit to implement K(s)

$$K(s) = \left(\frac{50(s+2)(s+6)}{s(s+7)}\right)$$

Rewrite as

$$10\left(\frac{s+2}{s+7}\right) \cdot 5\left(\frac{s+6}{s}\right)$$