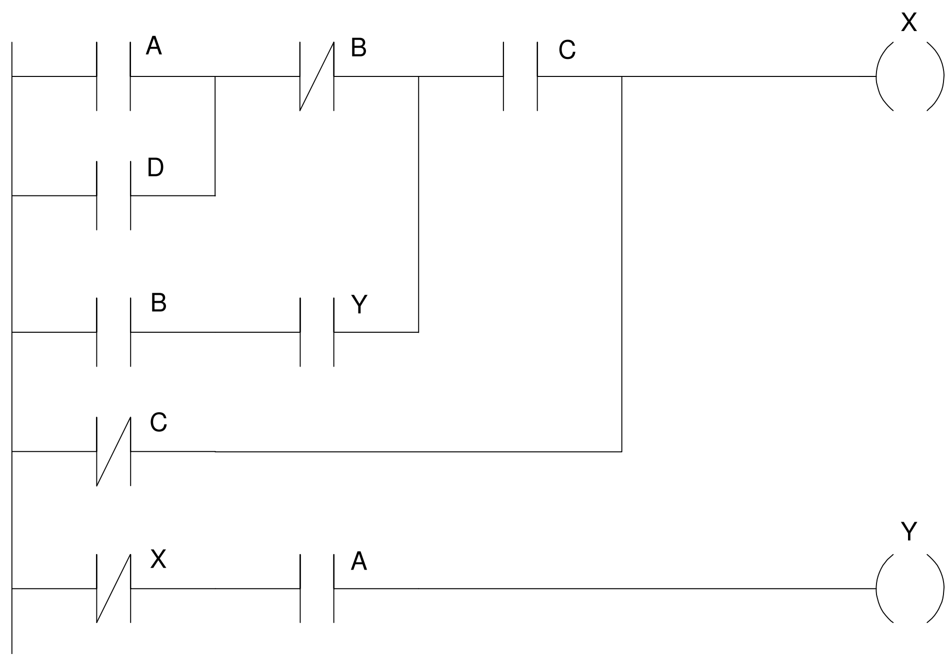
ECE 461/661 - Test #1: Name ____________________

Fall 2024

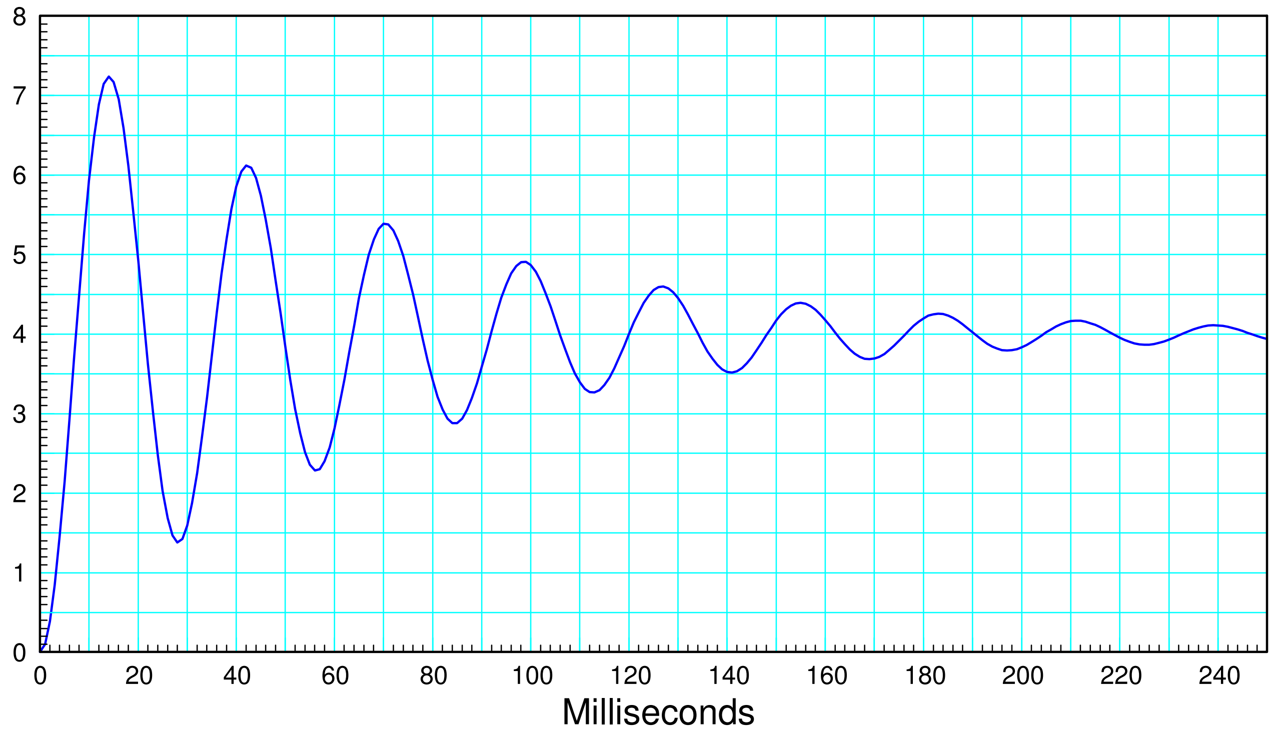
1) Determine the functions for X and Y according to the following ladder diagram. (you don't need to simplify)

$$X = \left((A+D)\overline{B} + BY \right) C + \overline{C}$$

$$Y = \overline{X} A$$



2) Give the transfer function for a system with the following response to a unit step input:



This is a second order system (oscillates meaning complex poles)

$$G(s) = \frac{a}{(s+b+jc)(s+b-jc)}$$

Ts = 240ms (approx)

$$b = \frac{4}{0.24} = 16.67$$

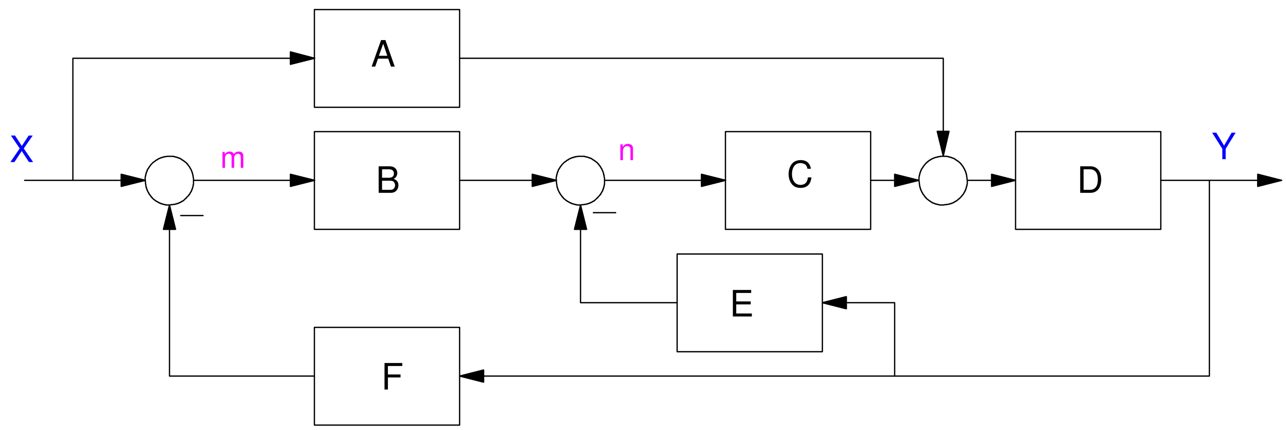
Ths frequency of oscillation is c:

$$c = \left(\frac{6 \text{ cycles}}{168\text{ms}}\right) 2\pi = 224.4$$

Pick 'a' to make the DC gain equal to 4.000

$$G(s) \approx \left(\frac{202{,}532}{(s+16.67+j224.4)(s+16.67-j224.4)}\right)$$

3) Find the transfer funciton from X to Y



Shortcut:

$$Y = \left(\frac{AD+BCD}{1+CDE+BCDF}\right)X$$

Long Way:

$$m = X - FY$$

$$n = Bm - EY$$

$$Y = DCn + DAX$$

Substituting

$$n = B(X - FY) - EY$$

$$Y = DC(B(X - FY) - EY) + DAX$$

Simplify

$$Y = DCBX - DCBFY - DCEY + DAX$$

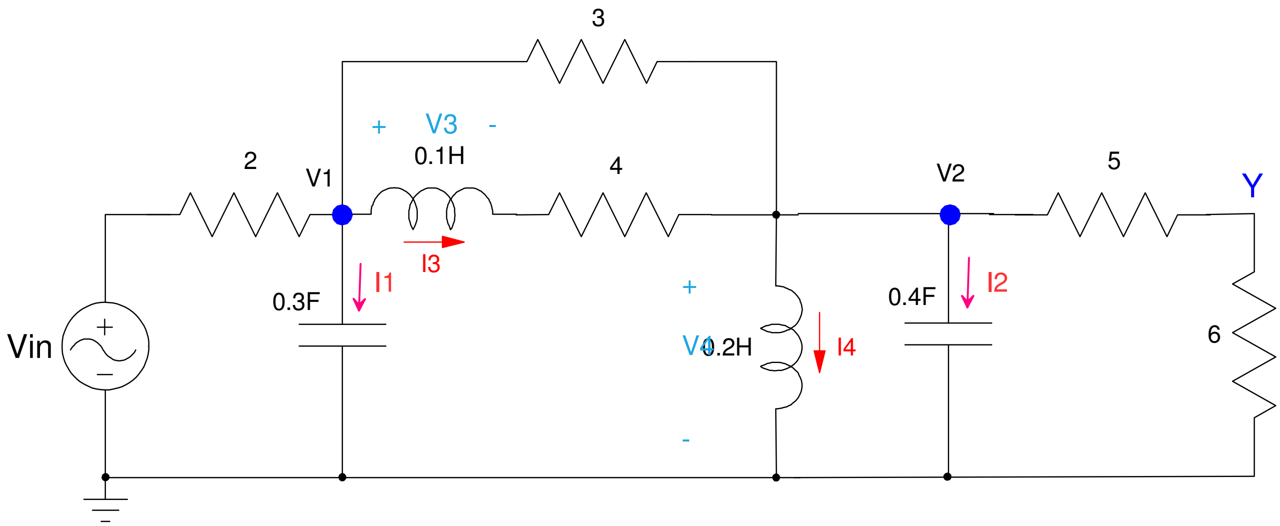
$$Y + DCBFY + DCEY = DCBX + DAX$$

$$(1 + DCBF + DCE)Y = (DCB + DA)X$$

$$Y = \left(\frac{DCB+DA}{1+DCBF+DCE}\right)X$$

4) For the following RLC circuit:

- Write the dynamics of this system as four compled differential equations in terms of {Vin, V1, V2, I3, I4}
- You don't need to solve or put in state-space form (that's a different problem on the test)



$$I_1 = 0.3sV_1 = \left(\frac{V_{in} - V_1}{2}\right) - I_3 - \left(\frac{V_1 - V_2}{3}\right)$$

$$I_2 = 0.4sV_2 = \left(\frac{V_1 - V_2}{3}\right) + I_3 - I_4 - \left(\frac{V_2}{11}\right)$$

$$V_3 = 0.1sI_3 = V_1 - 4I_3 - V_2$$

$$V_4 = 0.2sI_4 = V_2$$

5) Assume the dynamics of an RLC circuit are:

$$0.1sV_1 = \left(\frac{V_{in}-V_1}{2}\right) + \left(\frac{V_2-V_1}{3}\right)$$

$$0.2sV_2 = \left(\frac{V_1-V_2}{4}\right) - 2I_3$$

$$0.5sI_3 = V_1 - 2V_2 - 3I_3$$

$$Y = 4V_2 - 5I_3$$

Give the state-space representation for the dynamics.

Simplify

$$sV_1 = 5V_{in} - 5V_1 + 3.33V_2 - 3.33V_1$$

$$sV_2 = 1.25V_1 - 1.25V_2 - 10I_3$$

$$sI_3 = 2V_1 - 4V_2 - 6I_3$$

In matrix form

$$s\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -8.33 & 3.33 & 0 \\ 1.25 & -1.25 & -10 \\ 2 & -4 & -6 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} V_{in}$$

$$Y = \begin{bmatrix} 0 & 4 & -5 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ I_3 \end{bmatrix} + [0]V_{in}$$