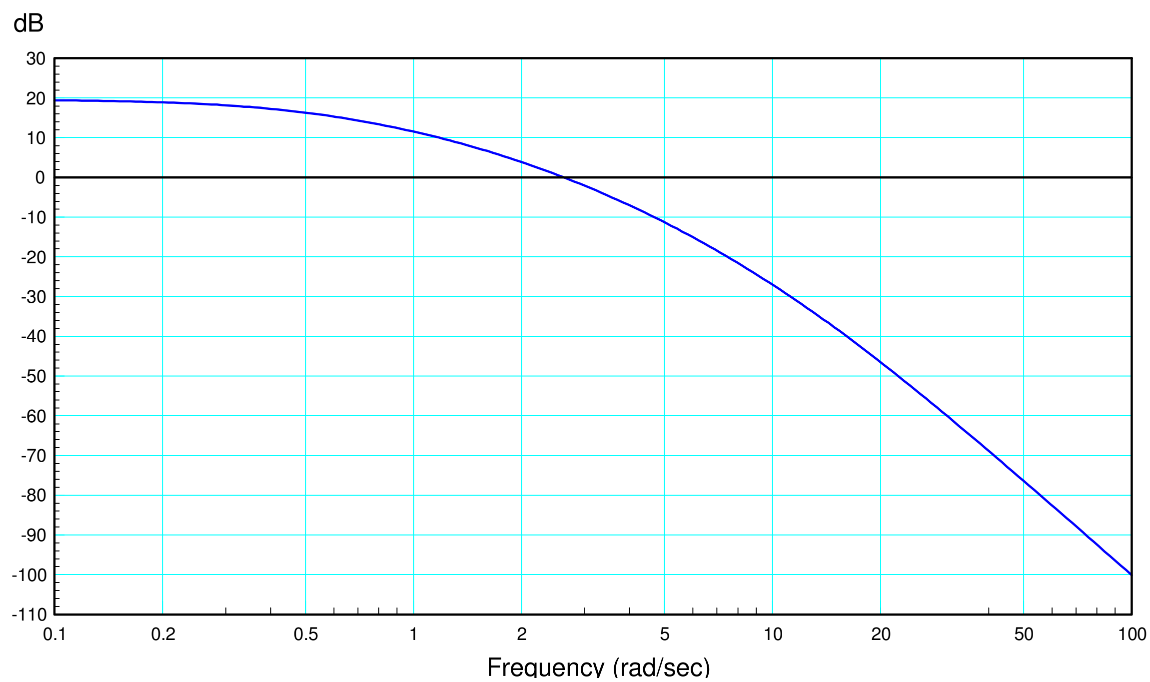
# Homework #12: ECE 461/661

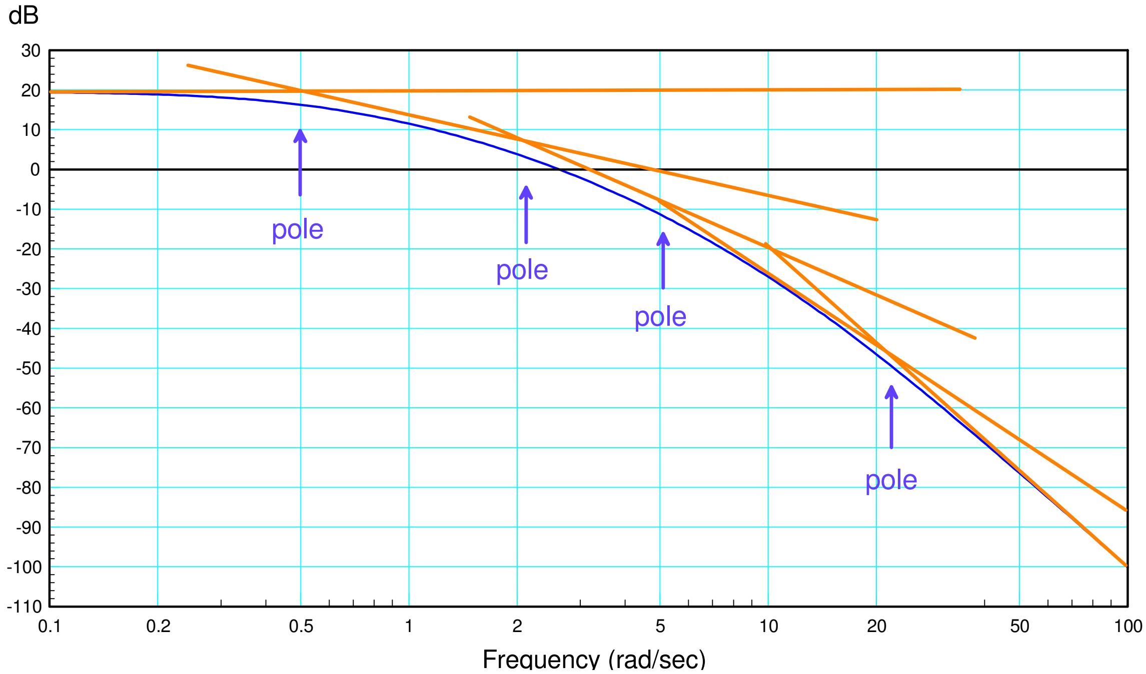
Bode Plots. Nichols charts and gain & lead compensation. Due Monday, December 2nd

## Bode Plots

1) Determine the system, G(s), with the following gain vs. frequency



Step 1: Draw in the straight-line asyptotes. Make sure each is at a multiple of 20dB/decade




Each corner (where the asymptotes meet) is a pole

$$G(s) = \left( \frac{k}{(s+0.5)(s+2.1)(s+5)(s+21)} \right)$$

Pick 'k' to match the DC gain (approx the same as the gain at s = j0.1)

$$\left( \frac{k}{(s+0.5)(s+2.1)(s+5)(s+21)} \right)_{s=0} = 20dB = 10$$

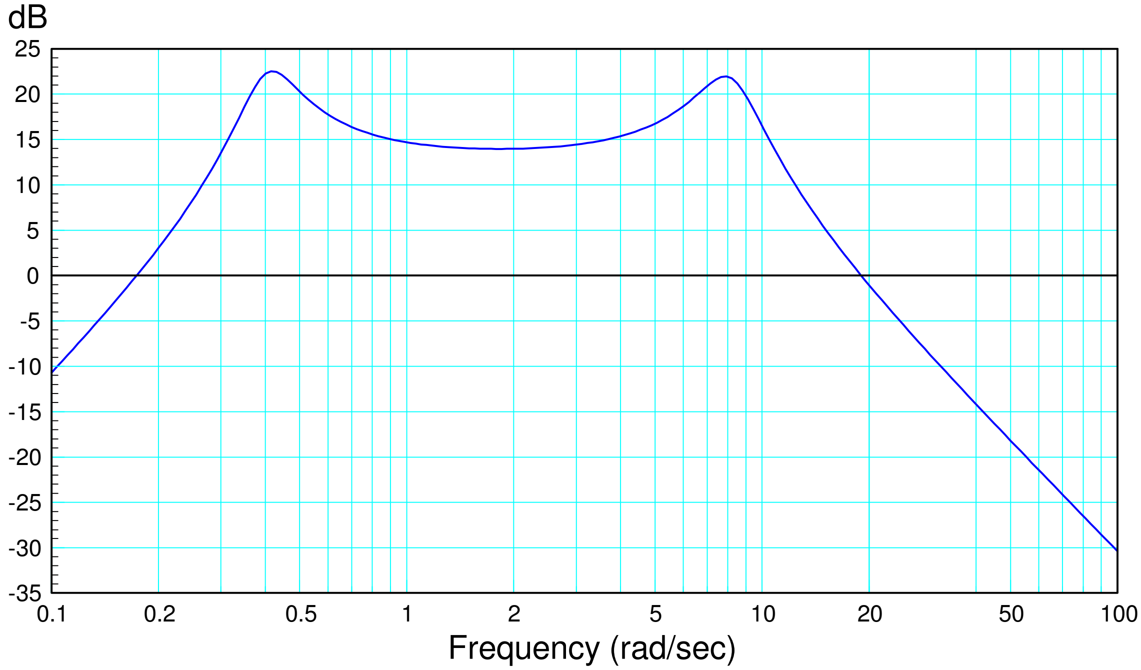
$$k = 1102.5$$

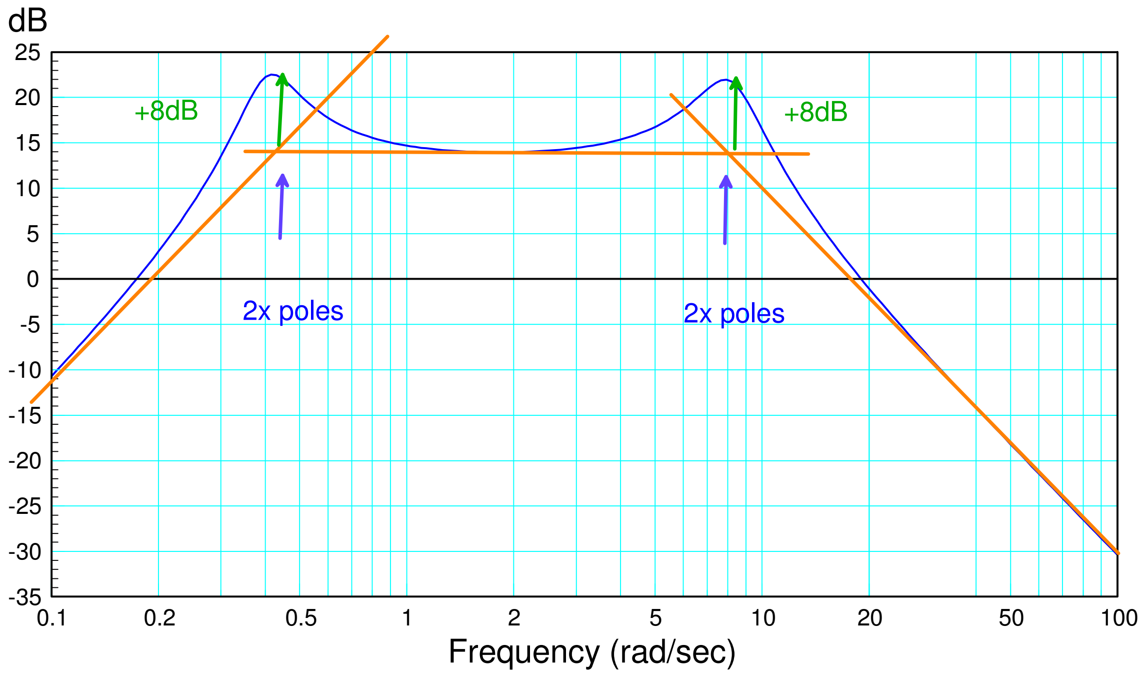
so

$$G(s) \approx \left( \frac{1102.5}{(s+0.5)(s+2.1)(s+5)(s+21)} \right)$$

*answers will vary*

2) Determine the system, G(s), with the following gain vs. frequency






Draw in the straight-line asymptotes at multiples of 20dB/decade

- shown on graph

The corners tell you the magnitude of the poles & zeros

- Two zeros left of 0.1 (assume at s = 0)
- Two poles at 0.42 rad/sec
- Two poles at 8 rad/sec

$$G(s) \approx \left(\frac{ks^2}{(s+0.42\angle\pm\theta)(s+8\angle\pm\phi)}\right)$$

The gain at the corner tells you the damping ratio

gain at corner = +8dB = 2.5119

$$\frac{1}{2\zeta} = 2.5119$$

$$\zeta = 0.199$$

$$\theta = 78.52^0$$

Same for $\phi$

To find k, pick k to match the gain somewhere, like s = j2

$$\left(\frac{ks^2}{\left(s+0.42\angle\pm78.52^0\right)\left(s+8\angle\pm78.52^0\right)}\right)_{s=j2} = 14dB = 5.012$$

$$k =$$

In Matlab:

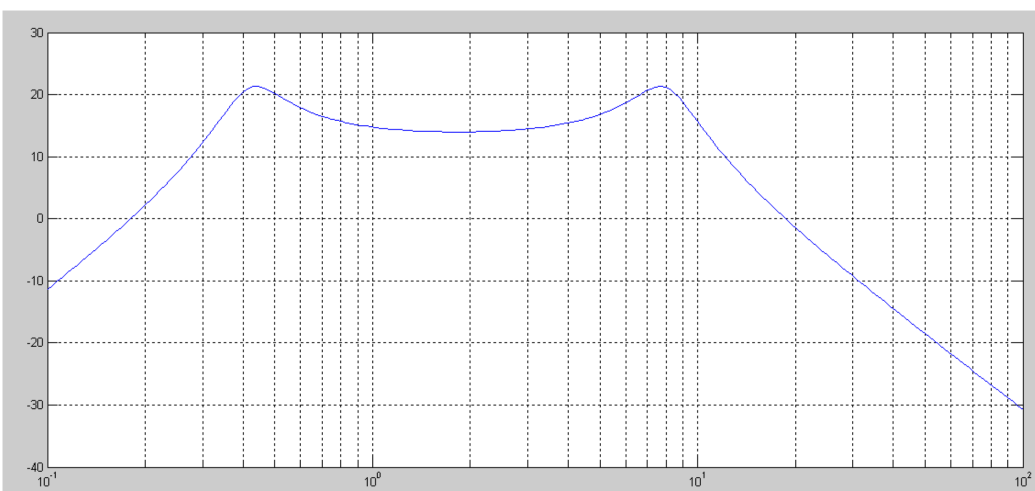
```
>> p1 = -0.42 * exp(j*78.52*pi/180);
>> p2 = conj(p1);
>> p3 = -8*exp(j*78.52*pi/180);
>> p4 = conj(p3);
>> G = zpk([0,0],[p1,p2,p3,p4],1);
>> evalfr(G,j*2)

ans =   0.0173 - 0.0003i

>> k = 5.012 / abs(ans)

k =  290.1763
```

so

$$G(s) \approx \left(\frac{290.1763s^2}{\left(s+0.42\angle\pm78.52^0\right)\left(s+8\angle\pm78.52^0\right)}\right)$$

Checking in Matlab

```
>> G = zpk([0,0],[-p1,-p2,-p3,-p4],290.1763)

Zero/pole/gain:
                    290.1763 s^2
--------------------------------------------
(s^2 + 0.1672s + 0.1764) (s^2 + 3.184s + 64)

>> Gw = Bode2(G,w);
>> semilogx(w,20*log10(abs(Gw)));
>> grid on
```



## Nichols Charts

3) The gain vs. frequency of a system is measured

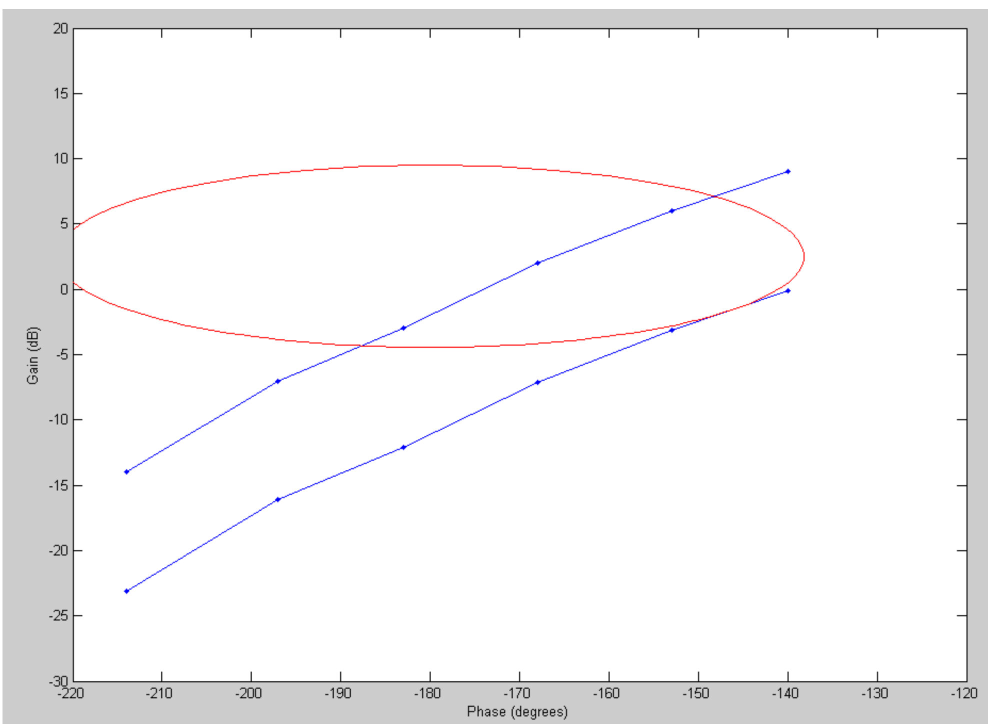
| w (rad/sec) | 2 | 3 | 4 | 5 | 6 | 10 |
|---|---|---|---|---|---|---|
| Gain (dB) | 9 | 6 | 2 | -3 | -7 | -14 |
| Phase (deg) | -140 | -153 | -168 | -183 | -197 | -214 |

Using this data

- Transfer it to a Nichols chart
- Determine the maximum gain that results in a stable system
- Determine the gain, k, that results in a maximum closed-loop gain of Mm = 1.5

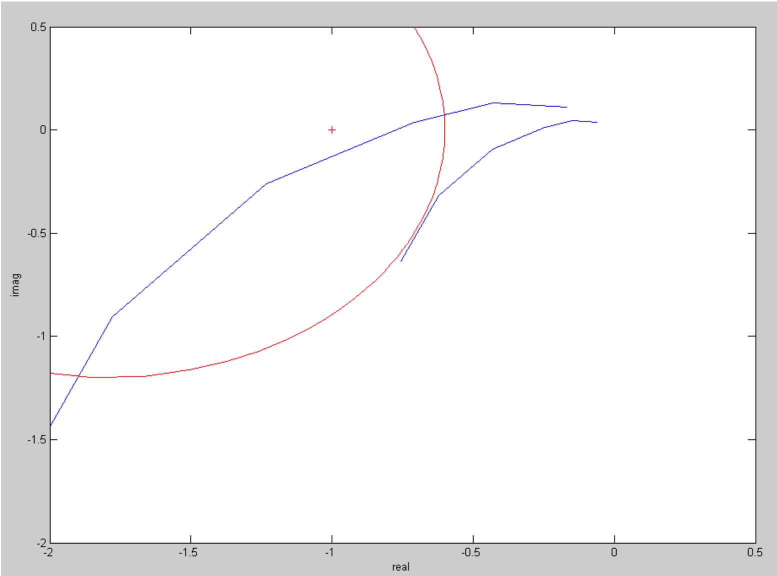
```
>> w = [2,3,4,5,6,10]';
>> GdB = [9,6,2,-3,-7,-14]';
>> Gp = [-140,-153,-168,-183,-197,-214]';
>> Gw = 10 .^(GdB/20) .* exp(j*Gp*pi/180)
>> nichols2(Gw*[1,0.5],1.5);
>> nichols2(Gw*[1,0.3],1.5);
>> nichols2(Gw*[1,0.34],1.5);
>> nichols2(Gw*[1,0.35],1.5);
>>
```

**ans: k = 0.35**



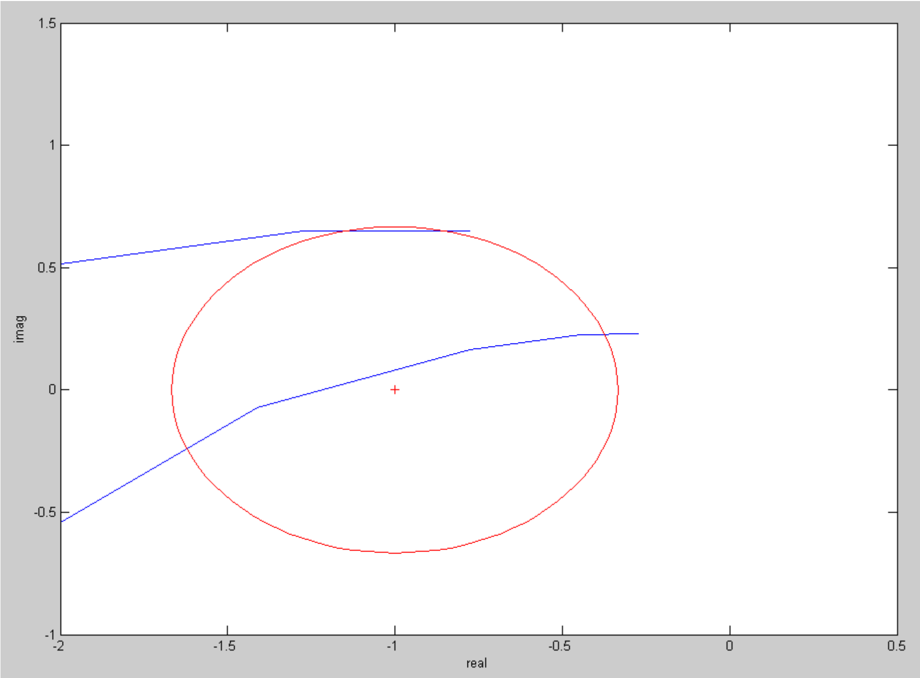
Sidelight: It's the same procedure with a Nyquist diagram

```
>> Nyquist2(Gw*[1,0.35],1.5);
```




or inverse-Nyquist diagram

```
InverseNyquist2(Gw*[1,0.35],1.5);
```




*it would look better with more points (take more measurements or do a spline curve fit to add more points)*

## Gain and Lead Compensation

Problem 4 & 5) Assume

$$G(s) = \left(\frac{300}{s(s+3)(s+8)(s+12)}\right)$$

4) Design a gain compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

For a 40 degree phase margin, at some frequency

$$GK(j\omega) = 1\angle - 140^0$$

Step 1: Find where G(jw) = -140 degrees

$$G(j1.7155) = 0.5120\angle - 140^0$$

so

$$k = \frac{1}{0.5120} = 1.9599$$

This should results in...

Phase margin = 40 degrees

$$M_m \approx \left(\frac{1\angle -140^0}{1+1\angle -140^0}\right) = 1.4619$$

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

$$\zeta = 0.3678$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 28.86\%$$

The closed-loop dominant pole is approximately:

$$s = -a + jb$$

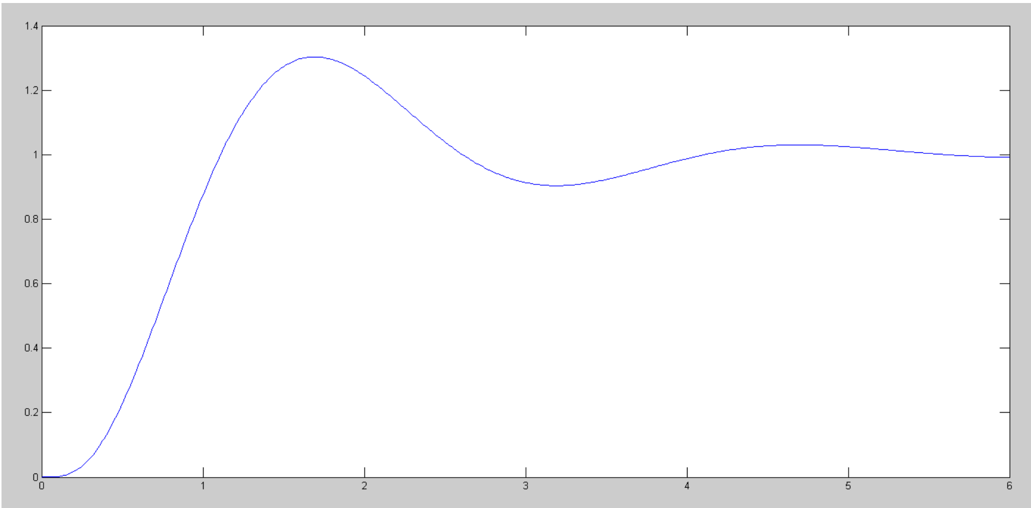
$$b = 1.7155$$ *the complex part of the pole is approximately the resonance freq*

$$a = \frac{b}{\tan(\arccos(\zeta))} = 0.6785$$

$$T_s = \frac{4}{a} = 5.89 \text{ seconds}$$

Ckecking in Matlab:

```
>> G = zpk([],[0,-3,-8,-12],300)

         300
--------------------
s (s+3) (s+8) (s+12)

>> k = 1.9599;
>> t = [0:0.01:6]';

>> Gcl = minreal(G*k / (1+G*k));
>> y = step(Gcl, t);
>> max(y)

ans =    1.3033

>> plot(t,y)
```




30.33% overshoot

- designed for 28.86% overshoot
- designing for a phase margin is usually slight agressive

2% settling time is about 5.89 seconds

5) Design a lead compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

$$G(s) = \left( \frac{300}{s(s+3)(s+8)(s+12)} \right)$$

Let

$$K(s) = k\left( \frac{s+3}{s+30} \right)$$

Find the frequency where the phase of GK = -140 degrees

$$GK = \left( \frac{300k}{s(s+8)(s+12)(s+30)} \right)$$

$$GK(j3.7836) = 0.0235k\angle -140^0$$

Pick k to make the gain one at this frequency

$$k = \frac{1}{0.0235} = 42.46$$

so

$$\boxed{K(s) = 42.46\left( \frac{s+3}{s+30} \right)}$$

The closed-loop response should have

- 28.86% overshoot (same phase margin)

$$s = -a + jb$$

$$b = 3.7876$$

$$a = \frac{b}{\tan(\arccos(\zeta))} = 1.4981$$

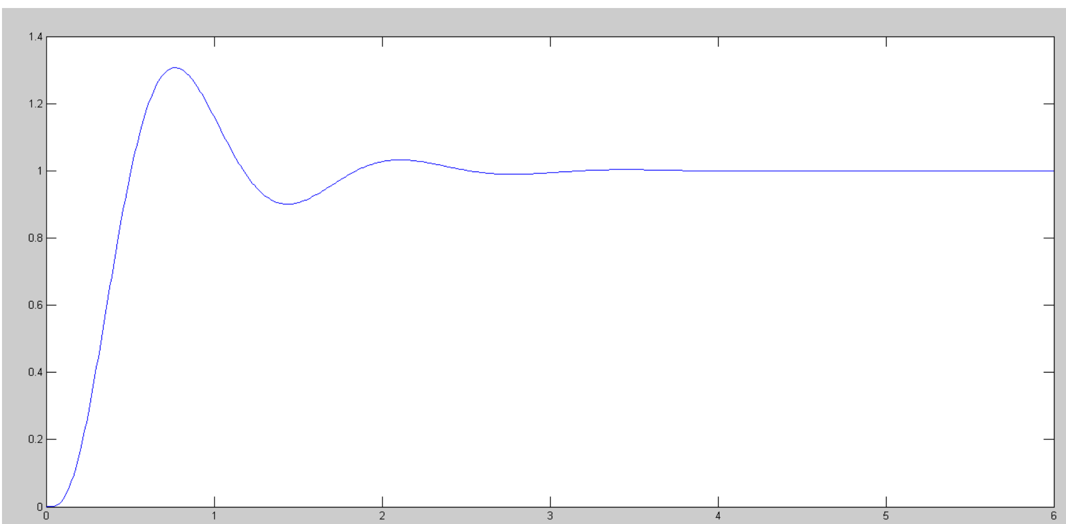
$$T_s = \frac{4}{1.4981} = 2.67 \text{ seconds}$$

Checking in Matlab

```
>> G = zpk([],[0,-3,-8,-12],300)

           300
--------------------
s (s+3) (s+8) (s+12)

>> K = zpk(-3,-30,42.46)

42.46 (s+3)
-----------
  (s+30)

>> Gcl = minreal(G*K / (1+G*K));
>> t = [0:0.01:6]';
>> y = step(Gcl, t);
>> max(y)

    1.3065

>> plot(t,y)
```

The step response is about as expected:

- Settling time is about 2.67 seconds as expected
- Overshoot is also about 28.86%



Problem 6 & 7) Assume a 200ms delay is added

$$G(s) = \left(\frac{300}{s(s+3)(s+8)(s+12)}\right) e^{-0.2s}$$

6) Design a gain compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

Same procedure as before. Search along the jw axis until the phase is -140 degrees

$$G(j1.2052) = 0.7891\angle -140^0$$

Pick k to make the gain one

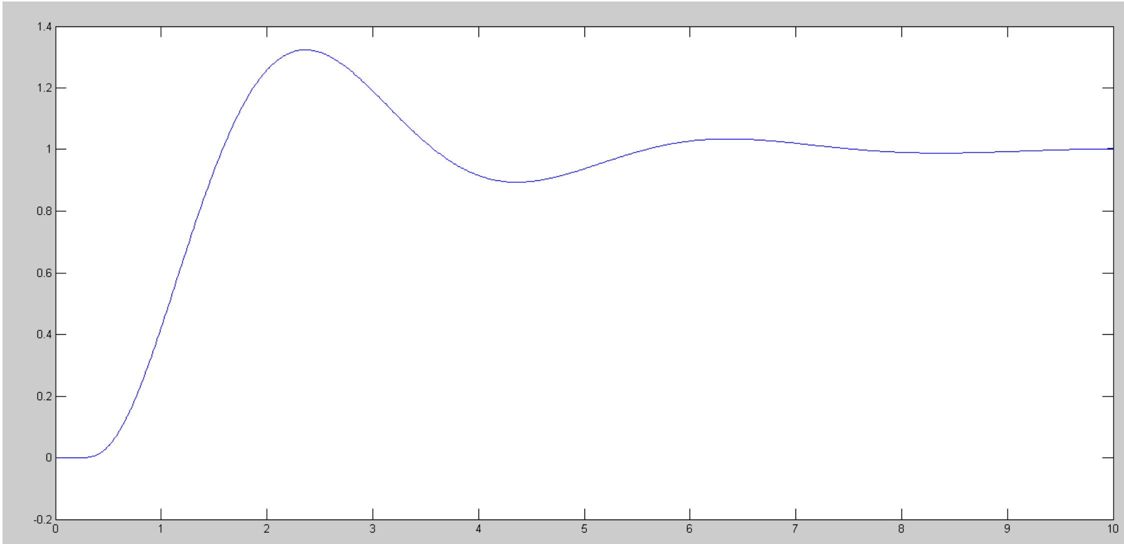
$$k = \frac{1}{0.7891} = 1.2673$$

This should result in

- $OS = 28.88\%$ (same phase margin as befoe)
- $real(s) = \frac{1.2052}{\tan(\arccos(\zeta))} = 0.4767$
- $T_s = \frac{4}{0.4767} = 8.39$ sec

Checking in Matlab (use a Pade approximation for a delay)

```
>> G = zpk([],[0,-3,-8,-12],300)

Zero/pole/gain:
        300
--------------------
s (s+3) (s+8) (s+12)

>> [num,den] = pade(0.2,6);
>> D = tf(num,den);
>> k = 1.2673;
>> Gcl = minreal(G*D*k / (1+G*D*k));
>> t = [0:0.01:10]';
>> y = step(Gcl, t);
>> max(y)

ans =    1.3235

>> plot(t,y)
>>
```




The results are about as expected

- 32.3% overshoot (vs. 28.8%)
- Ts is about 8.39 seconds

Note: Delays are not an issue when working in the frequency domain (unlike root-locus)

7) Design a lead compensator that results in a 40 degree phase margin.

- Check the resulting step response in Matlab

$$G(s) = \left(\frac{300}{s(s+3)(s+8)(s+12)}\right) e^{-0.2s}$$

Let

$$K(s) = k\left(\frac{s+3}{s+30}\right)$$

so

$$GK = \left(\frac{300k}{s(s+8)(s+12)(s+30)}\right) e^{-0.2s}$$

Search along the jw axis until the phase is -140 degrees

$$G(j1.9907) = 0.05\angle -140^0$$

meaning

$$k = \frac{1}{0.05} = 20.01$$

and

$$\boxed{K(s) = 20.01\left(\frac{s+3}{s+30}\right)}$$

This should result in

28.88% overshoot (40 degree phase margin)

$$real(s) = \frac{1.9907}{\tan(\arccos(\zeta))} = 0.7874$$

$$T_s = \frac{4}{0.7874} = 5.08 \text{ seconds}$$

Checking in Matlab

```
>> G = zpk([],[0,-3,-8,-12],300)

        300
--------------------
s (s+3) (s+8) (s+12)

>> K = zpk(-3,-30,20.01)

20.01 (s+3)
-----------
  (s+30)

>> Gcl = minreal(G*D*K / (1+G*D*K));
>> t = [0:0.01:10]';
>> y = step(Gcl, t);
>> max(y)

ans =    1.3434

>> plot(t,y)
```

The results are about as expected

- 34.34% overshoot (vs. 28.88% expected)
- Ts is about 5.08 seconds
- The delay forces you to back off on the gain giving you a slower system