Homework #11: ECE 461/661

Digital PID Control. Due Monday, November 18th

PID Control

Assume T = 0.1 seconds:

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

1) Design a digital I controller

$$K(z) = k\left(\frac{z}{z-1}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Solution: Use numerical methods to find where the phase is 180 degrees along the damping line:

$$G(s) \cdot ZOH \cdot K(z) = 1\angle 180^0$$

$$\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{kz}{z-1}\right) = 1\angle 180^0$$

Searching along the damping line: $\zeta = 0.4559$

$$s = -1.1363 + j2.2726$$

$$z = e^{sT} = 0.8696 + j0.2011$$

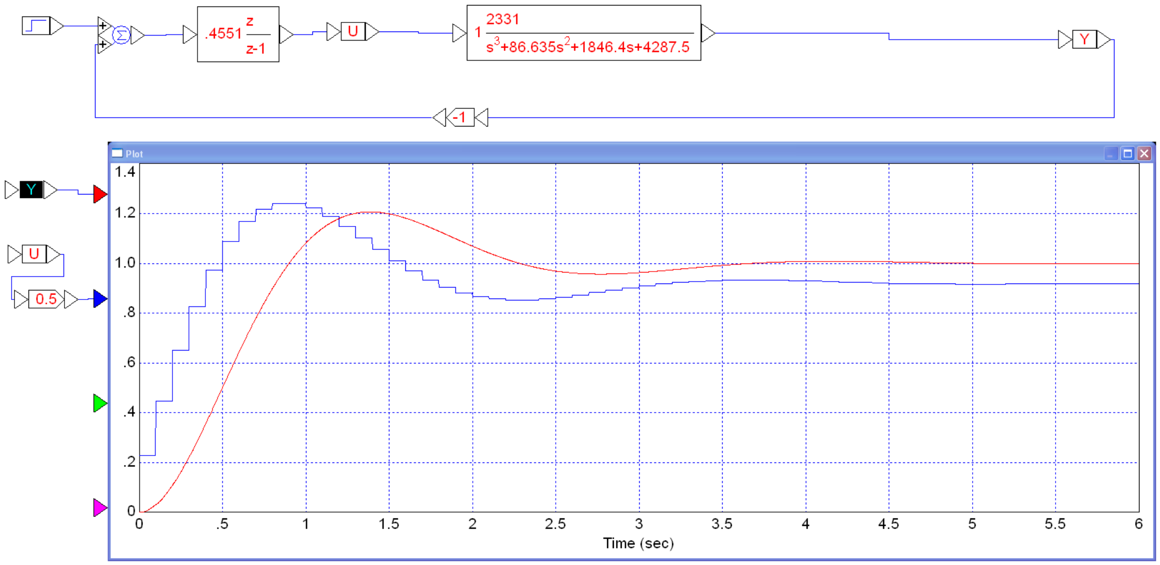
$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{kz}{z-1}\right)\right)_s = 2.1970k\angle 180^0$$

$$k = \frac{1}{2.1970} = 0.4551$$

so

$$\boxed{K(z) = 0.4551\left(\frac{z}{z-1}\right)}$$

Checking the step response in VisSim



Note:

- The overshoot is 20%
- The stead-state error is zero (type-1 system)

2) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Design a digital PI controller

$$K(z) = k\left(\frac{z-a}{z-1}\right)$$

that results in 20% overshoot in the step response. Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Pick 'a' to cancel the pole at s = -2.6388

$$z = e^{sT} = 0.7681$$

The plant + sample & hold + controller is then

$$\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{z-0.7681}{z-1}\right) = 1\angle 180^0$$

Search along the damping line until the angle is 180 degrees

$$s = -5.2219 + j10.4437$$

$$z = e^{sT} = 0.2981 + j0.5129$$

To find k

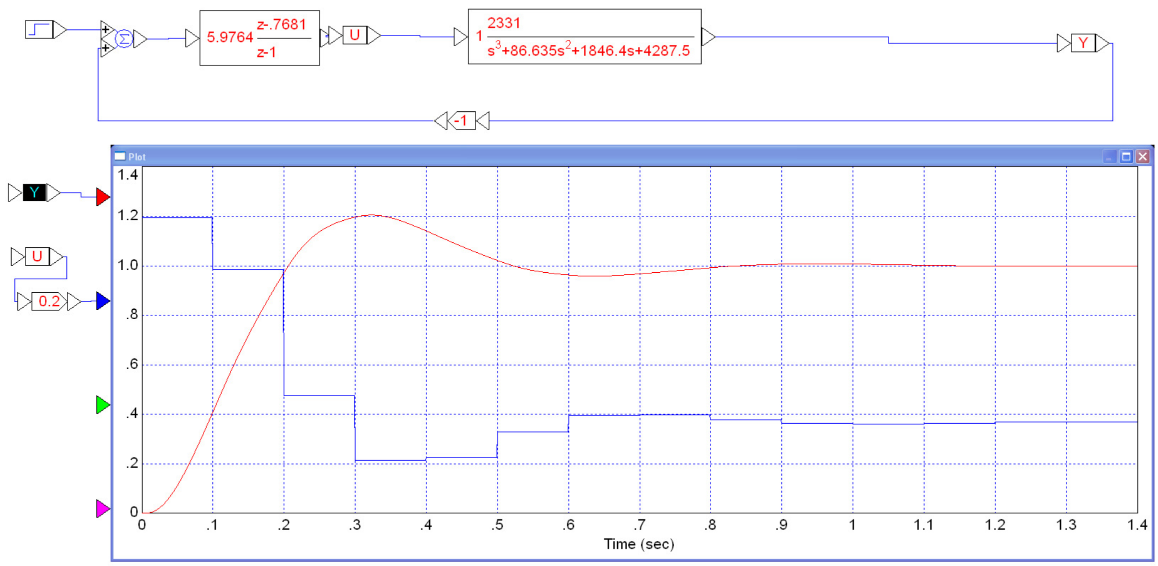
$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{z-0.7681}{z-1}\right)\right)_s = 0.1673k\angle 180^0$$

$$k = \frac{1}{0.1673} = 5.9764$$

and

$$\boxed{K(z) = 5.9764\left(\frac{z-0.7681}{z-1}\right)}$$

Checking in VisSim



Note:

- 20% overshoot
- 0.8 second settling time

3) Assume T = 0.1 seconds and

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Design a digital PID controller

$$K(z) = k\left(\frac{(z-a)(z-b)}{z(z-1)}\right)$$

that results in 20% overshoot in the step response.

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Pick the zeros to cancel the poles at {-2.6448, -30.2062}

$$s = -2.6448 \qquad z = e^{sT} = 0.7676$$

$$s = -30.2062 \qquad z = e^{sT} = 0.0488$$

K(z) is then of the form

$$K(z) = k\left(\frac{(z-0.7676)(z-0.0488)}{z(z-1)}\right)$$

note: This is almost pole-zero cancellation. This is telling you that you need to use a faster sampling rate for a PID controller to have much improvement over a PI)

The plant + sample & hold + controller is then

$$\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{(z-0.7676)(z-0.0488)}{z(z-1)}\right) = 1\angle 180$$

Search along the damping line until the angle is 180 degrees

$$s = -5.5801 + j11.1601$$

$$z = e^{sT} = 0.2514 + j0.5142$$

At this point

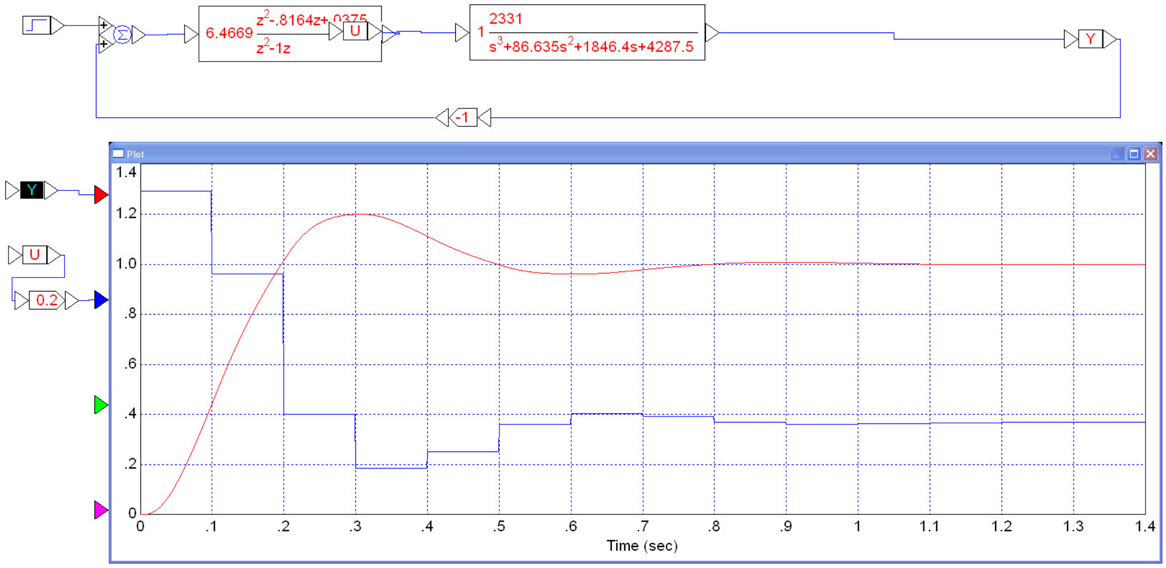
$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{(z-0.7676)(z-0.0488)}{z(z-1)}\right)\right)_s = -0.1546k$$

To find k

$$k = \frac{1}{0.1546} = 6.4669$$

so

$$\boxed{K(z) = 6.4669\left(\frac{(z-0.7676)(z-0.0488)}{z(z-1)}\right)}$$



6.4669 z^2-.8164z+.0375 / z^2-1z
U
1 2331 / s^3+86.635s^2+1846.4s+4287.5
Y
-1
Plot
Y
U
0.2
1.4
1.2
1.0
.8
.6
.4
.2
0
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1 1.1 1.2 1.3 1.4
Time (sec)

Meeting Design Specs

4) Assume a sampling rate of T = 0.1 seconds and

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Design a digital controller that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 1 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Translation:

- Make this a type-1 system
- Place the closed-loop dominant pole at s = -4 + j8
- Or in the z-plane: z = 0.4670 + j0.4809

Pick K(z) to be of the form

$$K(z) = k\left(\frac{(z-0.7676)(z-0.0488)}{(z-1)(z-a)}\right)$$

The open-loop system is then

$$G(s)\cdot ZOH \cdot K(z) = 1\angle 180^0$$

$$\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot \exp\left(\frac{-sT}{2}\right)\cdot k\left(\frac{(z-0.7676)(z-0.0488)}{(z-1)(z-a)}\right)$$

Analyze what we know

$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot \exp\left(\frac{-sT}{2}\right)\cdot\left(\frac{(z-0.7676)(z-0.0488)}{(z-1)}\right)\right)_{s=-4+j8}$$

$$= 0.1278\angle -115.6617^0$$

For the phase to add up to 180 degrees, the pole at z=a must provide a phase shift of

$$\angle(z-a) = 64.3383^0$$

Using some trig:

$$a = 0.4670 - \frac{0.4809}{\tan\left(64.3383^0\right)} = 0.2360$$

Analyzing what we know:

$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot \exp\left(\frac{-sT}{2}\right)\cdot\left(\frac{(z-0.7676)(z-0.0488)}{(z-1)(z-0.2360)}\right)\right)_{s=-4+j8}$$

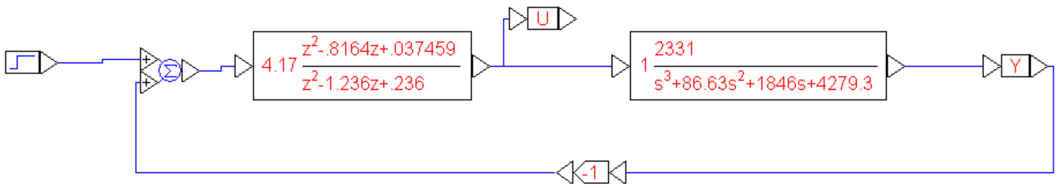
$$= 0.2396\angle 180^0$$

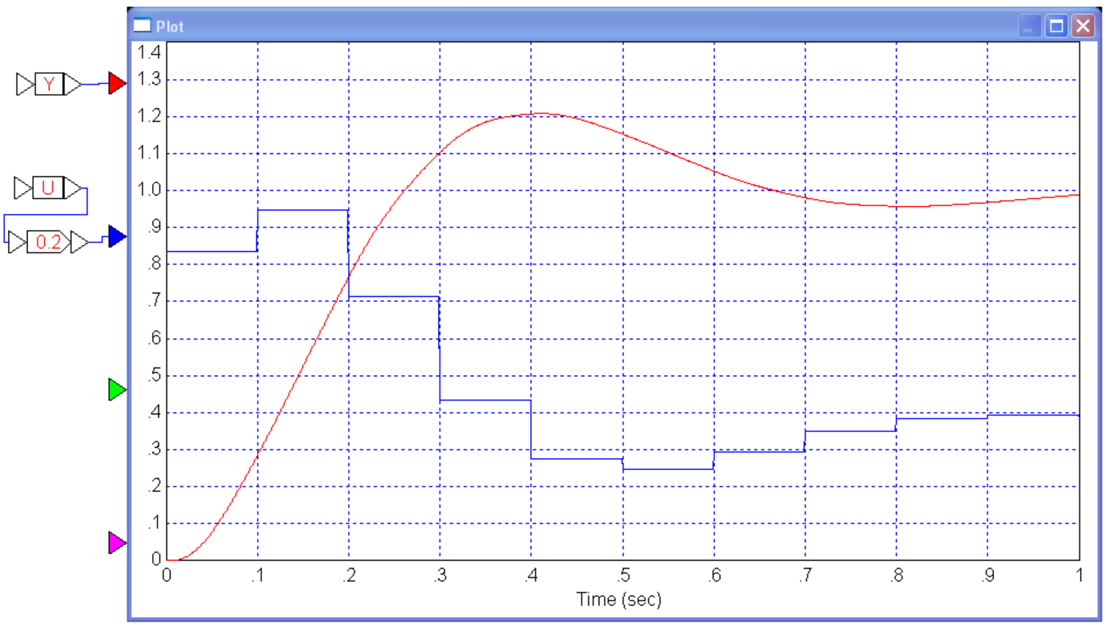
so

$$k = \frac{1}{0.2396} = 4.1737$$

and

$$K(z) = 4.1737\left(\frac{(z-0.7676)(z-0.0488)}{(z-1)(z-0.2360)}\right)$$





5) Assume

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Design a digital controller with T = 0.05 seconds that results in

- No error for a step input
- 20% overshoot for the step response, and
- A 2% settling time of 1 seconds

Simulate the step response of the closed-loop system (VisSim or Simulink preferred with K(z)*G(s))

Note: Changing the sampling rate is a big deal: it means a complete redesign of K(z)

Translation:

- Make this a type-1 system
- Place the closed-loop dominant pole at s = -4 + j8
- Or in the z-plane: z = 0.7541 + j0.3188

Pick K(z) to be of the form

$$K(z) = k\left(\frac{(z-0.8766)(z-0.2208)}{(z-1)(z-a)}\right)$$

note: The zeros moved since the sampling rate changed. They're still defined as z = exp(sT)

The open-loop system is then

$$\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot\exp\left(\frac{-sT}{2}\right)\cdot k\left(\frac{(z-0.8766)(z-0.2208)}{(z-1)(z-a)}\right)$$

Analyze what we know

$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot\exp\left(\frac{-sT}{2}\right)\cdot\left(\frac{(z-0.8766)(z-0.2208)}{(z-1)}\right)\right)_{s=-4+j8}$$

$$= 0.1092\angle-123.01^0$$

For the phase to add up to 180 degrees, the pole at z=a must provide a phase shift of

$$\angle(z-a) = 56.994^0$$

Using some trig:

$$a = 0.7541 - \frac{0.3188}{\tan\left(56.994^0\right)} = 0.5470$$

Analyzing what we know:

$$\left(\left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)\cdot \exp\left(\frac{-sT}{2}\right)\cdot\left(\frac{(z-0.8766)(z-0.2208)}{(z-1)(z-0.5470)}\right)\right)_{s=-4+j8}$$

$$=0.31848\angle 180^0$$

so

$$k=\frac{1}{0.31848}=3.14$$

and

$$K(z)=3.14\left(\frac{(z-0.8766)(z-0.2208)}{(z-1)(z-0.5470)}\right)$$

note:

- *The response (red line) is almost unchanged. It should be: it's the same design requirements*
- *It was a complete redesign for K(z) due to changing the sampling rate*