Homework #9: ECE 461/661

PID, Meeting Specs, Delays. Due Monday, October 28th
20 points per problem

PID:

A 3rd-order model for the following 10-stage RC filter is

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)} \right)$$

I Compensation

1) Design an I compensator, , which results in 20% overshoot for a step input. For this K(s), determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

Give an op-amp circuit to implement K(s)

Step 1) Sketch the root locus of G*K

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)} \right) \left(\frac{k}{s} \right)$$

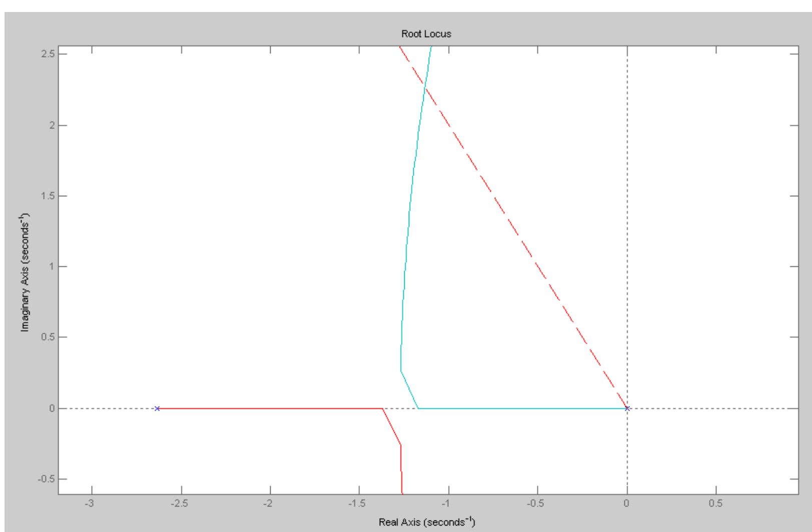
In Matlab

```
>> GK = zpk([],[0,-2.6338,-30.2062,-53.7896],2331)

                 2331
  --------------------------------
  s (s+2.634) (s+30.21) (s+53.79)

>> k = logspace(-2,2,200)';
>> rlocus(GK,k);
>> hold on
>> plot([0,-5],[0,10],'r--')
```

Step 2) Find the spot on the root locus where the damping is 0.4559

zooming in

s = -1.134 + j2.268



Step 3) Find k so that GK = -1 at this point

```
>> s = -1.134 + j*2.268;
>> evalfr(GK,s)
  -0.2200 + 0.0001i

>> k = 1/abs(ans)
k =     4.5459
```

so

$$K(s) = \left(\frac{4.5459}{s}\right)$$

Check the closed-loop step response

```
>> G = zpk([],[-2.6338,-30.2062,-53.7896],2331);
>> K = zpk([],0,4.459)

4.459
-----
  s

>> Gcl = minreal(G*K / (1 + G*K))
>> eig(Gcl)

  -1.1370 + 2.2394i
  -1.1370 - 2.2394i
 -30.7283
 -53.6273
```

Calculations:

The closed-loop dominant pole(s)

s = -1.1370 + 2.2394i

The 2% settling time,

$$T_s = \frac{4}{1.1370} = 3.518 \text{ seconds}$$

The error constant, Kp, and

Infinity (this is a type-1 system)

The steady-state error for a step input.

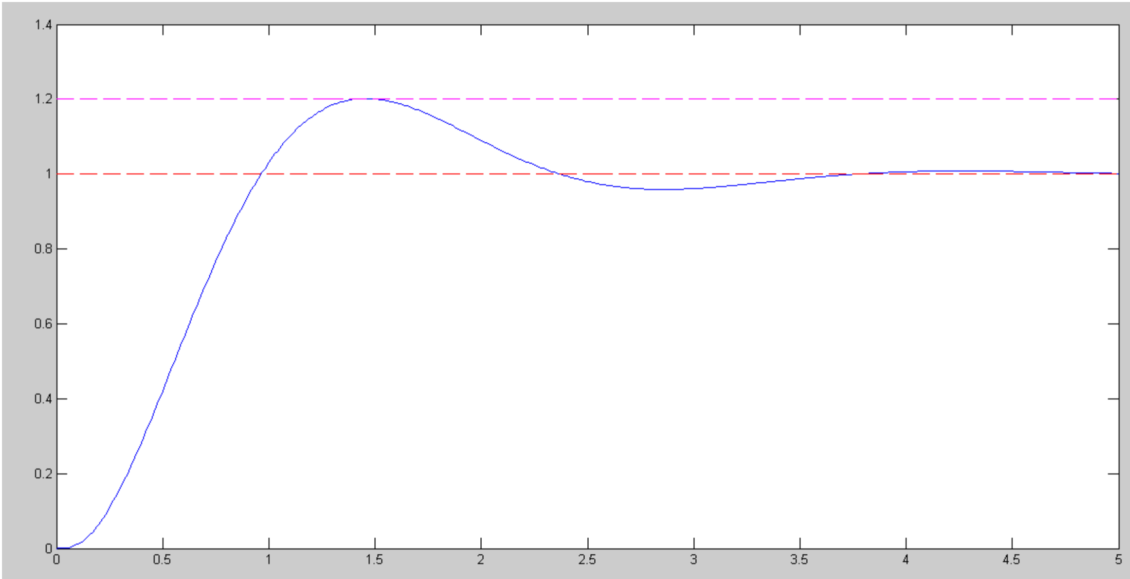
Zero (this is a type-1 system)

Taking the step response of the closed-loop system:

```
>> t = [0:0.01:5]';
>> y = step(Gcl, t);
>> max(y)
    1.2019
```

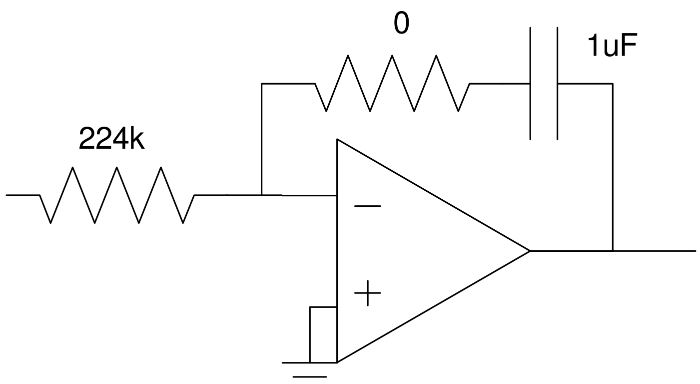
Overshoot is 20.19% (close)

```
>> plot(t,y,t,0*t+1,'r--',t,0*t+1.2,'m--')
```



Design a circuit to implement K(s)

$$K(s) = \left(\frac{4.5459}{s}\right)$$



PI Compensation

2) Design a PI compensator which results in 20% overshoot for a step input.

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Pick K(s) of the form

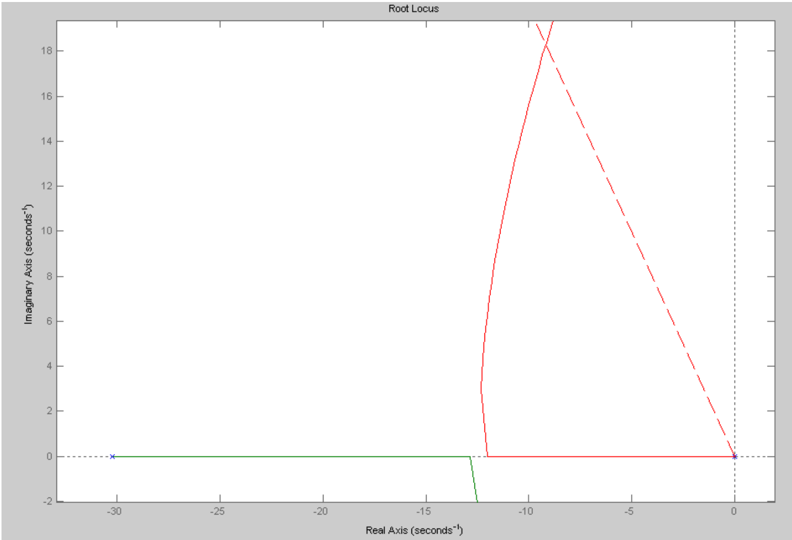
$$K(s) = k\left(\frac{s+2.6338}{s}\right)$$

resultingin GK being

$$GK = \left(\frac{2331k}{s(s+30.2062)(s+53.7896)}\right)$$

Sketch the root locus of GK and find the spot that hits the damping line:

```
>> GK = zpk([],[0,-30.2062,-53.7896],2331)

         2331
---------------------
s (s+30.21) (s+53.79)

>> k = logspace(-2,2,200)';
>> rlocus(GK,k);
>> hold on
>> plot([0,-10],[0,20],'r--')
```



Zoom in to find the point that hits the damping line

s = -9.171 + j18.342

Find k at this point

```
>> s = -9.171 + j*18.342;
>> evalfr(GK,s)

ans =  -0.0844 - 0.0000i
```

```
>> k = 1/abs(ans)

k =   11.8447

>> Gcl = minreal(GK*k / (1 + GK*k));
>> eig(Gcl)

 -65.6535
  -9.1712 +18.3421i
  -9.1712 -18.3421i
```

so

$$K(s) = 11.8447\left(\frac{s+2.6338}{s}\right)$$

Computations

The closed-loop dominant pole(s)

s = -9.1712 +j18.3421

The 2% settling time,

$$T_s = \frac{4}{9.1712} - 436ms$$

The error constant, Kp, and

infinity (type-1 system)

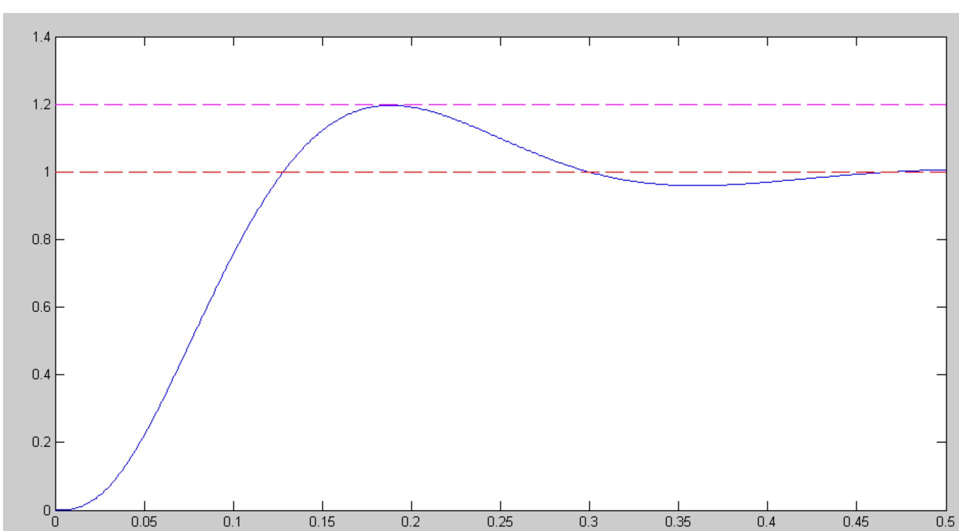
The steady-state error for a step input.

zero (type-1 system)

Closed-Loop Step Response

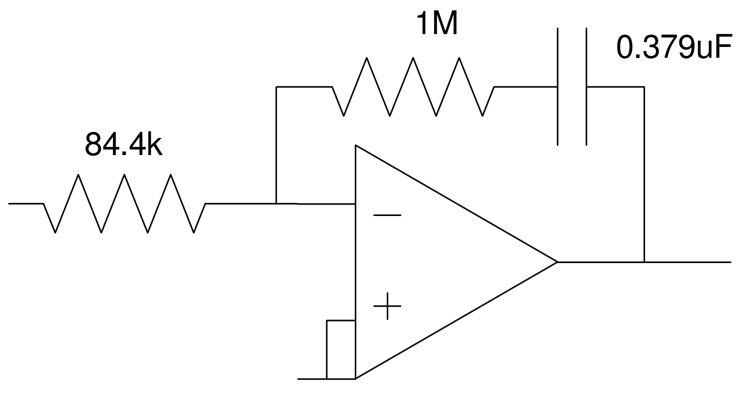
Check your design in Matlab or Simulink or VisSim

```
>> t = [0:0.001:0.5]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'r--',t,0*t+1.2,'m--')
```



Give an op-amp circuit to implement K(s)

$$K(s) = 11.8447\left(\frac{s+2.6338}{s}\right)$$



Meeting Design Specs

3) Design a compensator, K(s), that results in

- No error for a step input
- A 2% settling time of 1 second, and
- 20% overshoot for the step response

Translation:

- Place a pole at s=0 (making it type-1)
- Place the dominant closed-loop pole at s = -4 + j8

Start by cancelling poles until you're too fast

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

$$K(s) = k\left(\frac{(s+2.6338)(s+30.2062)}{s(s+a)}\right)$$

Taking what we know at the design point

$$GK = \left(\frac{2331}{s(s+53.7896)}\right)_{s=-4+j8} = 5.1680\angle -125.6931^0$$

The angle needs to add up to 180 degrees for s = -4+j8 to be on the root locus. This means

$$\angle s + a = 54.3069^0$$

or

$$a = 4 + \frac{8}{\tan\left(54.3069^0\right)} = 9.7471$$

This gives

$$K(s) = k\left(\frac{(s+2.6338)(s+30.2062)}{s(s+9.7471)}\right)$$

$$GK = \left(\frac{2331k}{s(s+53.7896)(s+9.7471)}\right)$$

Evaluate at the design point

$$\left(\frac{2331k}{s(s+53.7896)(s+9.7471)}\right)_{s=-4+j8} = 0.5247k\angle 180^0$$

Pick k to make the gain one

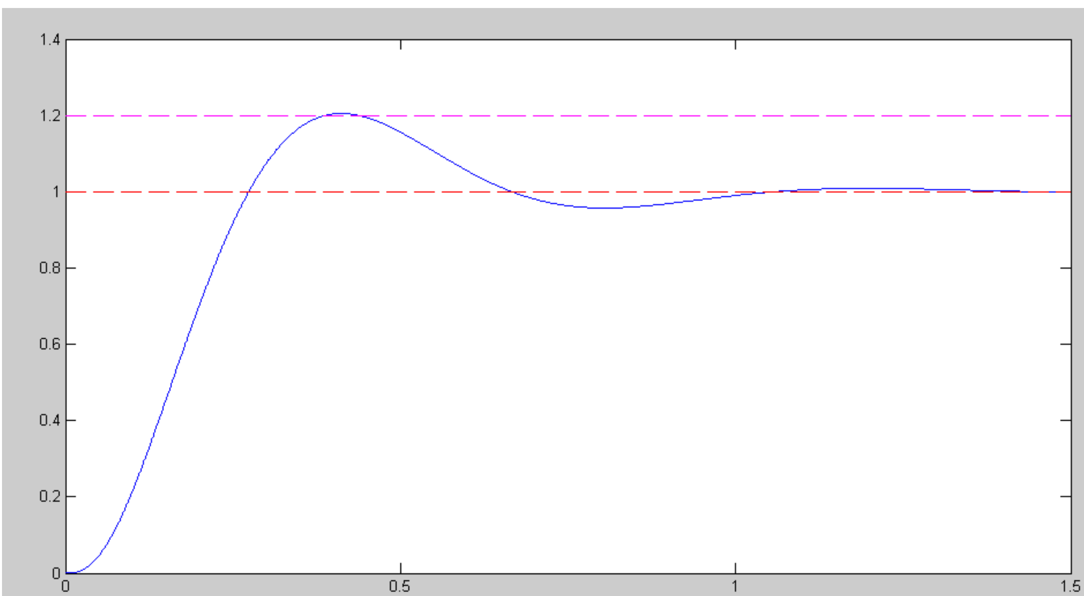
$$k = \frac{1}{0.5247} = 1.9060$$

so

$$\boxed{K(s) = 1.9060\left(\frac{(s+2.6338)(s+30.2062)}{s(s+9.7471)}\right)}$$

Check your design in Matlab or Simulink or VisSim

```
>> G = zpk([],[-2.6338,-30.2062,-53.7896],2331)

            2331
-----------------------------
(s+2.634) (s+30.21) (s+53.79)

>> K = zpk([-2.6338,-30.2062],[0,-9.7471],1.9060)

1.906 (s+2.634) (s+30.21)
-------------------------
       s (s+9.747)

>> Gcl = minreal(G*K / (1 + G*K));
>> eig(Gcl)

  -4.0000 + 7.9999i
  -4.0000 - 7.9999i
 -55.5367

>> t = [0:0.001:1.5]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'r--',t,0*t+1.2,'m--')
```

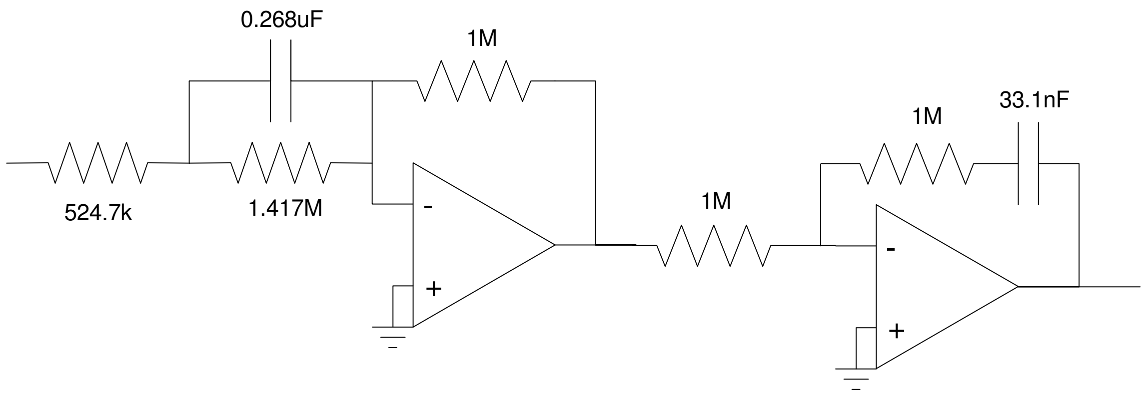


Give an op-amp circuit to implement K(s)

$$K(s) = 1.9060\left(\frac{(s+2.6338)(s+30.2062)}{s(s+9.7471)}\right)$$

Rewrite as a Lead * PI

$$K(s) = \left(1.9060\left(\frac{s+2.6338}{s+9.7471}\right)\right)\left(\frac{s+30.2062}{s}\right)$$



Systems with Delays

4) Assume a 100ms delay is added to the system

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)e^{-0.1s}$$

Design a compensator, K(s), that results in

- No error for a step input
- A 2% settling time of 1 second, and
- 20% overshoot for the step response

Same procedure as problem #3. Let

$$K(s) = k\left(\frac{(s+2.6338)(s+30.2062)}{s(s+a)}\right)$$

At s = -4 + j8

$$GK = \left(\left(\frac{2331k}{s(s+a)(s+53.7896)}\right)e^{-0.1s}\right)_{s=-4+j8} = 1\angle 180^0$$

Evaluate what we know:

$$GK = \left(\left(\frac{2331}{s(s+53.7896)}\right)e^{-0.1s}\right)_{s=-4+j8} = 7.7098\angle -171.5297^0$$

so

$$\angle(s+a) = 8.4703^0$$

$$a = 4 + \frac{8}{\tan\left(8.4703^0\right)} = 57.7198$$

and

$$K(s) = k\left(\frac{(s+2.6338)(s+30.2062)}{s(s+57.7198)}\right)$$

Find k to make the gain one at s = -4 + j8

$$GK = \left(\left(\frac{2331k}{s(s+53.7896)(s+57.7198)}\right)e^{-0.1s}\right)_{s=-4+j8} = 0.1425k\angle 180^0$$

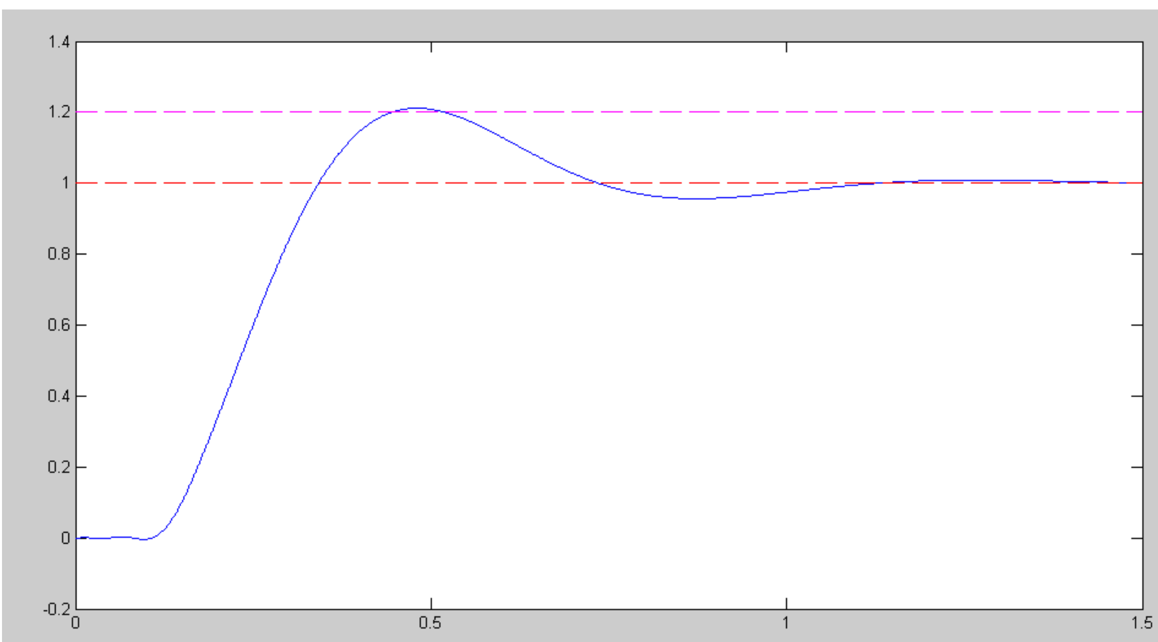
$$k = \frac{1}{0.1425} = 7.0189$$

and

$$\boxed{K(s) = 7.0189\left(\frac{(s+2.6338)(s+30.2062)}{s(s+57.7198)}\right)}$$

Check your design in Matlab or Simulink or VisSim

```
>> [num,den] = pade(0.1,4);
>> D = tf(num,den);
>> G = zpk([],[-2.6338,-30.2062,-53.7896],2331)

               2331
-----------------------------
(s+2.634) (s+30.21) (s+53.79)

>> K = zpk([-2.6338,-30.2062],[0,-57.7198],7.0189)

7.0189 (s+2.634) (s+30.21)
--------------------------
       s (s+57.72)

>> Gcl = minreal(G*D*K / (1 + G*D*K));
>> eig(Gcl)

  1.0e+002 *

  -0.0402 + 0.0798i
  -0.0402 - 0.0798i
  -0.2529 + 0.5164i
  -0.6784 + 0.6545i
  -0.2529 - 0.5164i
  -0.6784 - 0.6545i
  -1.1721

>> t = [0:0.001:1.5]';
>> y = step(Gcl,t);
>> plot(t,y,t,0*t+1,'r--',t,0*t+1.2,'m--')
```



Give an op-amp circuit to implement K(s)

$$K(s) = 7.0189\left(\frac{(s+2.6338)(s+30.2062)}{s(s+57.7198)}\right)$$

Rewrite as a lead * PI

$$K(s) = 7.0189\left(\frac{s+30.2062}{s+57.7198}\right) \cdot \left(\frac{s+2.6338}{s}\right)$$