Homework #8: ECE 461/661

Root Locus with Complex Poles, Gain, Lead. Due Monday, October 21st

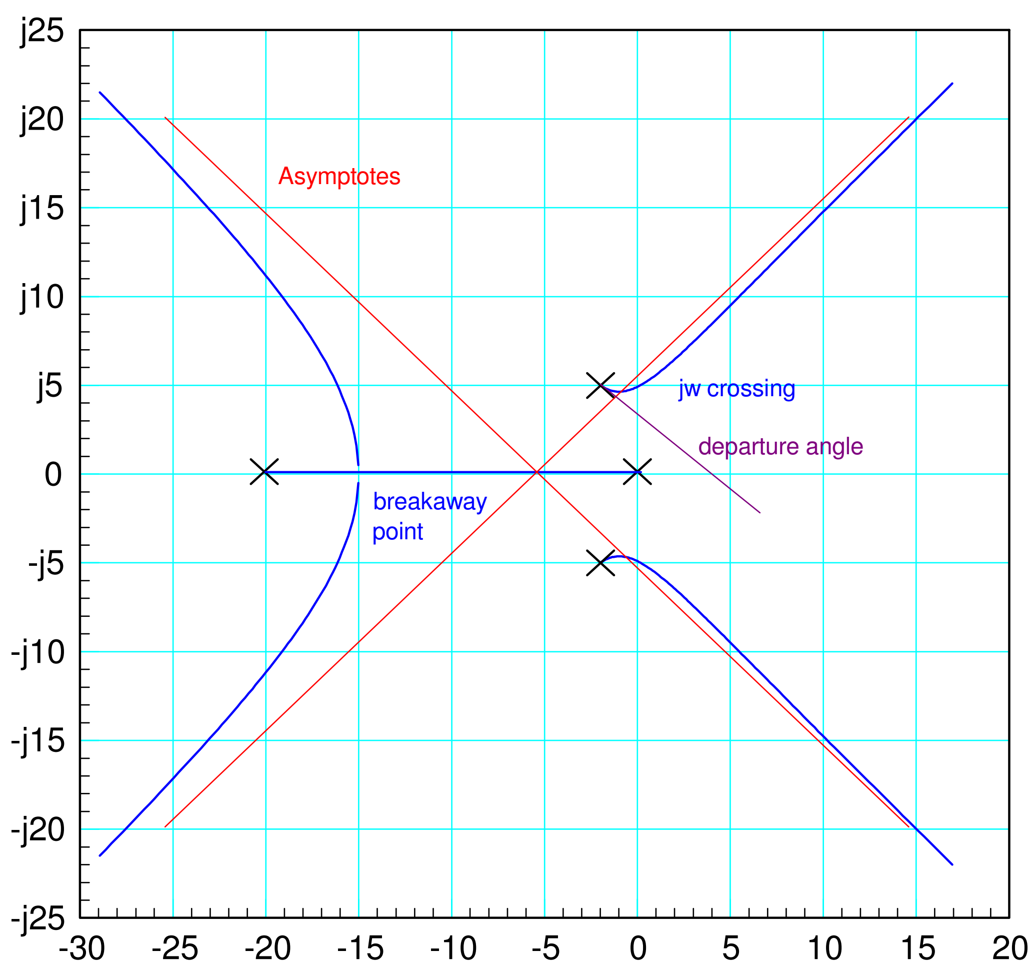
Root Locus with Complex Poles & Zeros

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), asymptotes, and departure/approach angle

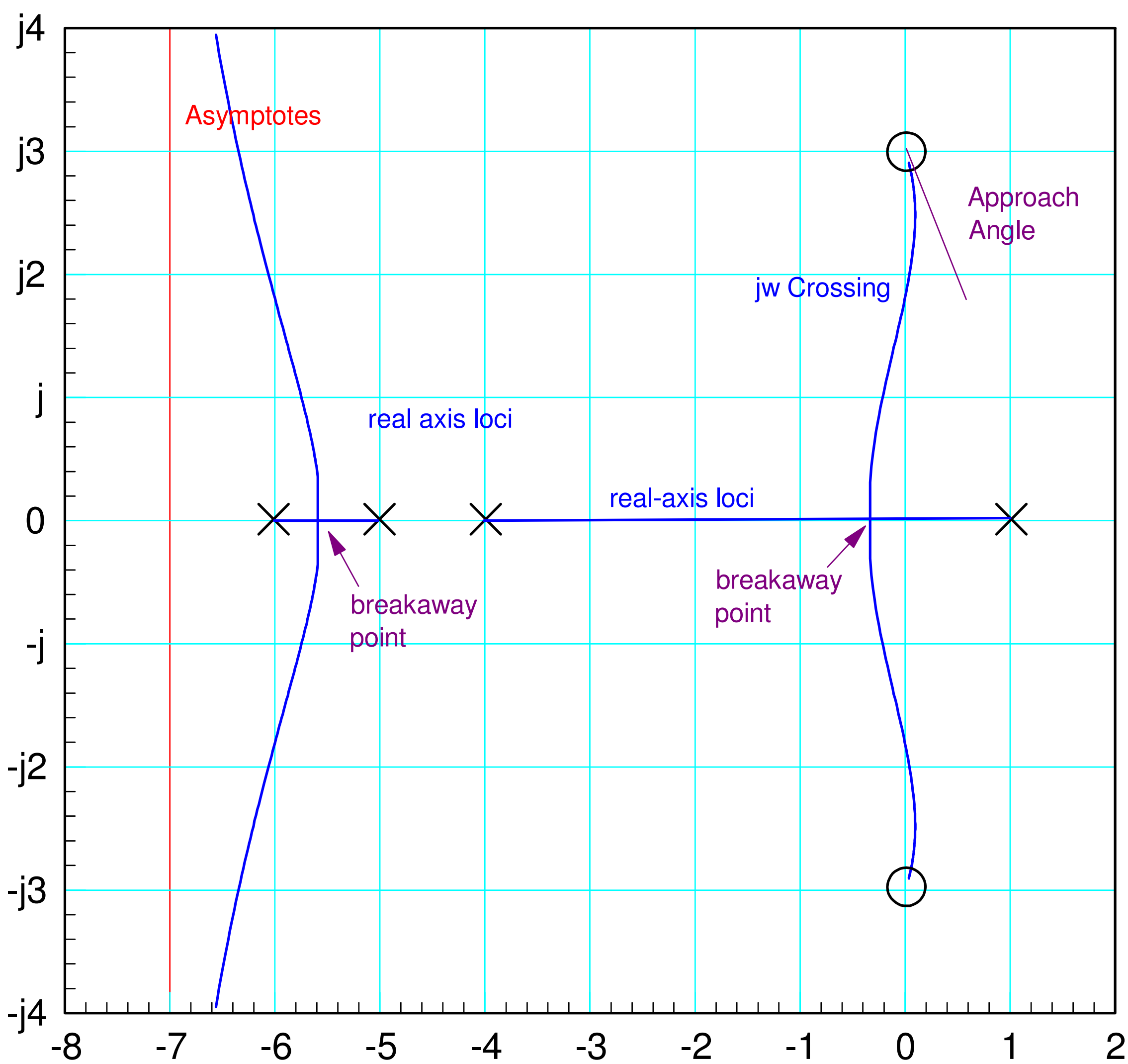
1) $$G(s) = \left(\frac{100}{s(s+20)(s+2+j5)(s+2-j5)}\right)$$

Open-Loop Poles:	0, -20, -2+j5, -2-j5
Real Axis Loci:	(0, -20)
Breakaway Points:	-15.014
jw Crossing	j4.9160, -j4.9160
Asymptotes	4
Asymptote Angle	+/- 45 degrees, +/- 135 degrees
Asumptote Intersect	-6
Departure Angle	-37.3255 degrees



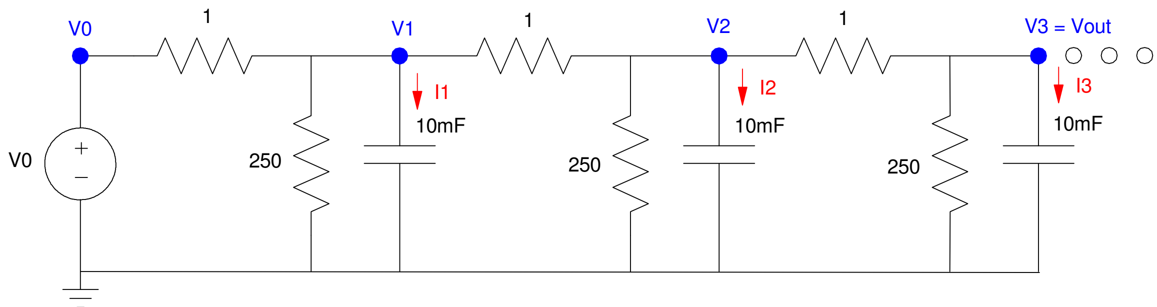
2) $G(s) = \left(\frac{(s+j3)(s-j3)}{(s-1)(s+4)(s+5)(s+6)} \right)$

Open-Loop Poles:	+1, -4, -5, -6
Real Axis Loci:	(+1, -4), (-5, -6)
Breakaway Points:	-5.5655 -0.3464
jw Crossing	j1.8127, -j1.8127
Asymptotes	2
Asymptote Angle	+/- 90
Asumptote Intersect	-7.0
Approach Angle	-67.166 degrees



A 3rd-order model for the following 10-stage RC filter is

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

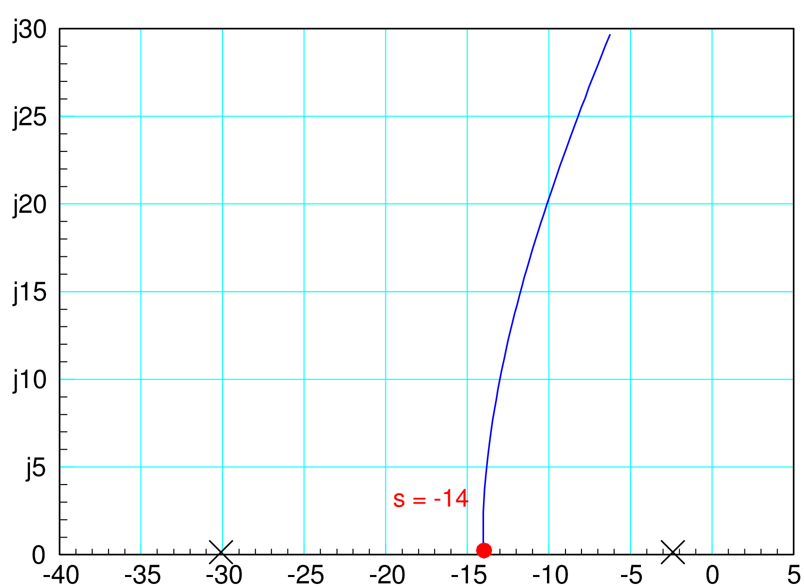


3) Design a gain compensator (K(s) = k) which results in
- The fastest system possible,
- With no overshoot for a step input (i.e. design for the breakaway point)

For this value of k, determine
- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

First, find the design point. Sketching the root locus with help from Matlab:



The breakaway point (highest gain with no overshoot is s = -14 (approx). Design a gain compensator to place the closed-loop dominant pole there.

```
>> G = zpk([],[-2.6338,-30.2062,-53.7896],2331)
```

```
             2331
------------------------------
(s+2.634) (s+30.21) (s+53.79)

>> s = -14;
>> evalfr(G,s)
ans =   -0.3180

>> k = 1/abs(ans)
k =    3.1443

>> Gcl = minreal(G*k / (1+G*k))

        7329.3601
---------------------------
(s+14) (s+14.19) (s+58.44)
```

Results:

Closed-Loop Dominant Pole(s):

s = -14, -14.19

2% settling time

$$T_s = \frac{4}{14}$$

Error Constant Kp

$$K_p = GK_{s=0} = \left(\left(\frac{2331}{(s+2.634)(s+30.21)(s+53.79)}\right) \cdot 3.1443\right)_{s=0}$$

```
>> Kp = evalfr(G,0) * k

Kp =  1.7127
```

Steady-State Error

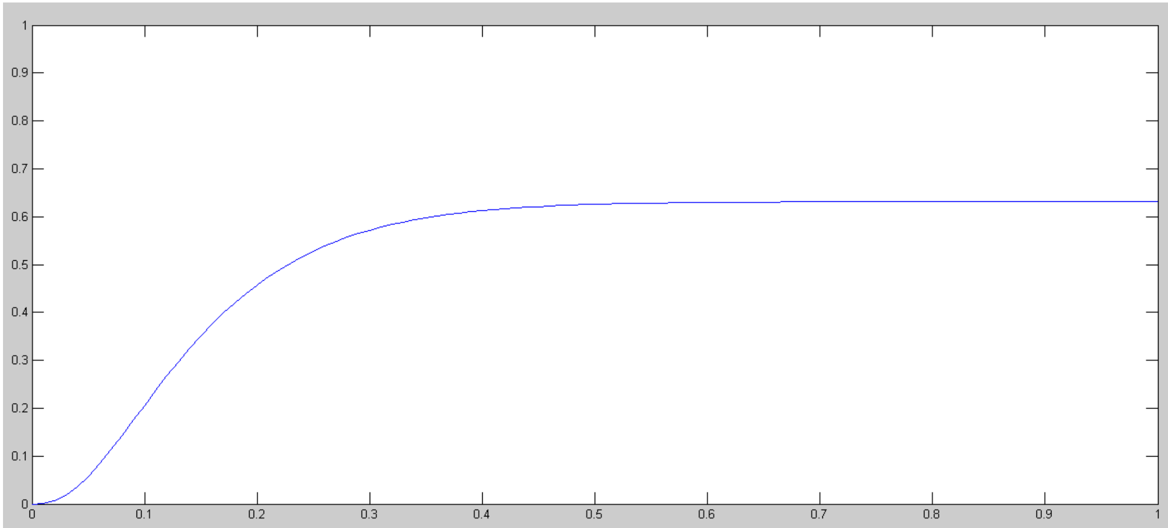
$$e = \frac{1}{K_p+1}$$

```
>> error = 1 / (Kp + 1)

error =    0.3686
```

Verify in Matlab (i.e. take the closed-loop system's step response)

```
>> t = [0:.01:1]';
>> y = step(Gcl, t);
>> plot(t,y)
>> ylim([0,1])
>>
```



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

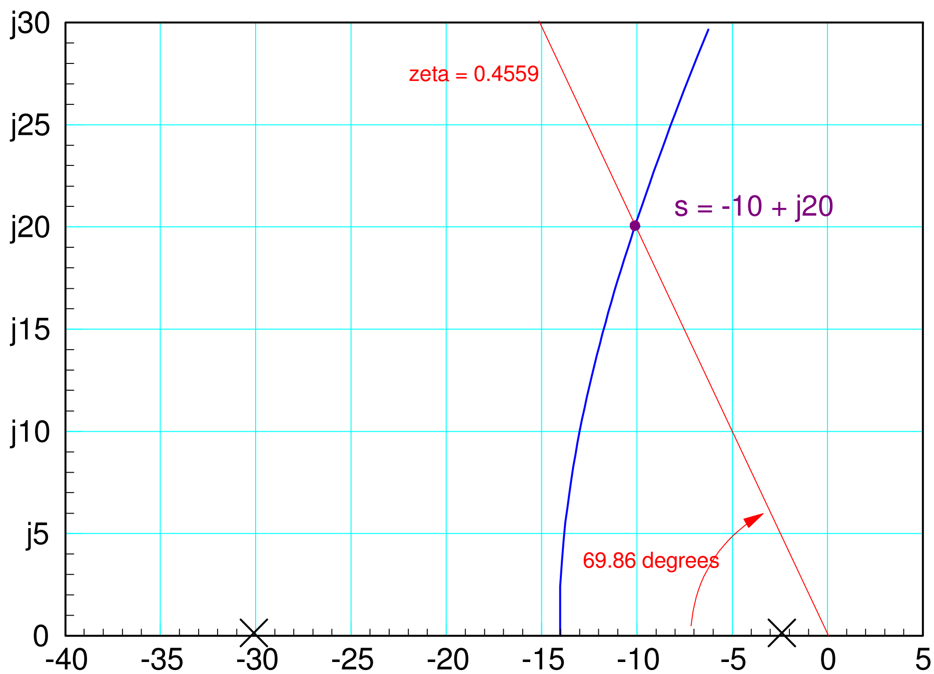
4) Design a gain compensator (K(s) = k) which results in 20% overshoot for a step input. For this value of k, determine

- The closed-loop dominant pole(s)
- The 2% settling time,
- The error constant, Kp, and
- The steady-state error for a step input.

Check your design in Matlab or Simulink or VisSim

First, find your design spot on the root locus plot. For 20% overshoot

- zeta = 0.4559
- angle = 69.86 degrees
- s = -10 + j20



Pick k to make GK = -1 at this point

```
>> s = -10 + j*20;
>> evalfr(G,s)

ans =  -0.0799 - 0.0007i

>> k = 1/abs(ans)

k =   12.5143
```

The closed-loop system is then

```
>> Gcl = minreal(G*k / (1+G*k))

              29170.8455
--------------------------------
(s+66.42) (s^2 + 20.21s + 503.6)
```

The roots are

```
>> eig(Gcl)

 -10.1054 +20.0376i
 -66.4187
 -10.1054 -20.0376i
```

The closed-loop dominant pole(s)

- -10.1053 + j20.0376
- -10.1053 - j20.0376

The 2% settling time,

$$T_s = \frac{4}{10.1054} = 0.3958 \text{ seconds}$$

The error constant, Kp, and

```
>> Kp = evalfr(G,0) * k

Kp =    6.8167
```

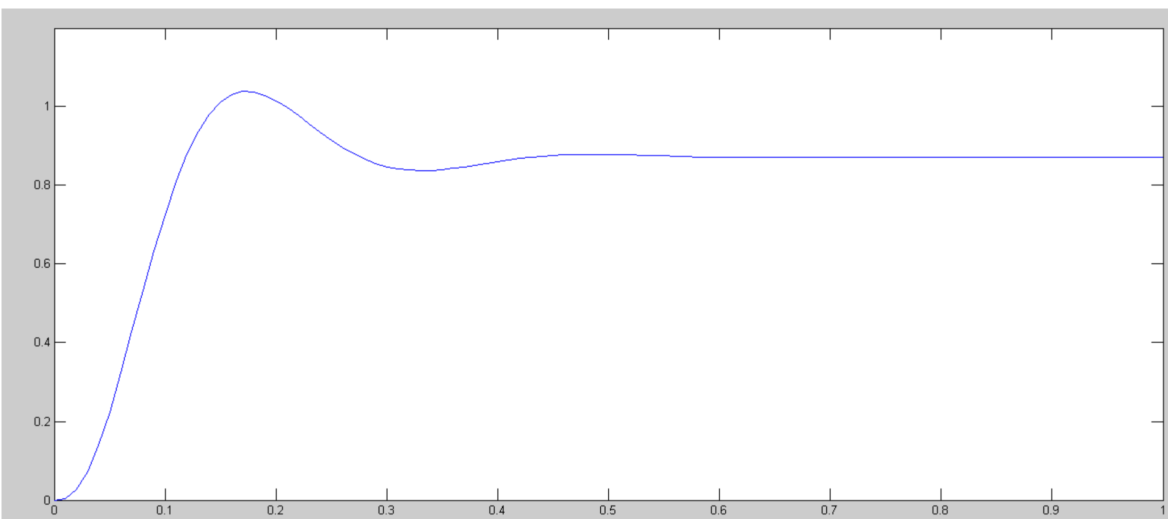
The steady-state error for a step input.

```
>> error = 1 / (Kp + 1)

error =    0.1279
```

Plot the closed-loop step response

```
>> t = [0:.01:1]';
>> y = step(Gcl, t);
>> plot(t,y)
>> ylim([0,1.2])
```




```
>>
```

5)) Design a lead compensator, $K(s) = k\left(\frac{s+a}{s+10a}\right)$, which results in 20% overshoot for a step input.

$$G(s) = \left(\frac{2331}{(s+2.6338)(s+30.2062)(s+53.7896)}\right)$$

Keep the pole at s = -2.63

Cancel the next slowest pole

Replace it with a pole 10x faster

$$K(s) = k\left(\frac{s+30.2}{s+302}\right)$$

resulting in

$$GK = \left(\frac{2331}{(s+2.6338)(s+53.7896)(s+30.2)}\right)$$

Find the spot on the root locus which has a damping ratio of 0.4559

s = -23.216 + j45.433 *found using numerical methods*

Find k at this point. In Matlab:

```
>> K = zpk(-30.2,-302,1)

(s+30.2)
--------
(s+302)

>> GK = G*K

            2331 (s+30.2)
-------------------------------------
(s+2.634) (s+30.21) (s+53.79) (s+302)

>> s = -23.216 + j*45.433;
>> evalfr(GK,s)
  -0.0030 - 0.0000i

k =  330.9893

>> K = zpk(-30.2,-302,k)

330.9893 (s+30.2)
-----------------
     (s+302)

>> Gcl = minreal(G*K / (1+G*K))

          771535.9994 (s+30.2)
----------------------------------------
(s+30.2) (s+311.7) (s^2 + 46.75s + 2613)

>> eig(Gcl)

  -30.20
  -23.38 + 45.46i
  -23.38 - 45.46i
  -311.68
```

The closed-loop dominant pole(s)

```
-23.38 + 45.46i
-23.38 - 45.46i
```

The 2% settling time,

$$T_s = \frac{4}{23.38} = 171ms$$

The error constant, Kp, and

```
>> Kp = evalfr(G,0) * evalfr(K,0)

Kp =   18.0293
```

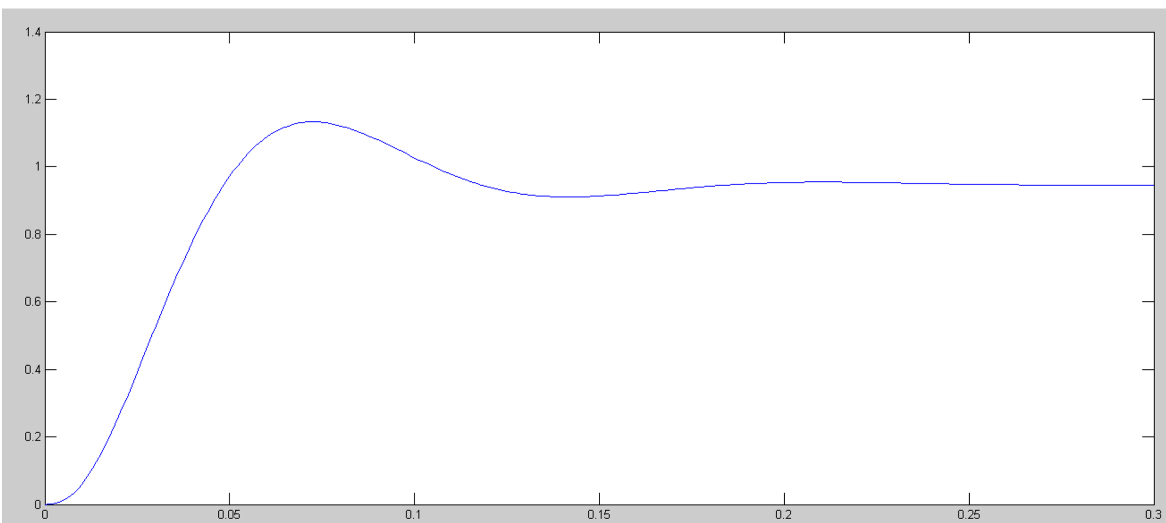
>> The steady-state error for a step input.

```
>> error = 1 / (Kp + 1)

error =     0.0526
```

Plot the closed-loop step response

```
>> t = [0:0.001:0.3]';
>> y = step(Gcl,t);
>> plot(t,y)
>> xlim([0,0.3])
>> ylim([0,1.2])
```



Design a circuit to implent K(s)

$$K(s) = 330.9893\left(\frac{s+30.2}{s+302}\right)$$