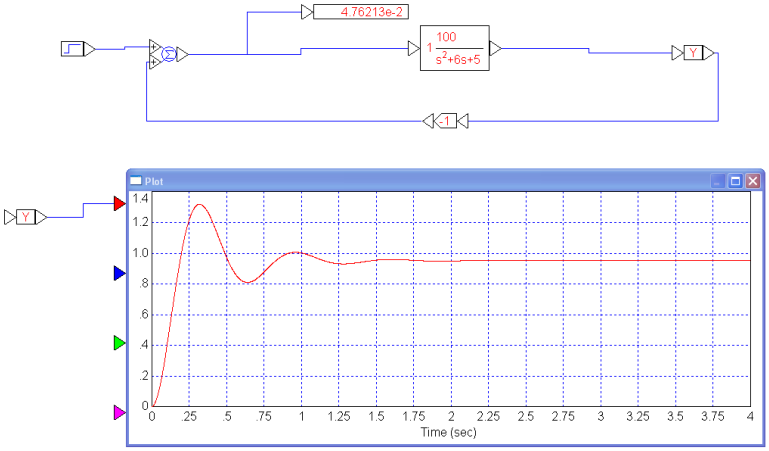
Homework #7: ECE 461/661

Error Constants, Routh Criteria, Skething a Root Locus. Due Monday, October 14th

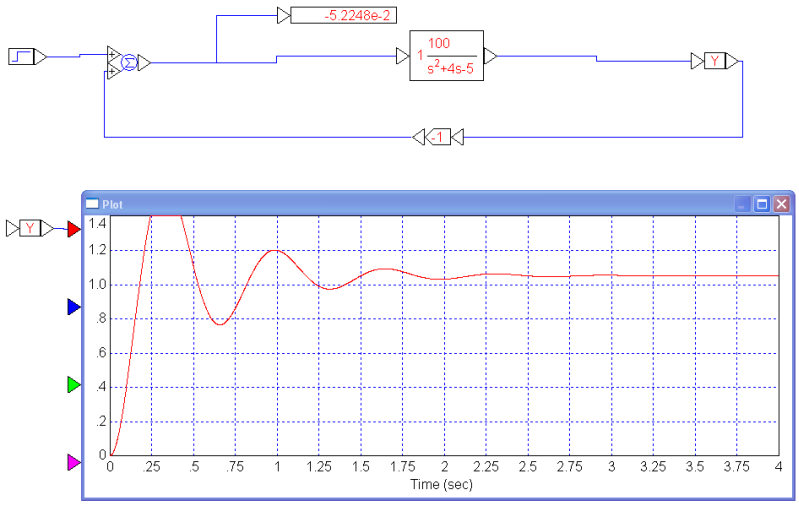
Error Constants

1) Determine the error constants and steady-state error for the following systems

G(s)	System Type	Kp	Kv	Error for a unit step input
$\left(\frac{100}{(s+1)(s+5)}\right)$	0	20	0	1 / 21
$\left(\frac{100}{s(s+1)(s+5)}\right)$	1	infinity	20	0
$\left(\frac{100(s+2)}{s^2(s+1)(s+5)}\right)$	2	infinity	infinity	0
$\left(\frac{100}{(s-1)(s+5)}\right)$	0	-20	0	-1 / 19



Part a)



Part d)

Routh Criteria

Determine the range of k that results in a negative definite polynomial (i.e. a stable system)

2) $(s-1)(s+6)(s+12)+3k=0$

Multiply Out

```
>> poly([1,-6,-12])

ans =     1    17    54   -72
```

$$s^3+17s^2+54s-72+3k=0$$

Set up a Routh table

1	54	0	
17	3k - 72	0	
(a)	0	0	k < 330
3k - 72	0	0	k > 24
0	0	0	

(a)

$$\frac{-\begin{vmatrix} 1 & 54 \\ 17 & 3k-72 \end{vmatrix}}{17} = 58.2353 - 0.1765k$$

For stability, there can be no sign flips

$$58.2353 - 0.1765k > 0$$

$$k < 330$$

$$3k - 72 > 0$$

$$k > 24$$

Result: **24 < k < 330**

3) $s(s+4)(s+6)(s+12)+3k=0$

Multiply out

```
>> poly([0,-4,-6,-12])

ans =     1    22   144   288     0
```

$$s^4+22s^3+144s^2+288s+3k=0$$

Set up a Routh table

1	144	3k	
22	288	0	
130.9091 (a)	3k	0	
288-0.5042k (b)	0	0	k < 571.23
3k	0	0	k > 0
0	0	0	

ans: 0 < k < 571.23

(a)

$$\frac{-\begin{vmatrix} 1 & 144 \\ 22 & 288 \end{vmatrix}}{22}=130.9091$$

(b)

$$\frac{-\begin{vmatrix} 22 & 288 \\ 130.9091 & 3k \end{vmatrix}}{130.9091}=288-0.5042k$$

For no sign flips

$$288-0.5042k>0$$

$$k<571.23$$

$$3k>0$$

$$k>0$$

Sketching a Root Locus

Sketch the root locus plot for the following systems for 0 < k < infinity. Also plot the

- real axis loci, break away points, jw crossings (if any), and asymptotes

4) $(s-1)(s+6)(s+12)+3k=0$

Open-Loop Poles: {+1, -6, -12}

Real Axis Loci: (+1, -6), (-12, -infinity)

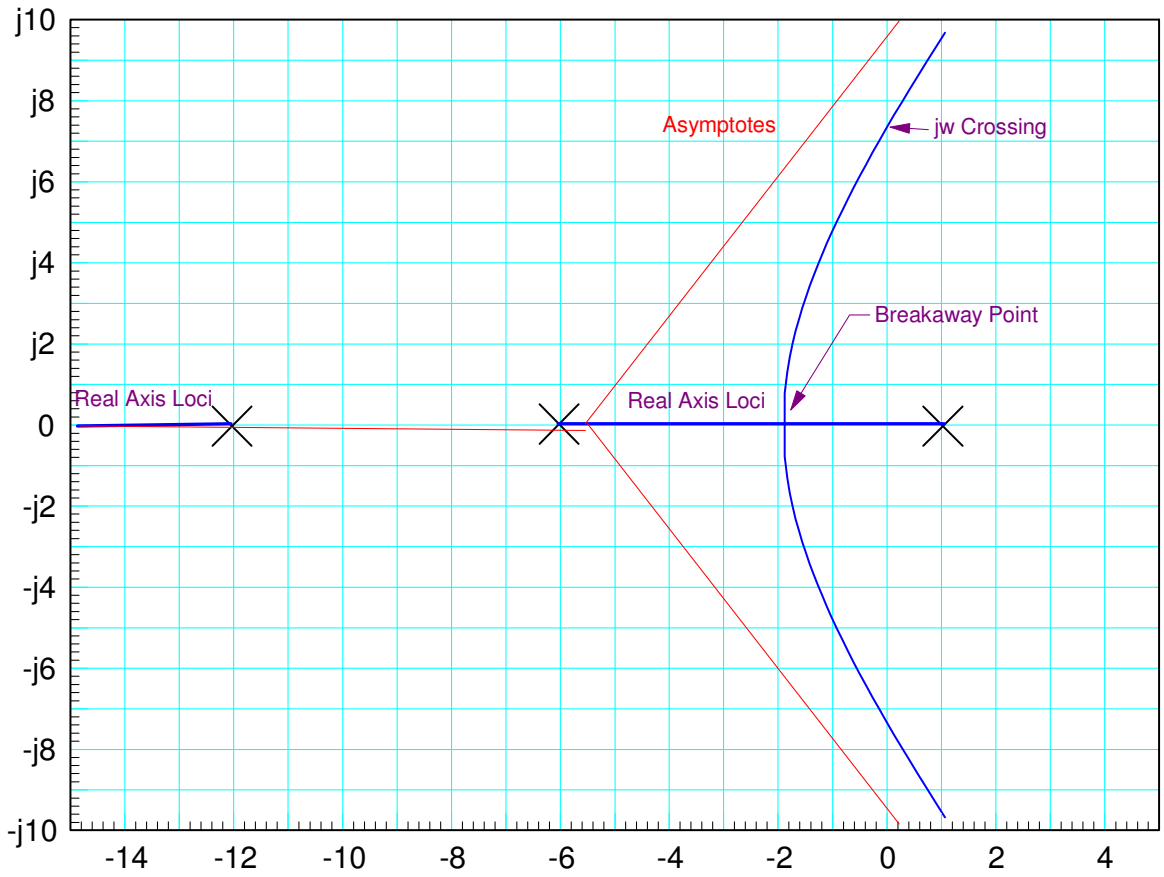
Breakaway Point: s = -1.9102 *found using numerical methods*

jw Crossing s = j7.3485 *found using numerical methods*

Asymptotes 3 asymptotes

+60 degrees, -60 degrees, 180 degrees

Intercept = -17/3



5) $s(s+4)(s+6)(s+12)+3k=0$

Open-Loop Poles: {0, -4, -6, -12}

Real Axis Loci: (0, -4), (-6, -12)

Breakaway Point: s = -1.42454, -10.0469 *found using numerical methods*

jw Crossing s = j3.6181 *found using numerical methods*

Asymptotes 4 asymptotes

+/- 45 degrees, +/- 135 degrees

Intercept = -5.5