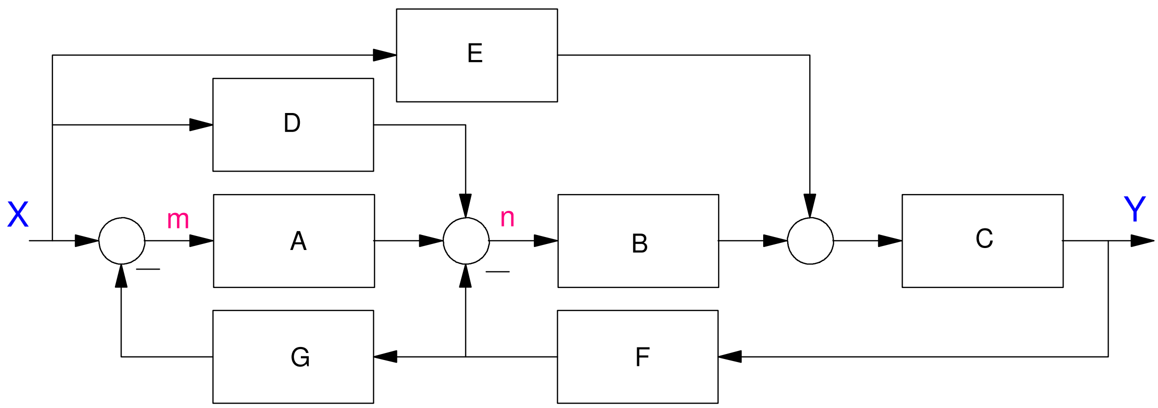
Homework #5: ECE 461/661

Block Diagrams, Canonical Forms, Electrical Circuits.   Due Monday, September 23rd

Block Diagrams

1) Determine the transfer function from X to Y



Shortcut:

$$Y = \left(\frac{ABC+DBC+EC}{1+BCF+ABCFG}\right)X$$

Long Way:

m = X - GFY

n = DX + Am - FY

Y = CEX + CBn

Simplifying...

n = DX + A(X - GFY) - FY

Y = CEX + CB(DX + A(X - GFY) - FY)

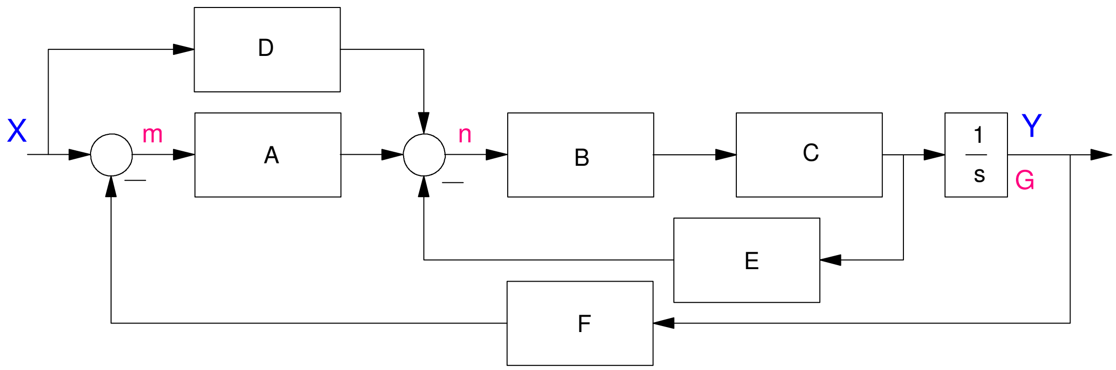
Y = CEX + CBDX + CBAX - CBAGFY - CBFY

(1 + CBAGF + CBF)Y = (CE + CBD + CBA)X

$$Y = \left(\frac{CE+CBD+CBA}{1+CBAGF+CBF}\right)X$$

The answers match

2) Determine the transfer funciton from X to Y



Shortcut

$$Y = \left(\frac{ABCG+DBCG}{1+BCE+ABCGF}\right)X$$

Long Way

m = X - FY

n = DX + Am - ECBn

Y = GCBn

Simplifying

n = DX + A(X - FY) - ECBn

$$(1+ECB)n = DX + AX - AFY$$

$$n = \left(\frac{DX+AX-AFY}{1+ECB}\right)$$

$$Y = GCB\left(\frac{DX+AX-AFY}{1+ECB}\right)$$

$$(1+ECB)Y = GCB(DX+AX-AFY)$$

$$(1+ECB+GCBAF)Y = GCBDX + GCBAX$$

$$Y = \left(\frac{GCBD+GCBA}{1+ECB+GCBAF}\right)X$$

same answer

Canonical Forms

3) Give two different state-space models that produce the following transfer function

$$Y = \left(\frac{s+30}{s(s+2)(s+10)}\right) U$$

Controller Form: Multiply out

$$Y = \left(\frac{s+30}{s^3+12s^2+20s}\right) U$$

In State Space:

$$sX = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -20 & -12 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 30 & 1 & 0 \end{bmatrix} X + [0] U$$

Check using Matlab

```
>> A = [0,1,0;0,0,1;0,-20,-12];
>> B = [0;0;1];
>> C = [30,1,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

     (s+30)
  --------------
  s (s+2) (s+10)
```

In Jordan form. Do a partial fraction expansion

$$Y = \left(\left(\frac{1.5}{s}\right) + \left(\frac{-1.75}{s+2}\right) + \left(\frac{0.25}{s+10}\right)\right) U$$

In State-Space

$$sX = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -10 \end{bmatrix} X + \begin{bmatrix} 1.5 \\ -1.75 \\ 0.25 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} X + [0] U$$

```
>> A = [0,0,0;0,-2,0;0,0,-10];
>> B = [1.5;-1.75;0.25];
>> C = [1,1,1];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

     (s+30)
  --------------
  s (s+10) (s+2)
```

In Cascade form

$$Y = \left(\left(\frac{a}{s+10} \right) + \left(\frac{b}{(s+10)(s+2)} \right) + \left(\frac{c}{(s+10)(s+2)(s)} \right) \right) U$$

Putting over a common denominator, the numerator becomes

$$s(s+2)a + bs + c = s + 30$$

a = 0, b = 1, c = 30

$$sX = \begin{bmatrix} -10 & 0 & 0 \\ 1 & -2 & 0 \\ 0 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} U$$

$$Y = \begin{bmatrix} 0 & 1 & 30 \end{bmatrix} X + [0] U$$

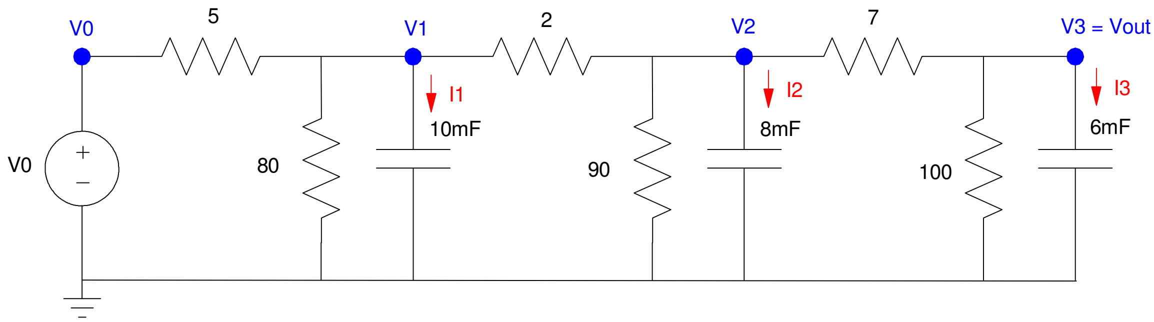
In Matlab

```
>> A = [-10,0,0;1,-2,0;0,1,0];
>> B = [1;0;0];
>> C = [0,1,30];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

     (s+30)
--------------
s (s+2) (s+10)
```

Electrical Ciruits

4) Using state-space methods, find the transfer function from V0 to V3



For capacitors, I = C dV/dt

$$I_1 = 0.01sV_1 = \left(\frac{V_0 - V_1}{5}\right) + \left(\frac{V_2 - V_1}{2}\right) - \left(\frac{V_1}{80}\right)$$

$$I_2 = 0.008sV_2 = \left(\frac{V_1 - V_2}{2}\right) + \left(\frac{V_3 - V_2}{7}\right) - \left(\frac{V_2}{90}\right)$$

$$I_3 = 0.006sV_3 = \left(\frac{V_2 - V_3}{7}\right) - \left(\frac{V_3}{100}\right)$$

Simplify

$$sV_1 = 20V_0 - 71.25V_1 + 50V_2$$

$$sV_2 = 62.5V_1 - 81.746V_2 + 17.857V_3$$

$$sV_3 = 23.809V_2 - 25.476V_3$$

Place in state-space form

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sV_3 \end{bmatrix} = \begin{bmatrix} -71.25 & 50 & 0 \\ 62.5 & -81.746 & 17.857 \\ 0 & 23.809 & -25.476 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$Y = V_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + [0]V_0$$

In Matlab:

```
>> A = [-71.25,50,0;62.5,-81.746,17.857;0,23.809,-25.476]

  -71.2500   50.0000         0
   62.5000  -81.7460   17.8570
         0   23.8090  -25.4760

>> B = [20;0;0]

    20
     0
     0

>> C = [0,0,1]

     0     0     1

>> D = 0

     0

>> G = ss(A,B,C,D);
>> zpk(G)


          29761.25
-----------------------------
(s+134.8) (s+35.66) (s+8.003)
```

5) Using state-space methods, find the transfer function from V0 to V1

A and B remain unchanged. C and D change

$$Y = V_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} + [0] V_0$$

In Matlab

```
>> C = [1,0,0];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)

   20 (s+88.49) (s+18.73)
-----------------------------
(s+134.8) (s+35.66) (s+8.003)
```

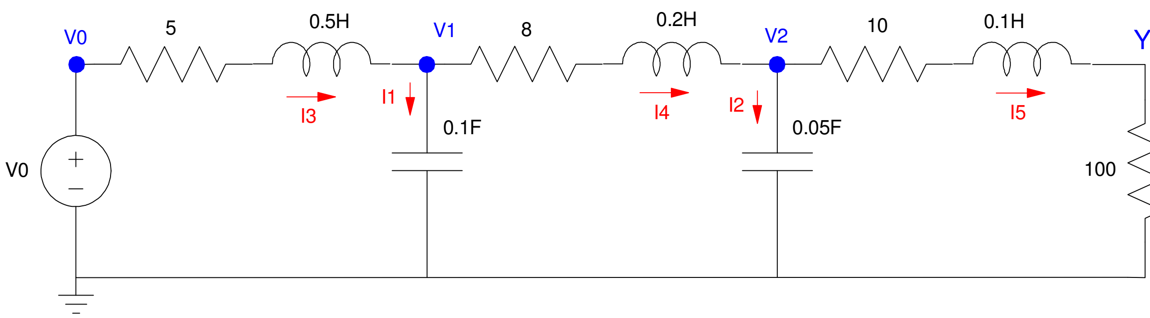
Note:

- When you change the output, the numerator changes (zeros)
- The denominator remains unchanged.

The denominator tells you where the poles are. These determine how the energy is dissipated. Changing the output doesn't change that.

6) Express the dynamics for the following RLC circuit in state-space form.

- Find the transfr function from V0 to V3



Note:

- V = L dI/dt
- I = C dV/dt

Writing the dynamics

$$I_1 = 0.1sV_1 = I_3 - I_4$$

$$I_2 = 0.05sV_2 = I_4 - I_5$$

$$V_3 = 0.5sI_3 = V_0 - 5I_3 - V_1$$

$$V_4 = 0.2sI_4 = V_1 - 8I_4 - V_2$$

$$V_5 = 0.1sI_5 = V_2 - 10I_5 - 100I_5$$

Solve for the derivatives

$$sV_1 = 10I_3 - 10I_4$$

$$sV_2 = 20I_4 - 20I_5$$

$$sI_3 = 2V_0 - 10I_3 - 2V_1$$

$$sI_4 = 5V_1 - 40I_4 - 5V_2$$

$$sI_5 = 10V_2 - 100I_5 - 1000I_5$$

Place in matrix form

$$\begin{bmatrix} sV_1 \\ sV_2 \\ sI_3 \\ sI_4 \\ sI_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 10 & -10 & 0 \\ 0 & 0 & 0 & 20 & -20 \\ -2 & 0 & -10 & 0 & 0 \\ 5 & -5 & 0 & -40 & 0 \\ 0 & 10 & 0 & 0 & -1100 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \\ 0 \\ 0 \end{bmatrix} V_0$$

$$Y = 100I_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} + [0]V_0$$

Put into Matlab

```
>> A = [0,0,10,-10,0;0,0,0,20,-20;-2,0,-10,0,0;5,-5,0,-40,0;0,10,0,0,-1100]

          0            0           10          -10            0
          0            0            0           20          -20
         -2            0          -10            0            0
          5           -5            0          -40            0
          0           10            0            0        -1100

>> B = [0;0;2;0;0]

     0
     0
     2
     0
     0

>> C = [0,0,0,0,100];
>> D = 0;
>> G = ss(A,B,C,D);
>> zpk(G)


                      2000000
--------------------------------------------------
(s+1100) (s+35.83) (s+1.306) (s^2 + 13.05s + 47.8)
```

7) Assume V0 = 0. Specify the initial conditions so that

- The transients decay as slow as possible (I3(0) = 1A)
- The transients decay as fast as possible (I5(0) = 1A)

```
>> [M,V] = eig(A)

M =

  -0.0000         -0.2317           -0.4883           -0.7330            -0.7330
  -0.0182          0.4760           -0.8640            0.4875 + 0.2577i   0.4875 - 0.2577i
  -0.0000         -0.0179            0.1123            0.2941 - 0.1937i   0.2941 + 0.1937i
  -0.0001         -0.8482            0.0486           -0.1841 - 0.0259i  -0.1841 + 0.0259i
  -0.9998          0.0045           -0.0079            0.0045 + 0.0023i   0.0045 - 0.0023i

>> eig(A)'

  -1099.8         -35.8             -1.3              -6.5 - 2.3i         -6.5 + 2.3i

>> X0 = M(:,3)

X0 =

   -0.4883
   -0.8640
    0.1123
    0.0486
   -0.0079

>> X0 = X0 / X0(3)
```

```
V1   -4.3470
V2   -7.6924
I3    1.0000
I4    0.4323
I5   -0.0700
```

Fast Mode

```
>> X0 = M(:,1);
>> X0 = X0 / X0(5)

V1    0.0000
V2    0.0182
I3    0.0000
I4    0.0001
I5    1.0000
```