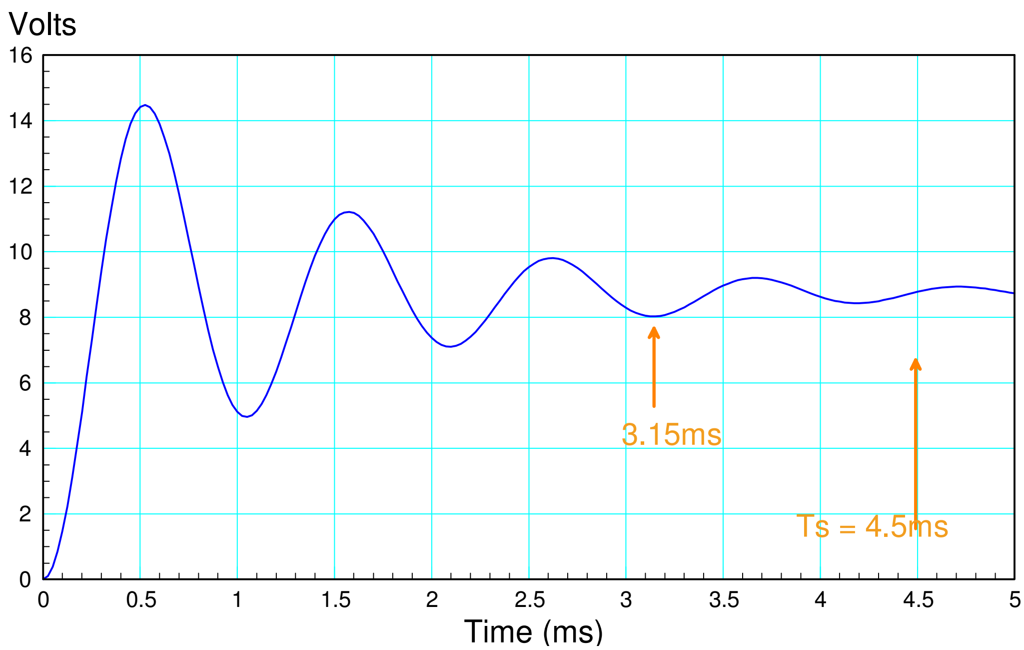
ECE 461 - Final: Name ____________________

Fall - 2024

1) Give the transfer function for a system with the following step response:



This is a 2nd-order system, so

$$G(s) \approx \left(\frac{a}{(s+b+jc)(s+b-jc)} \right)$$

The DC gain is about 8.5

The 2% settling time is about 4.5ms

$$b \approx \frac{4}{4.5ms} = 889$$

The oscillations are three cycles in 3.15ms

$$c \approx \left(\frac{3 \text{ cycles}}{3.15ms} \right) 2\pi = 5984 \frac{rad}{\sec}$$

so

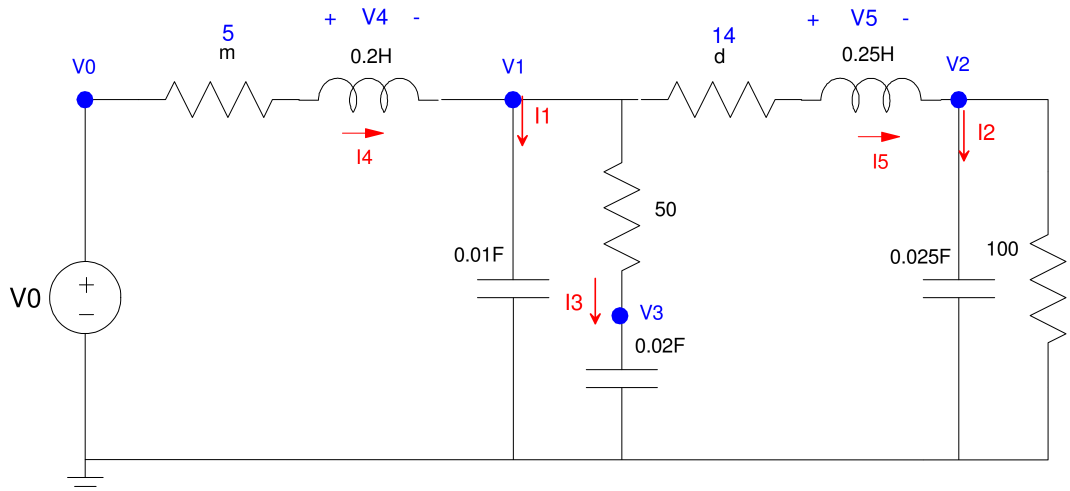
$$G(s) \approx \left(\frac{a}{(s+889+j5984)(s+889-j5984)} \right)$$

Pick 'a' so that the DC gain is 8.5

$$G(s) \approx \left(\frac{311{,}087{,}904}{(s+889+j5984)(s+889-j5984)} \right)$$

2) Write the differential equations which describe the following circuit (i.e. write the N differential equations which correspond to the voltage node equations). Assume

- m is your birth month (1..12) Ohms
- d is your birth date (1..31) Ohms



The base equations are

$$I = CsV \qquad \textit{capacitors}$$

$$V = LsI \qquad \textit{inductors}$$

The 5 differential equations that describe this circuit are then

$$I_1 = 0.01sV_1 = I_4 - I_5 - \left(\frac{V_1 - V_3}{50}\right)$$

$$I_2 = 0.025sV_2 = I_5 - \frac{V_2}{100}$$

$$I_3 = 0.02sV_3 = \left(\frac{V_1 - V_3}{50}\right)$$

$$V_4 = 0.2sI_4 = V_0 - 5I_4 - V_1$$

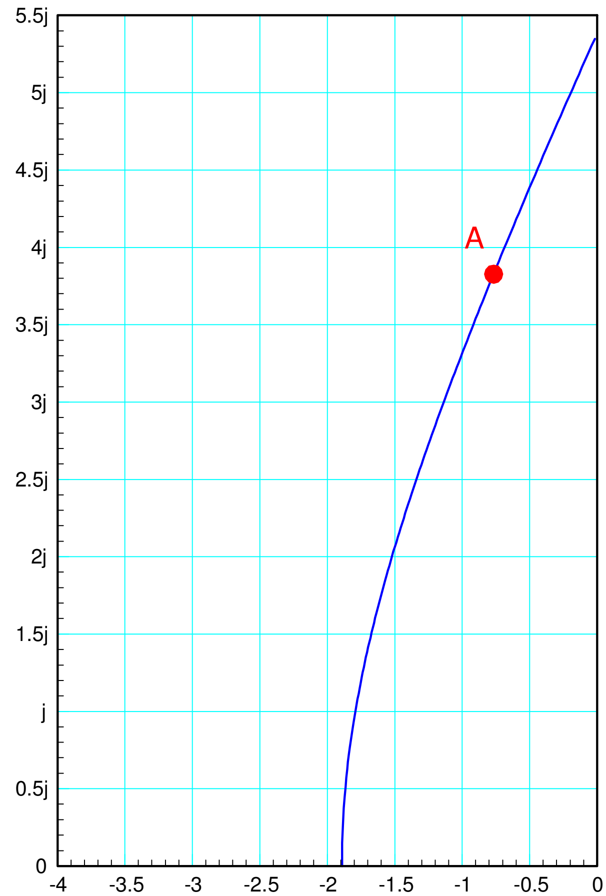
$$V_5 = 0.25sI_5 = V_1 - 14I_5 - V_2$$

3) The root locus for

$$G(s) = \left(\frac{5}{(s+0.5)(s+4)(s+6)}\right)$$

is shown to the right. Determine the following:

A value of point A x + jy	**-0.8 + j3.8**
k for placing the closed-loop pole at A	**24.39**
Resulting 2% settling time	**5.00 sec**
Resulting % Overshoot	**51.16%**
Resulting Error Constant (kp)	**10.16**



a) Value of A

- read off the graph

b) k for placing the pole at A

$$\left(\frac{5}{(s+0.5)(s+4)(s+6)}\right)_{s=-0.8+j3.8} = 0.0410\angle 179.428^0$$

almost 180 degrees

$$k = \frac{1}{0.0410} = 24.39$$

c) Ts

$$T_s = \frac{4}{real(s)} = \frac{4}{0.8} = 5.00$$

d) % Overshoot

$$s = -0.8 + j3.8 = -3.8833\angle 78.111^0$$

$$\zeta = \cos(78.111^0) = 0.2060$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 51.16\%$$

e) kp (DC gain)

$$k_p = (G \cdot K)_{s=0} = \left(\frac{5}{(s+0.5)(s+4)(s+6)}\right)_{s=0} \cdot 24.39 = 10.16$$

4) Given the following stable system where 'd' is your birth date (1..31) and m is your birth month (1..12).

$$G(s) = \left(\frac{5+d}{(s+0.5)(s+2)(s+m)}\right) = \left(\frac{19}{(s+0.5)(s+2)(s+5)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- No error for a step input, and
- A closed-loop dominant pole at s = -3 + j4

Pick K(s) of the form

$$K(s) = k\left(\frac{(s+0.5)(s+2)(s+5)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{19k}{s(s+a)^2}\right)$$

Evaluate what we know

$$\left(\frac{19}{s}\right)_{s=-3+j4} = 3.800\angle -126.87^0$$

so to make the angles add up to 180 degrees

$$\angle(s+a)^2 = 53.13^0$$

$$\angle(s+a) = 26.565^0$$

$$a = 3 + \frac{4}{\tan\left(26.565^0\right)} = 11.00($$

At this point

$$GK = \left(\frac{19k}{s(s+11)^2}\right)_{s=-3+j4} = 0.0475k\angle 180^0$$

$$k = \frac{1}{0.0475} = 21.052$$

so

$$\boxed{K(s) = \left(\frac{21.052(s+0.5)(s+2)(s+5)}{s(s+11)^2}\right)}$$

answers vary with your birth date

5) Given the following stable system where 'd' is your birth date (1..31) and m is your birth month (1..12).

$$G(s) = \left(\frac{5+d}{(s+0.5)(s+2)(s+m)}\right) = \left(\frac{19}{(s+0.5)(s+2)(s+5)}\right)$$

Determine a digital compensator, K(z), which results in the closed-loop system having

- No error for a step input,
- A closed-loop dominant pole at s = -3 + j4 (z = 0.682 + j0.288), and
- A sampling rate of T = 0.1

Pick K(s) and K(z) of the form where s converts to the z-plane as exp(sT)

$$K(s) = k\left(\frac{(s+0.5)(s+2)(s+5)}{s(s+a)^2}\right)$$

$$K(z) = k\left(\frac{(z-0.9512)(z-0.8187)(z-0.6065)}{(z-1)(z-a)^2}\right)$$

The open-loop system is then

$$(G(s) \cdot zoh \cdot K(z))_{z=-3+j4} = 1\angle 180^0$$

$$\left(\frac{19}{(s+0.5)(s+2)(s+5)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot k\left(\frac{(z-0.9512)(z-0.8187)(z-0.6065)}{(z-1)(z-a)^2}\right) = 1\angle 180^0$$

Evaluating what we know at s = -3 + j4

$$\left(\frac{19}{(s+0.5)(s+2)(s+5)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{(z-0.9512)(z-0.8187)(z-0.6065)}{(z-1)}\right) = 0.0222\angle -115.26^0$$

Meaning to make the angles add up to 180 degrees

$$\angle(z-a)^2 = 64.73^0$$

$$\angle(z-a) = 32.26^0$$

$$a = 0.682 - \frac{0.288}{\tan\left(32.26^0\right)} = 0.2272$$

To find k, evaluate what we now know at s = -3 + j4:

$$\left(\frac{19}{(s+0.5)(s+2)(s+5)}\right) \cdot \exp\left(\frac{-sT}{2}\right) \cdot \left(\frac{(z-0.9512)(z-0.8187)(z-0.6065)}{(z-1)(z-0.2272)^2}\right) = 0.0763\angle 180^0$$

so

$$k = \frac{1}{0.0763} = 13.11$$

and

$$K(s) = 13.11\left(\frac{(z-0.9512)(z-0.8187)(z-0.6065)}{(z-1)(z-0.2272)^2}\right)$$

6) Given the following stable system where 'd' is your birth date (1..31) and m is your birth month (1..12).

$$G(s) = \left(\frac{5+d}{(s+0.5)(s+2)(s+m)}\right) = \left(\frac{19}{(s+0.5)(s+2)(s+5)}\right)$$

Determine a compensator, K(s), which results in the closed-loop system having

- A closed-loop DC gain of 1.000 (i.e. no error for a step input),
- A 0dB gain frequency of 4 rad/sec, and
- A phase margin of 57 degrees

Let K(s) be of the form

$$K(s) = k\left(\frac{(s+0.5)(s+2)(s+5)}{s(s+a)^2}\right)$$

$$GK = \left(\frac{19k}{s(s+a)^2}\right)$$

For a phase margin of 57 degrees at 4 rad/sec,

$$GK(j4) = 1\angle -123^0$$

Evaluate what we know

$$\left(\frac{19}{s}\right)_{s=j4} = 4.75\angle -90^0$$

For the total phase to add up to -123 degrees

$$\angle(s+a)^2 = 33^0$$

$$\angle(s+a) = 16.5^0$$

$$a = \frac{4}{\tan\left(16.5^0\right)} = 13.5038$$

Evaluate what we now know at s = j4

$$GK = \left(\frac{19k}{s(s+13.5038)^2}\right)_{s=j4} = 0.0239k\angle -123^0$$

$$k = \frac{1}{0.0239} = 41.758$$

and

$$K(s) = 41.758\left(\frac{(s+0.5)(s+2)(s+5)}{s(s+13.5038)^2}\right)$$