ECE 376 - Homework #7

Data Collection & Student t-Test.

Data Collection (population A): 100uF Electrolitic Capacitor

1) Measure one of the following with at least two data sets and 20+ data points per run:

- The voltage across a capacitor as it discharges
- The temperature of a cup (or can) of hot water as it cools off
- The temperature of a can of cold water as it warms up
- Other

Plot the resulting data vs. time.

Procedure:

- Connect the capacitor from RA1 to ground
- (100k resistor is in parallel: soldered onto the PIC board)
- Charge to 5.00V
- Start the data collection (sample RA1 every 10ms)
- Remove the wire and let the capacitor discharge
- Copy the resulting data to Matlab

Main Loop in the C Program:

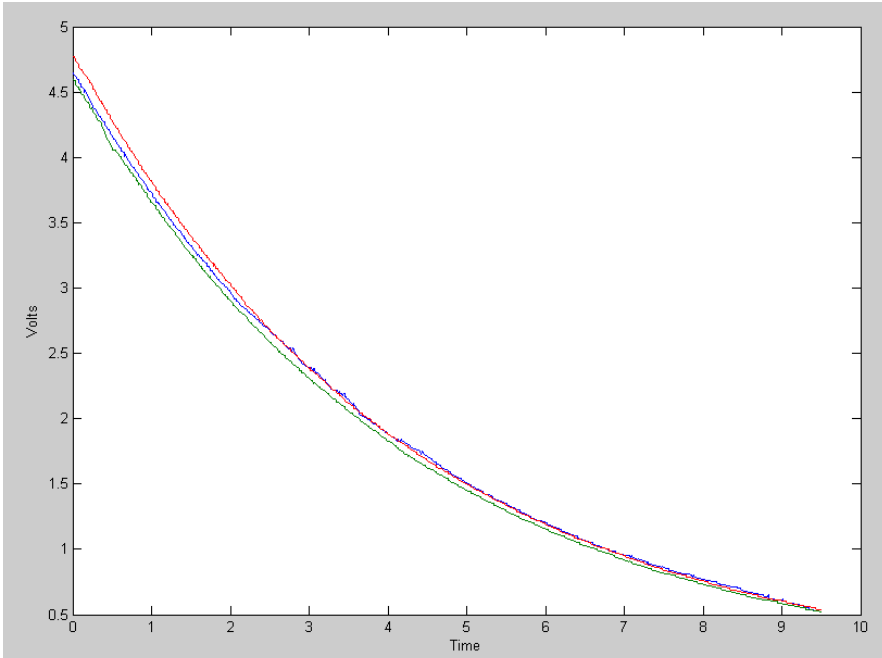
```
while(1) {
   RC1 = !RC1;
   A2D = A2D_Read(1);
   V1 = 0.488*A2D;
   if(V1 < 4.9) {
       SCI_Out(V1, 3, 2);
       SCI_CRLF();
       }
   Wait_ms(10);
   }
```

Measuring the frequency on RC0 gives 23.82Hz for a loop time of

$$T = \frac{1}{2} \cdot \frac{1}{23.82Hz} = 21.0ms$$

Matlab: Plotting the results

```
>> V1 = V1(50:1000);
>> V2 = V2(50:1000);
>> V3 = V3(50:1000);
>> T = 0.021;
>> t = [1:length(V1)]' * T;
>> plot(t,V1,t,V2,t,V3)
>> xlabel('Time')
>> ylabel('Volts');
```



Resulting voltages vs. time

2) Determine the time constant from your data using least-squares

$$V = ae^{-bt} \qquad\qquad T = ae^{-bt} + T_{amb}$$

$$\ln(V) = \ln(a) - bt \qquad\qquad \ln(T - T_{amb}) = \ln(a) - bt$$

```
>> B = [t, t.^0];
>> A1 = inv(B'*B)*B'*log(V1)

   -0.1080
    1.5447

>> A2 = inv(B'*B)*B'*log(V2)

   -0.1094
    1.5253

>> A3 = inv(B'*B)*B'*log(V3)

   -0.1100
    1.5652

>> Data = [A1(1),A2(1),A3(1)]
```

Data =   -0.1080   -0.1094   -0.1100

```
>> uFa = -[10/A1(1), 10/A2(1), 10/A3(1)]
```

uFa =   92.63   91.423   90.909

uF is the corresponding value of C assuming

- R = 100k
- t = 21ms sampling rate

Note: The readings may be off due to an additional current draw on the PIC board (the PIC's analog input has an impedance less than infinity). The resistance you need to bring the computed capacitance to 100uF would be

$$slope = -0.11 = \frac{-1}{RC}$$

If C = 100uF

$$R = 90.9k\Omega$$

This is the 100k resistor in parallel with the resistance of the PIC

$$R = 90.9k = 100k || R_{PIC}$$

Solving gives

$$R_{PIC} = 998.9k\Omega$$

So it looks like the input impedance of the A/D converter is about 1M Ohm

3) Use a student t-test to determine the 90% confidnence interval for your time constant (b).

Assume R(PIC) = 1M. Then

$$R = 100k || 1M = 90.909k\Omega$$

$$\frac{1}{RC} = |slope|$$

$$C = \frac{1}{|slope| \cdot R}$$

C is then { 101.85uF, 100.54uF, 100.00uF }

Individual Test: What will the next reading of C be with p=0.9?

```
>> A = [101.85, 100.54, 100.00];
>> Xa = mean(A)
Xa =  100.7967

>> Sa = std(A)
Sa =    0.9513
```

From a t-Table with 2 degrees of freedom, 5% tails corresponds to a t-score of 2.92

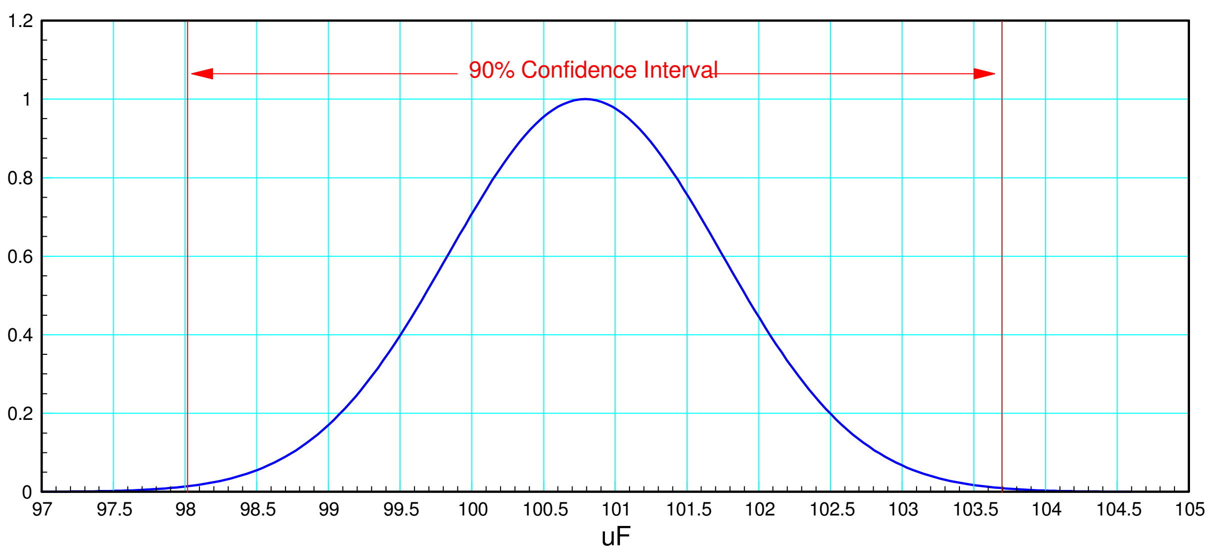
The 90% confidence interval is thus

$$\bar{x} - 2.92s < b < \bar{x} + 2.92s$$

```
>> Xa + 2.92*Sa
ans =  103.5746

>> Xa - 2.92*Sa
ans =   98.0188
```

I'm 90% certain that my next reading of C will be in the range of (98.02uF ... 103.57uF)



pdf for the next reading of C (individual test)

Population: What is the actual value of C with p=0.9?

This is a population question (the more data I have, the more certain I am). In this case, divide the variance by the sample size

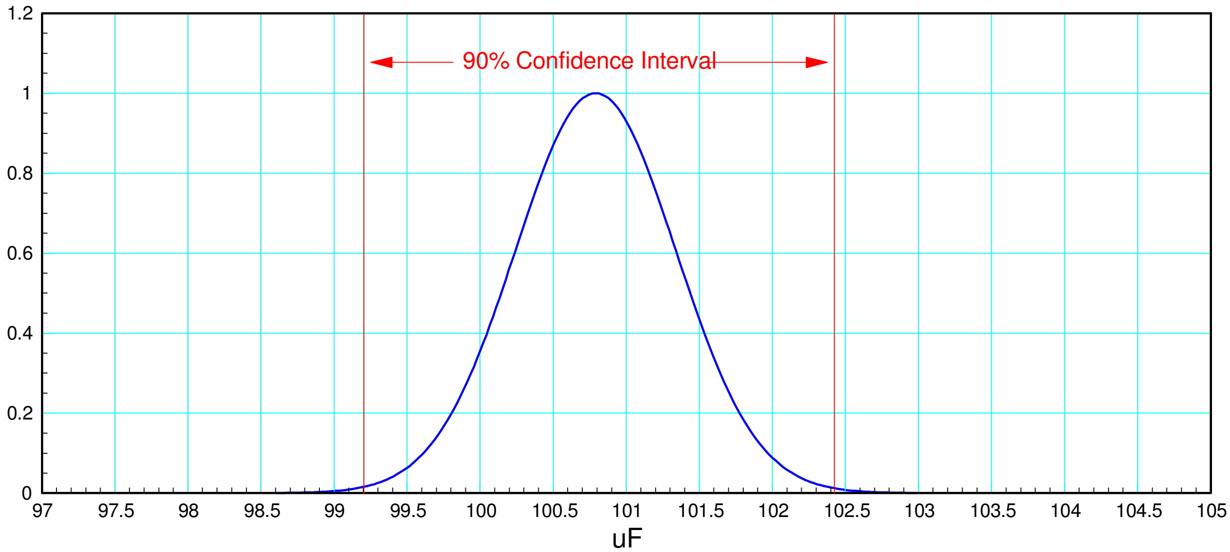
```
>> A = [101.85, 100.54, 100.00];
Xa = mean(A)
Xa =  100.7967

>> Sa = std(A) / sqrt(3)
Sa =    0.5493

>> Xa + 2.92*Sa
  102.4005

>> Xa - 2.92*Sa
   99.1929
```

I'm 90% certain that the capacitance in in the range of (99.19uf .. 102.40uF)



90% Confidence Interval for the Population's Mean (actual value of C)

Data Collection (population B)

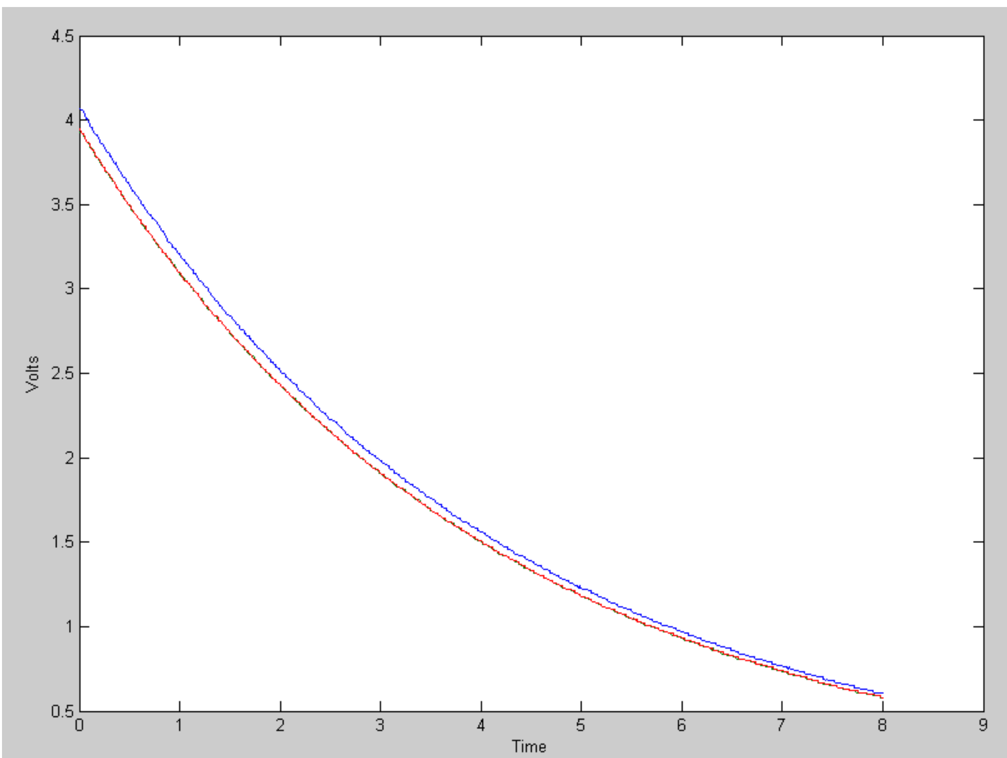
Change something in your experiment

- **Use a different 100uF capacitor**

4) Take a second set of data with the change.

Plot the resulting data vs. time

```
>> V1 = Data1(100:900);
>> V2 = Data2(100:900);
>> V3 = Data3(100:900);
>> t = [1:length(V1)]' * 0.021;
>> plot(t,V1,t,V2,t,V3)
>> xlabel('Time')
>> ylabel('Volts');
```



5) Determine the time constant from your data using least-squares

```
>> B = [t, t.^0];
>> A1 = inv(B'*B)*B'*log(V1)

   -0.1137
    1.4019

>> A2 = inv(B'*B)*B'*log(V2)

   -0.1142
    1.3679

>> A3 = inv(B'*B)*B'*log(V3)

   -0.1140
    1.3678

>> Data2 = [C1(1), C2(1), C3(1)]
```

Data2 =   -0.1137   -0.1142   -0.1140

```
>> uF2 = -[11/C1(1), 11/C2(1), 11/C3(1)]
```

uF2 =   96.7458   96.4067   96.4912

This is the measured capacitange of the second 100uF capacitor

6) Use a student t test to determine the 90% confidnence interval for your time constant (b).

Again, with a sample size of three (meaning two degrees of freedom), 5% tails corresponds to a t-score of 2.92

Individual

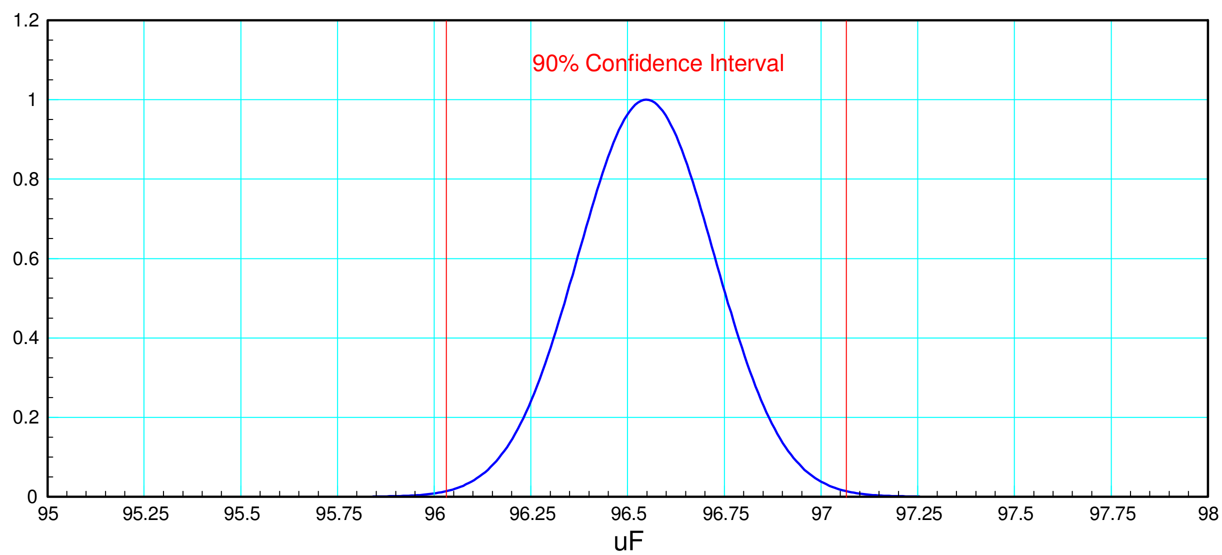
```
>> B = [96.7458    96.4067     96.4912];
>> Xb = mean(B)
Xb =    96.5479

>> Sb = std(B)
Sb =     0.1765

>> Xb + 2.92*Sb
ans =    97.0633

>> Xb - 2.92*Sb
ans =    96.0325
```

I'm 90% certain that the next reading of the second 100uF capacitor will be in the range of (96.03uF ... 97.06uF)



90% Confidence Interval for the Next Reading of C (individual)

Population:

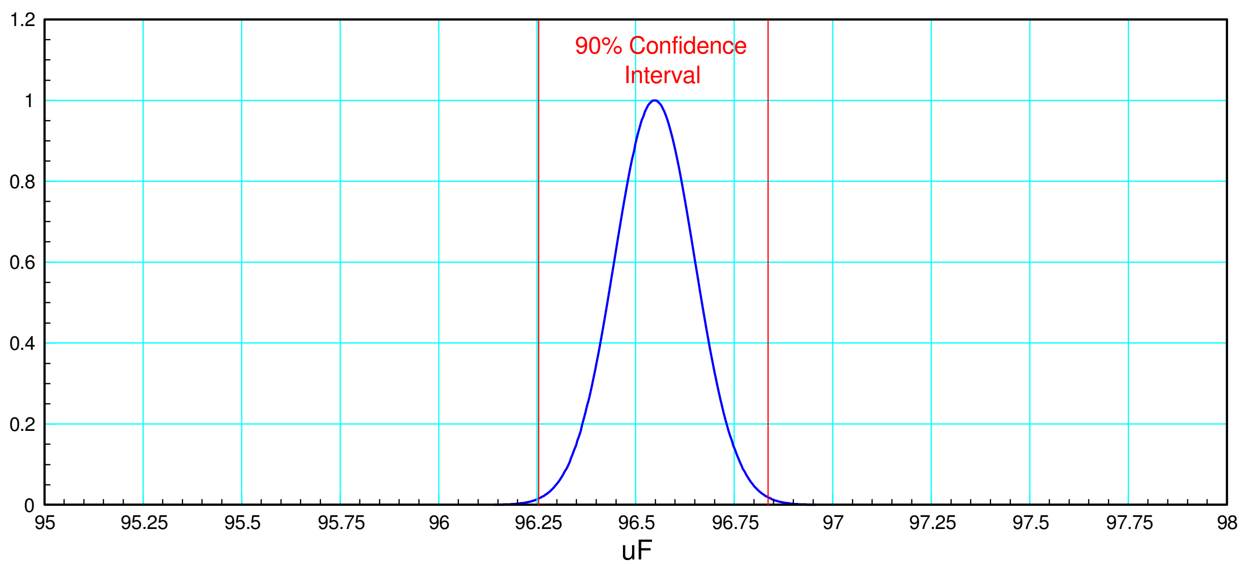
```
>> B = [96.7458   96.4067    96.4912];
Xb = mean(B)
Xb = 96.5479

>> Sb = std(B)/sqrt(3)
Sb =    0.1019

>> Xb + 2.92*Sb
ans =   96.8455

>> Xb - 2.92*Sb
ans =   96.2503
```

I'm 90% certain that the actual value of B is in the range of (96.25uF .. 96.84uF)



90% Confidence Interval for Population B's Mean (true value of capacitor B)

Comparison of Means Test (A vs. B)

7) Do a comparison of means test to determine the probability that

- The next measurement from A will have a higher value than the next meaurement from B
- Population A has a higher mean than population B

Individual: Create a new variable W = A - B

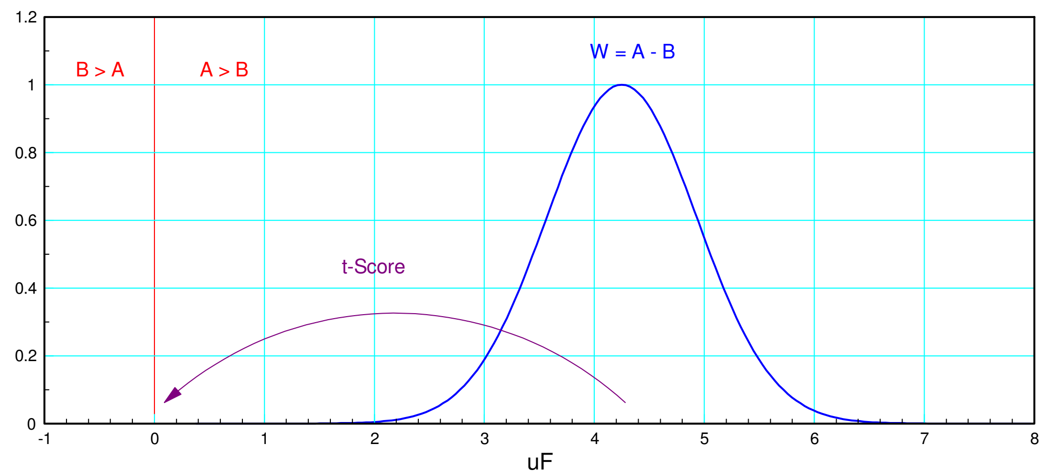
```
>> Xw = Xa - Xb
Xw =     4.2488

>> Sw = sqrt(Sa^2 + Sb^2)
Sw =     0.9676

>> t = Xw / Sw
t =     4.3912
```

From StatTrek, a t-score of 4.3055 with 2 dof corresponds to a probability of 0.975

I'm 97.5% certain the the next reading of capacitor A will be higher than the next reading of capacitor B



Population: (take sample size in to account): Create a new variable, W = A - B

```
>> Xw = Xa - Xb
Xw =    4.2488

>> >> Sw = sqrt( (Sa^2)/3 + (Sb^2)/3 )
Sw =    0.5586

>> t = Xw / Sw
t =    7.6057
```

From StatTrek, this corresponds to a probability of 0.991

I'm 99.1% certain that capacitor A's mean is larger than capacitor B's mean