## ECE 341-Test \#2

Continuous Probability - Summer 2024

## 1) Continuous PDF

A 5\% tolerance rersistor often has a pdf as shown below (resistors which are within $2 \%$ of rated value are removed and sold as $2 \%$ or $1 \%$ resistors).

a) Determine a scalar so that this is a valid pdf (i.e. the total area $=1.0000$ )
b) Determine the moment generating function (i.e. LaPlace transform)

## 2) Uniform Distribuitions

Let $\mathrm{A}, \mathrm{B}$, and C be continuous uniform distributions

- $\mathrm{A}=$ uniform over the interval of $(3,7)$
- $B=$ uniform over the interval of $(1,2)$,
- $\mathrm{X}=\mathrm{A}+\mathrm{B}$

Use moment generating functions to determine the pdf for X (i.e. LaPlace Transforms)

## 3) Gamma CDF

Let A, B be continuous exponential distributions:

- A has a mean of 2 seconds $\quad a(t)=\frac{1}{2} e^{-t / 2} u(t)$

$$
\begin{aligned}
& A(s)=\left(\frac{1 / 2}{s+1 / 2}\right) \\
& B(s)=\left(\frac{1 / 5}{s+1 / 5}\right)
\end{aligned}
$$

- B has a mean of 5 seconds $\quad b(t)=\frac{1}{5} e^{-t / 5} u(t)$

The moment generating function for the cdf of $\mathrm{Y}=$ two A's and one B happens is

$$
Y=\left(\frac{1}{s}\right)\left(\frac{1 / 2}{s+1 / 2}\right)^{2}\left(\frac{1 / 5}{s+1 / 5}\right)
$$

Determine the equation for the cdf (i.e. take the inverse LaPlace transform)

## 4) Central Limit Theorem

The Dungeons and Dragons spell Meteor Swarm does 20-120 damage (the sum of twenty 6-sided dice)

$$
y=20 d 6
$$

Use a normal approximation to determine the probability that the total damage is more than 99.5

| die | d 4 | d 6 | d 8 | d 10 | d 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mean | 2.5000 | 3.5000 | 4.5000 | 5.5000 | 6.5000 |
| variance | 1.2500 | 2.9167 | 5.2500 | 8.2500 | 11.9167 |


| mean of y | standard deviation of y | z -score for sum $=99.5$ | $\mathrm{p}($ sum $>99.5)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |

## 5) Testing with Normal pdf

Two wizards in Dungeons and Dragons cast spells. Let

- A be the damage done by a Flame Strike spell (the sum of eight 6-sided dice: 8d6)
- B be the damage done by a Firestorm spell (sum of seven 10 -sided dice: 7d10)

| die | d 4 | d 6 | d 8 | d 10 | d 12 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| mean | 2.5000 | 3.5000 | 4.5000 | 5.5000 | 6.5000 |
| variance | 1.2500 | 2.9167 | 5.2500 | 8.2500 | 11.9167 |

Use a normal approximation to determine the probability that $\mathrm{A}>\mathrm{B}$

|  | $\mathrm{A}=8 \mathrm{~d} 6$ | $\mathrm{~B}=7 \mathrm{~d} 10$ | $\mathrm{~W}=\mathrm{A}-\mathrm{B}$ |
| :---: | :---: | :---: | :---: |
| mean |  |  |  |
| variance |  |  |  |
| z -Score |  |  |  |
| $\mathrm{p}(\mathrm{A}>\mathrm{B})$ |  |  |  |

