ECE 341 - Test #1

Combinations, Permitations, and Discrete Probability - Summer 2024

Open-Book, Open Notes. Calculators & Tarot cards allowed. Chegg or other people *not* allowed.

1) Permutations & Combinations in Bison Poker

Assume a 65-card deck of playing cards

- 13 card values (ace .. king)
- Five suits (clubs, diamonds, hearts, spades, bison)

Each player is dealt six cards. The best 5-card hand makes up your band in Bison poker.

Calculate the odds of being dealt three of a kind:

- best five cards include a 3-of-a-kind
- hand = {xxx abc},
- {x, a, b, c} all have different values, suit doesn't matter.

M = (13c1 1 for x)(5c3 for three x's)(12c3 for abc)(5c1 for a)(5c1 for b)(5c1 for c)

$$M = \binom{13}{1}\binom{5}{3}\binom{12}{3}\binom{5}{1}\binom{5}{1}\binom{5}{1}$$

$$M = 3,575,000$$

The number of 6-card hands:

$$N = \binom{65}{6} = 82,598,880$$

The odds of being dealt a three of a kind:

$$p = \frac{M}{N} = 0.043281$$

2) Conditional Probability

Assume you play the following game:

- Flip a coin. (heads or tails)
- If the coin is a heads, then roll two 6-sided dice
- If the coin is a tails, then roll two 8-sided dice

Your score is the sum of the die rolls.

Determine the probability that the sum of the dice is three.

Case 1: You flip a heads

$$p(B) = 0.5$$

$$p(A|B) = \left(\frac{2}{36}\right)$$ (two ways to get a three: 1+2 and 2+1)

$$p(A) = p(A|B)p(B) = \frac{1}{36}$$

Case 2: You flip a tails

$$p(B) = 0.5$$

$$p(A|B) = \left(\frac{2}{64}\right)$$ (two ways to get a three: 1+2 and 2+1)

$$p(A) = p(A|B)p(B) = \frac{1}{64}$$

The total odds are the sum of the two

$$p(3) = \frac{1}{36} + \frac{1}{64} = 0.043403$$

3. Binomial Distribution

Let X be the number of 1's you get when rolling fourty 4-sided dice.

- die roll = {1}   1 point
- die roll = {2, 3, 4}   0 points

Determine the probability that X = m where m is your birth date (1..31)

m birth date (1..31)	probability X = m with fourty die rolls
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This is a binomial distribution

$$p(x) = \binom{40}{x}\left(\frac{1}{4}\right)^{x}\left(\frac{3}{4}\right)^{40-x}$$

x	p(x)
1	0.0001
2	0.0009
3	0.0037
4	0.0113
5	0.0272
6	0.0530
7	0.0857
8	0.1179
9	0.1397
10	0.1444
11	0.1312
12	0.1057
13	0.0759
14	0.0488
15	0.0282
16	0.0147
17	0.0069
18	0.0029
19	0.0011
20	0.0004
21	0.0001
22	0.0000
23	0.0000
24	0.0000
25	0.0000
26	0.0000
27	0.0000
28	0.0000
29	0.0000
30	0.0000
31	0.0000

4. Convolution

Use convolution by hand (i.e. not with Matlab or similar programs) to determine the product of two polynomials:

$$y(x) = (5x^2 + 3x + 7)(2x + 3) = a(x) ** b(x)$$

Note: Show your work to get full credit

ans: $y(x) = 21 + 23x + 21x^2 + 10x^3$

a) x^0 term (determine using convolution) **= 21**

-3	-2	-1	0	1	2	3	4	5
a(x)	-	-	7	3	5	-	-	-
b(k-x)	-	2	3	-	-	-	-	-
product			21					

b) x^1 term (determine using convolution) **= 23**

-3	-2	-1	0	1	2	3	4	5
a(x)	-	-	7	3	5	-	-	-
b(k-x)	-	-	2	3	-	-	-	-
product			14	9				

c) x^2 term (determine using convolution) **= 21**

-3	-2	-1	0	1	2	3	4	5
a(x)	-	-	7	3	5	-	-	-
b(k-x)	-	-	-	2	3	-	-	-
product				6	15			

d) x^3 term (determine using convolution) **= 10**

-3	-2	-1	0	1	2	3	4	5
a(x)	-	-	7	3	5	-	-	-
b(k-x)	-	-	-	-	2	3	-	-
product					10			

5. Geometric & z-Transforms

Let

- X be the number of rolls of an 10-sided die until you get a number from 1..3 {1, 2, 3}:

  $$X = \left(\frac{0.3}{z-0.7}\right)$$

- Y be the number of rolls a 10-sided die until you get a number from 1..4: {1, 2, 3, 4}:

  $$Y = \left(\frac{0.4}{z-0.6}\right)$$

Determine the pdf for W = X + Y using z-transforms

$$W = XY$$

$$W = \left(\frac{0.3}{z-0.7}\right)\left(\frac{0.4}{z-0.6}\right)$$

do partial fractions

$$W = \left(\frac{1.2}{z-0.7}\right) + \left(\frac{-1.2}{z-0.6}\right)$$

Multiply by z and take the inverse z-transform

$$zW = \left(\frac{1.2z}{z-0.7}\right) + \left(\frac{-1.2z}{z-0.6}\right)$$

$$zw(k) = \left(1.2(0.7)^k - 1.2(0.6)^k\right)u(k)$$

Divide by z (delay by one)

$$w(k) = \left(1.2(0.7)^{k-1} - 1.2(0.6)^{k-1}\right)u(k-1)$$

Other valid ways to write this are:

$$w(k) = \left(1.714(0.7)^k - 2(0.6)^k\right)u(k-1)$$

$$w(k) = \left(0.84(0.7)^{k-2} - 0.72(0.6)^{k-2}\right)u(k-2)$$