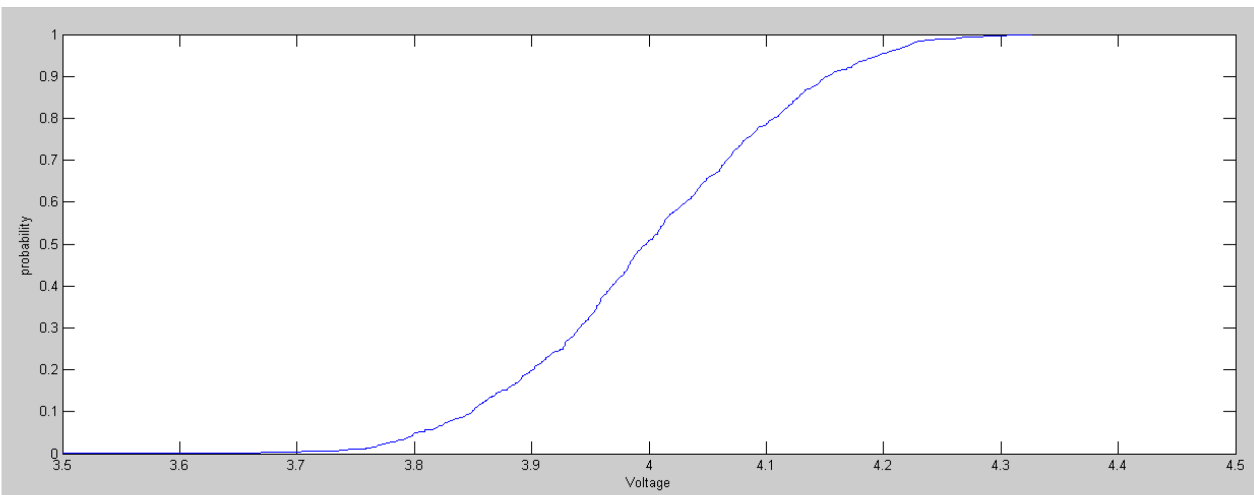
ECE 341 - Homework #9

Weibull Distribution, Central Limit Theorem. Summer 2024

Weibull Distribution

1) Determine and plot the cdf for the voltage, Y, in homework set #7 problem #3

```
Data = 0;
for i = 1:1e3
   R1 = (1 + 0.05*(rand*2-1)) * 1000;
   R2 = (1 + 0.05*(rand*2-1)) * 4000;
   R3 = (1 + 0.05*(rand*2-1)) * 1000;
   R4 = (1 + 0.05*(rand*2-1)) * 4000;

   A = 3;
   B = 2;

   Y = (R3+R4)/R3 * R2/(R1+R2) * A - (R4/R3)*B;

   Data = [Data;Y];
end

Data = sort(Data);
p = [1:length(Data)]';
p = p / max(p);

plot(Data, p);
```



2) Determine and plot the pdf for this voltage using a Weibull approximation for the cdf

Save the data from problem #1

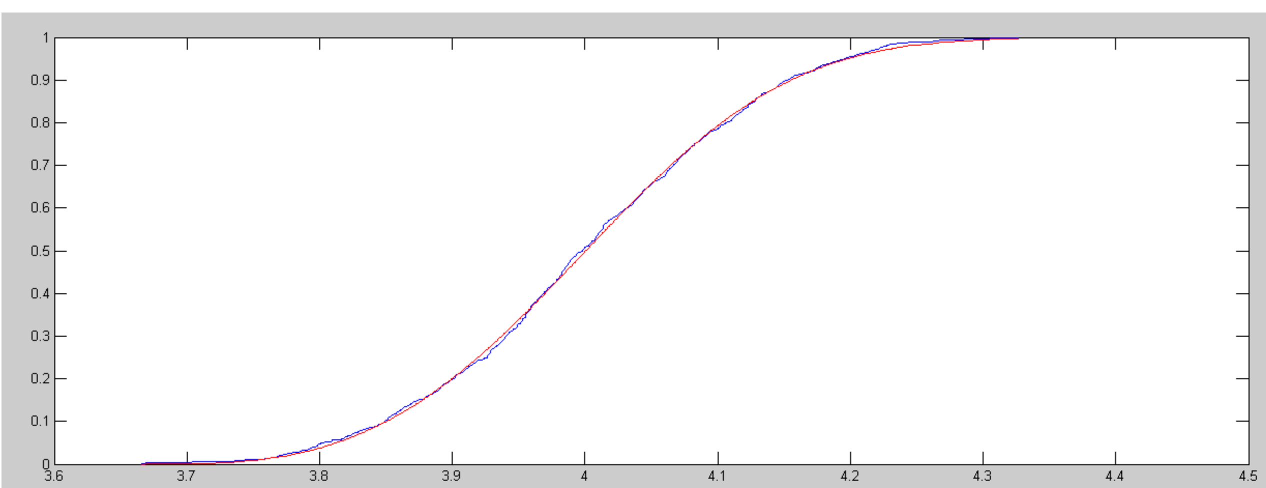
```
>> Volts = Volts(2:1001);
>> p = p(2:1001);
>> save HW9 Volts p
```

Create a funciton to return the sum-squared error between the actual CDF and a Weibull CDF

```
function y = Weibull(z)
% Weibull distribution curve fit
k = z(1);
L = z(2);

load HW9

x = Volts;
x0 = min(x);

% p(Vce) = target
W = 1 - exp( -( ((x-x0)/L) .^ k ));

e = p - W;

plot(Volts, p, 'b', Volts, W, 'r');
pause(0.01);
y = sum(e.^2);

end
```

Optimize with fminsearch

```
>> [Z,e] = fminsearch('Weibull',[1,1])
         k          L
Z =    3.1595    0.3761
e =    0.0505
```



So, the CDF is approximately

$$F(x) \approx \left(1 - \exp\left(-\left(\frac{x}{3.1595}\right)^{0.3761}\right)\right) + 3.6657$$

The pdf is then

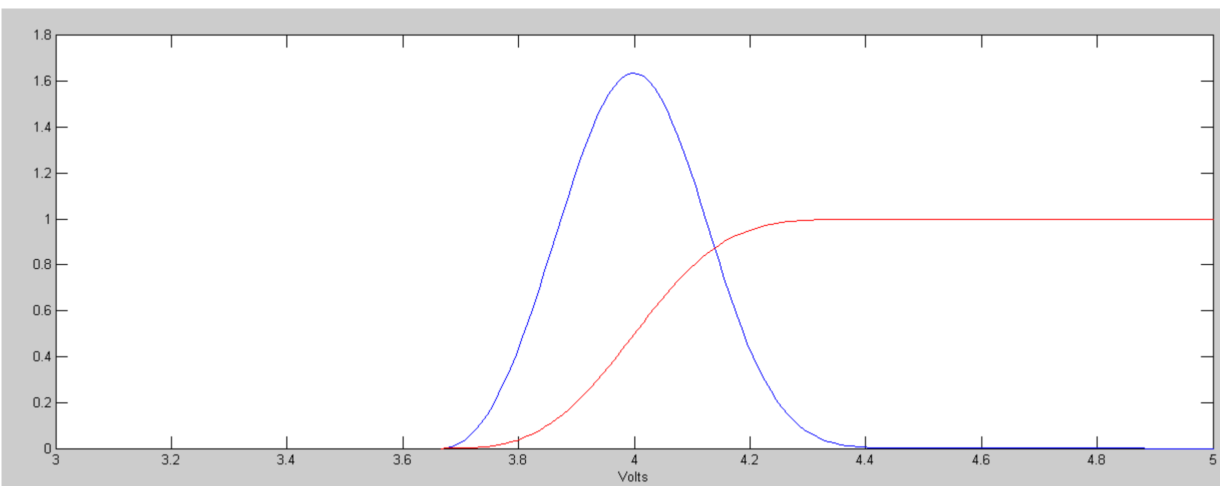
$$f(x) \approx \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left(-\left(\frac{x}{\lambda}\right)^{k}\right) u(k)$$

Shift by x0 (3.6657 Volts)

$$f(x - x_0) \approx \frac{3.1595}{0.3761}\left(\frac{x}{0.3761}\right)^{3.1595-1} \exp\left(-\left(\frac{x}{0.3761}\right)^{3.1595}\right) u(k)$$

```
>> k = Z(1);
>> L = Z(2);
>> x0 = min(Volts);
>> x = [0:0.01:10]';
>> pdf = (k/L) * (x/L).^(k-1) .* exp(-(x/L).^k);
>> cdf = 1 - exp(-(x/L).^k);
>> plot(x+x0,pdf,'b',x+x0,cdf,'r');
>> xlabel('Volts');
>> xlim([3,5])
>> plot(x+x0,pdf/2,'b',x+x0,cdf,'r');
>> xlim([3,5])
>> xlabel('Volts');
```

>>



pdf (blue) & cdf (red)

Central Limit Theorem

The mean and standard deviation for a 4, 6, and 8-sided die are

Die	d4	d6	d8	d10	d12
mean	2.5	3.5	4.5	5.5	6.5
standard deviation	1.1180	1.7078	2.2191	2.8723	3.4521
variance	1.2500	2.9166	5.2487	8.25	11.9167

3) Let Y be the sum of rolling four 10-sided dice (homework #4 problem 4):

Y = 4d10

a) What is the mean and standard deviation of Y?

$$\mu_y = 4\mu_{d10} = 22$$

$$\sigma_y^2 = 4\sigma_{d10}^2 = 33$$

$$\sigma_y = \sqrt{33} = 5.7445$$

Note: Homework #4 found the mean and standard deviation using convolution:

- Mean = 22.0000
- StDev = 5.7446

The Central Limit theroem gets you the same answer with a lot less work.

b) Using a normal approximation, what is the 90% confidence interval for Y?

5% tails corresponds to a z-score of 1.64485

$$\mu_y - 1.64485\sigma_y < roll < \mu_y + 1.64485\sigma_y$$

$$12.5511 < roll < 31.4489$$

c) Using a normal approximation, what is the probability that the sum the dice will be more than 29.5?

Find the z-score

$$z = \left(\frac{29.5-\mu_y}{\sigma_y}\right) = \left(\frac{29.5-22}{5.7445}\right) = 1.3056$$

From StatTrek, this z-score corresponds to a tail of **0.09584**

d) Compare these results to the actual odds (from homework #4)

```
>> sum(IceStorm(31:41))
ans =    0.0857
```

Close but a little off. The error comes from where you mark the cutoff between 29 and 30 (29.1? 29.5? 29.9?)

4) Let Y be the sum of rolling four 6-sided dice and two 8-sided dice (homework #4 problem 5):

Y = 4d6 + 2d8

a) What is the mean and standard deviation of Y?

$$\mu_y = 4\mu_{d6} + 2\mu_{d8}$$

$$\mu_y = 23$$

$$\sigma_y^2 = 4\sigma_{d6}^2 + 2\sigma_{d8}^2$$

$$\sigma_y^2 = 22.1638$$

$$\sigma_y = 4.7078$$

b) Using a normal approximation, what is the 90% confidence interval for Y?

$$\mu_y - 1.64485\sigma_y < roll < \mu_y + 1.64485\sigma_y$$

$$15.2564 < roll < 30.7436$$

c) Using a normal approximation, what is the probability that the sum the dice will be more than 29.5?

$$z = \left(\frac{29.5-\mu}{\sigma}\right)$$

$$z = \left(\frac{29.5-22}{4.7078}\right) = 1.3807$$

From StatTrek, this corresponds to a probability of 0.08522

d) Compare these results to the actual odds (from homework #4)

```
>> Mean = sum(IceStorm .* n)

Mean =   23.0000

>> Variance = sum( IceStorm .* (n - Mean).^2 )

Variance =   22.1667

>> StDev = sqrt(Variance)

StDev =    4.7081

>> sum(IceStorm(31:41))

ans =    0.0857
```

Same results with a lo tless work

5) Let Y be the sum of rolling five dice (homework #4 problem 6)

Y = d4 +d6 + d8 + d10 + d12

a) What is the mean and standard deviation of Y?

$$\mu_y = \mu_{d4} + \mu_{d6} + \mu_{d8} + \mu_{d10} + \mu_{d12}$$

$$\mu_y = 22.5$$

$$\sigma_y^2 = \sigma_{d4}^2 + \sigma_{d6}^2 + \sigma_{d8}^2 + \sigma_{d10}^2 + \sigma_{d12}^2$$

$$\sigma_y^2 = 29.5820$$

$$\sigma_y = \sqrt{29.5820} = 5.4389$$

b) Using a normal approximation, what is the 90% confidence interval for Y?

$$\mu_y - 1.64485\sigma_y < roll < \mu_y + 1.64485\sigma_y$$

$$13.5538 < roll < 31.4462$$

c) Using a normal approximation, what is the probability that the sum the dice will be more than 29.5?

$$z = \left(\frac{29.5-\mu}{\sigma}\right)$$

$$z = \left(\frac{29.5-22.5}{5.4389}\right) = 1.27870$$

From StatTrek:

p = 0.09905

d) Compare these results to the actual odds (from homework #4)

Same mean and standard deviation as homework #4

Almost the same odds:

```
>> sum(Prob6(31:41))

ans =    0.1039
```

Almost the same - the difference comes from where you draw the line between 29 and 30