ECE 341 - Homework #6

LaPlace Transforms, Continuous Probability Density Functions. Summer 2024

LaPlace Transforms

1) Let X and Y be related by the following transfer function:

$$Y = \left(\frac{2s+30}{(s+5)(s+8)}\right) X$$

a) What is the differential equation relating X and Y?

Cross multiply and multiply out

$$(s+5)(s+8)Y = (2s+30)X$$

$$(s^2+13s+40)Y = (2s+30)X$$

Note that 'sY' means 'the derivative of y'

$$y'' + 13y' + 40y = 2x' + 30x$$

b) Determine y(t) assuming

$$x(t) = 3\cos(5t) + 2\sin(5t)$$

Use phasors:

$$s = j5$$

$$X = 3 - j2$$

$$Y = \left(\frac{2s+30}{(s+5)(s+8)}\right)_{s=j5} \cdot (3 - j2)$$

$$Y = -0.0674 - 1.7079i$$

meaning

$$y(t) = -0.0674\cos(5t) + 1.7079\sin(5t)$$

c) Determine y(t) assuming x(t) is the unit step function (0 for t<0, 1 for t>0)

$$x(t) = u(t)$$

$$Y = \left(\frac{2s+30}{(s+5)(s+8)}\right)X$$

$$Y = \left(\frac{2s+30}{(s+5)(s+8)}\right)\left(\frac{1}{s}\right)$$

$$Y = \left(\frac{0.75}{s}\right) + \left(\frac{-1.3333}{s+5}\right) + \left(\frac{0.5833}{s+8}\right)$$

taking the inverse LaPlace transform

$$y(t) = (0.75 - 1.3333e^{-5t} + 0.5833e^{-8t})u(t)$$

2) Let X and Y be related by the following transfer funciton

$$Y = \left(\frac{2s+30}{(s+1+j6)(s+1-j6)}\right)X$$

a) What is the differential equation relating X and Y?

Cross multiply and multiply out

$$(s+1+j6)(s+1-j6)Y = (2s+30)X$$

$$(s^2+2s+37)Y = (2s+30)X$$

sY means *the derivative of y*

$$y'' + 2y' + 37y = 2x' + 30x$$

b) Determine y(t) assuming

$$x(t) = 3\cos(5t) + 2\sin(5t)$$

Use phasors

$$s = j5$$

$$X = 3 - j2$$

$$Y = \left(\frac{2s+30}{(s+1+j6)(s+1-j6)}\right)_{s=j5} \cdot (3-j2)$$

$$Y = 4.1803 - j5.9836$$

meaning

$$y(t) = 4.1803\cos(5t) + 5.9836\sin(5t)$$

c) Determine y(t) assuming x(t) is the unit step function (0 for t<1, 1 for t>0)

$$x(t) = u(t)$$

$$Y = \left(\frac{2s+30}{(s+1+j6)(s+1-j6)}\right)X$$

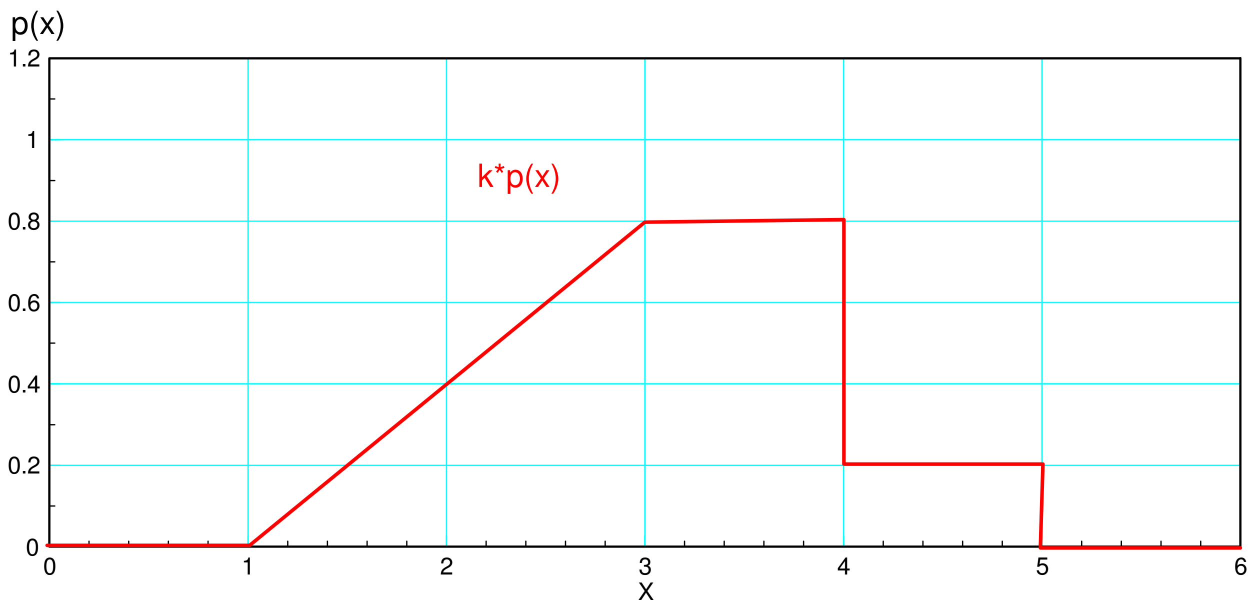
$$Y = \left(\frac{2s+30}{(s+1+j6)(s+1-j6)}\right)\left(\frac{1}{s}\right)$$

$$Y = \left(\frac{0.8108}{s}\right) + \left(\frac{0.4173\angle 166^0}{s+1+j6}\right) + \left(\frac{0.4173\angle -166^0}{s+1-j6}\right)$$

takin gthe inverse LaPlace transform

$$y(t) = 0.8108 + 0.8347e^{-t}\cos(6t - 166^0) \qquad t > 0$$

Continuous Probability Density Functions



3) Determine the scalar so that the above function is a valid pdf (i.e. the total area is 1.000)

The current area is 0.8 + 0.8 + 0.2 = 1.8

Divide by 1.8 to make the total area one

$$k = \frac{1}{1.8} = 0.5556$$

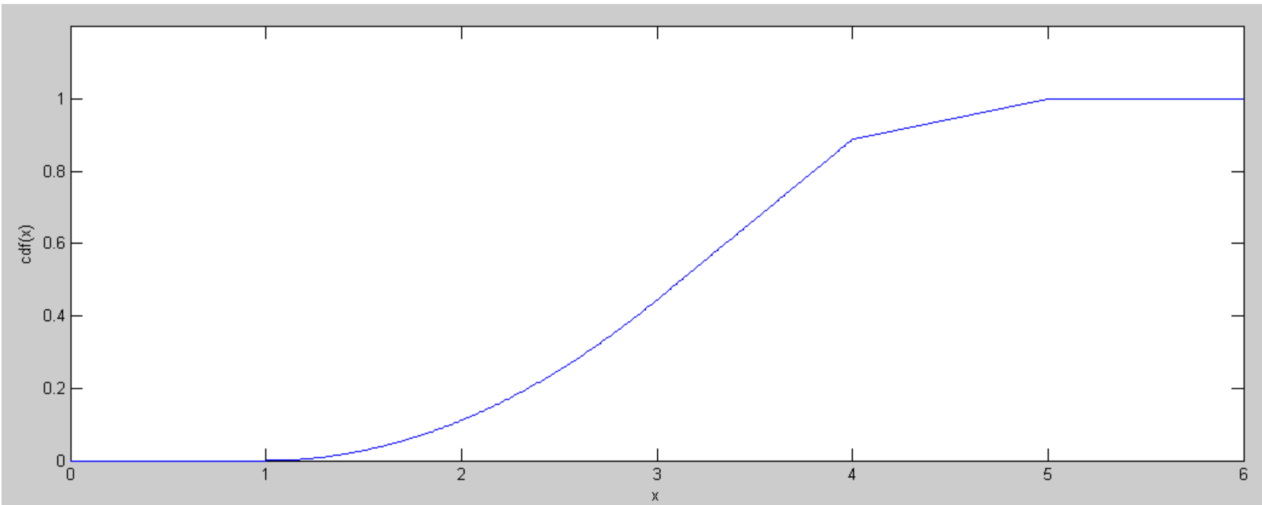
4) Determine the corresponding cdf

$$1.8 \cdot pdf = \begin{cases} 0 & x < 1 \\ 0.4(x-1) & 1 < x < 3 \\ 0.8 & 3 < x < 4 \\ 0.2 & 4 < x < 5 \\ 0 & x > 5 \end{cases}$$

Integrate

$$1.8 \cdot cdf = \begin{cases} 0 & x < 1 \\ 0.2x^2 - 0.4x + 0.2 & 1 < x < 3 \\ 0.8x - 1.6 & 3 < x < 4 \\ 0.2x + 0.8 & 4 < x < 5 \\ 1.8 & x > 5 \end{cases}$$

```
>> x = [0:0.01:6]';
>> C = 0*x;
>> for i=1:length(x)
C(i) = cdf(x(i));
end
>> plot(x,C);
>> ylim([0,1.2]);
>> xlabel('x');
>> ylabel('cdf(x)');
>>
```



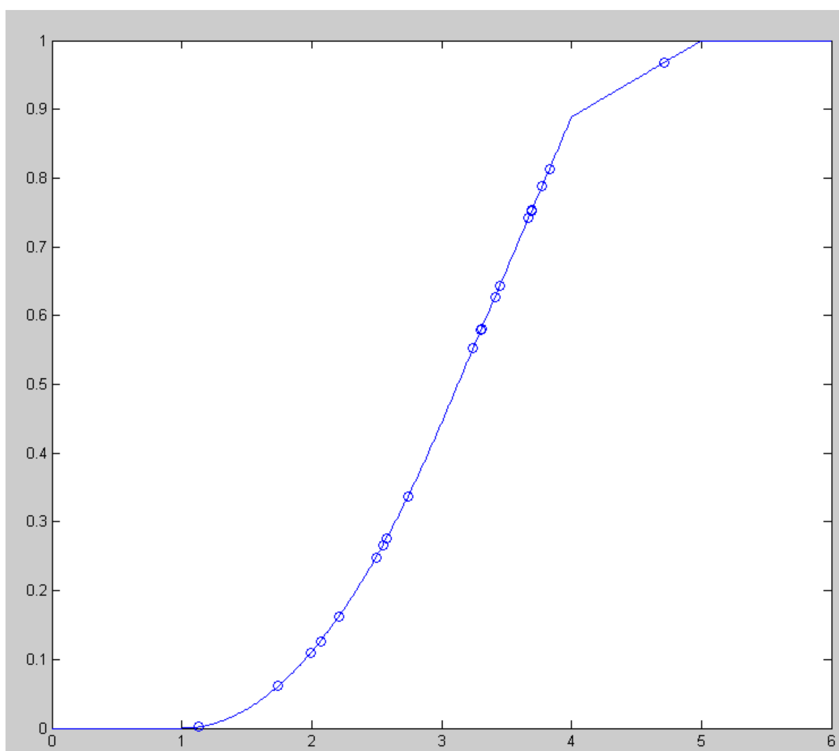
5) Using Matlab, find 20 random values of x for the above pdf

Solve using interval halving:

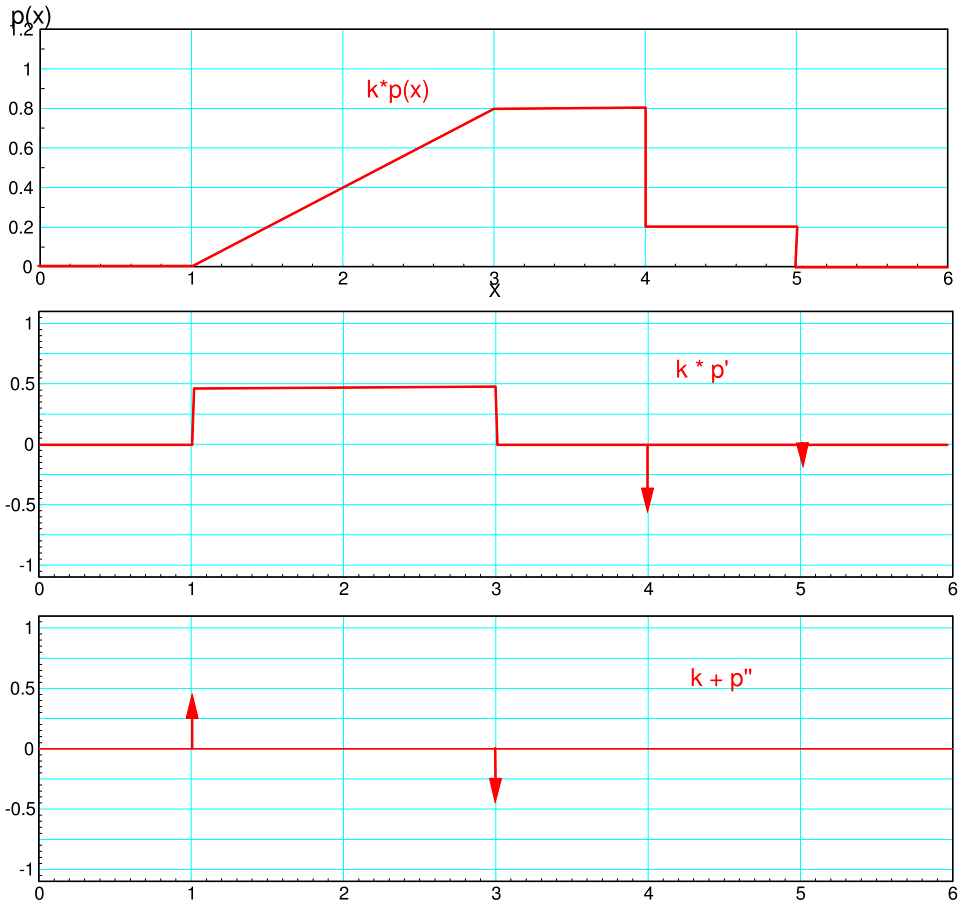
```
function [x] = Prob5(p)

x1 = 1;
p1 = cdf(x1) - p;
x2 = 5;
p2 = cdf(x2) - p;
for i=1:20
    x3 = (x1+x2)/2;
    p3 = cdf(x3) - p;

    if(p3<0) x1 = x3;
    else     x2 = x3;
    end

%    disp([x1,x2,cdf(x3)])
%    pause(0.5);
end
x = x3;
end
```

Generate 20 random probabilities and compute the correspnding X

```
>> p = rand(20,1);
x = 0*p;
for i=1:20
   x(i) = Prob5(p(i));
   end
plot(x,p,'x')
[p,x]

     p          x
   0.2755    2.5748
   0.7885    3.7742
   0.5801    3.3053
   0.0613    1.7429
   0.5523    3.2427
   0.1265    2.0672
   0.1098    1.9939
   0.7515    3.6910
   0.9682    4.7142
   0.7427    3.6711
   0.7534    3.6953
   0.5792    3.3031
   0.3376    2.7431
   0.1624    2.2091
   0.2666    2.5491
   0.6435    3.4480
   0.6269    3.4105
   0.0019    1.1300
   0.8136    3.8306
   0.2483    2.4948
```



6) Find the moment generating function for p(x)



$$\psi(s) = \left(\frac{1}{1.8s}\right)(-0.6e^{-4s} - 0.2e^{-5s}) + \left(\frac{1}{1.8s^2}\right)(0.4e^{-s} - 0.4e^{-3s})$$