ECE 341 - Homework #5

Geometric & Pascal Distributions. Summer 2024

Geometric Distributions

Let A be the number of times you roll a 10-sided die until you get a 1 (p = 1/10)

1) Determine the pdf of A using z-transforms. From this, compute

- The probabilty that A = 10
- The probability that A >= 10

$$A = \left(\frac{1/10}{z-9/10}\right)$$

$$zA = \left(\frac{0.1z}{z-0.9}\right)$$

$$za(k) = 0.1 \cdot (0.9)^k u(k)$$

$$a(k) = 0.1(0.9)^{k-1} u(k-1)$$

Probability of k = 10

$$a(10) = 0.03874$$

Probability k >= 10

This is an infinite series. Use Matlab to sum k=10 to 1000

```
>> k = [1:1000]';
>> A = (1/10) * (9/10) .^ (k-1);
>> A(10)

ans =    0.0387

>> sum(A(10:1000))

ans =    0.3874
```

2) Use a Monte-Carlo simulation with 100,000 A's. From your Monte-Carlo simulation, determine

- The probability that A = 10
- The probability that A >= 10

Matlab Code

```
Result = zeros(200,1);
for n=1:1e5
    N = 1;
    while(rand > 1/10)
        N = N + 1;
    end
    Result(N) = Result(N) + 1;
end

Result(10)

ans =         3845
```

Calculated odds were 3874 in 100,000 (close to the Monte-Carlo results)

```
sum(Result(10:200))

ans =       38994
```

Calculated odds were 38,742 (close to the Monte-Carlo results)

	Calculated	Monte-Carlo #1	Monte-Carlo #2
p(A = 10)	3,874	3,845	3,897
p(A >= 10)	38,742	38,994	38,502

Pascal Distribution

Let

- A be the number of times you roll an 10-sided die until you get a 1 (p = 1/10), and
- B be the number of times you roll an 10-sided die until you get a 1 or 2 (p = 1/5).
- X = A + B

3) Determine the pdf of X using z-transforms. From this comptue

- The probability that X = 20
- The probability that X >= 20

$$A = \left(\frac{1/10}{z-9/10}\right)$$

$$B = \left(\frac{2/10}{z-8/10}\right)$$

$$X = AB = \left(\frac{0.1}{z-0.9}\right)\left(\frac{0.2}{z-0.8}\right)$$

$$zX = \left(\frac{0.02}{(z-0.9)(z-0.8)}\right)z$$

$$zX = \left(\left(\frac{0.2}{z-0.9}\right) + \left(\frac{-0.2}{z-0.8}\right)\right)z$$

$$zX = \left(\frac{0.2z}{z-0.9}\right) + \left(\frac{-0.2z}{z-0.8}\right)$$

$$zx(k) = \left(0.2(0.9)^k - 0.2(0.8)^k\right)u(k)$$

$$x(k) = \left(0.2(0.9)^{k-1} - 0.2(0.8)^{k-1}\right)u(k-1)$$

p(X = 20)

```
>> k = [1:1000]';
>> x = 0.2 * ( (0.9).^(k-1) - 0.8.^(k-1) );
>> x(20)

ans =    0.0241
```

p(X >= 20)

```
>> sum(x(20:1000))

ans =    0.2558
```

Check: total probability is one:

```
>> sum(x)

ans =     1
```

4) Determine the pdf of X using convolution. From this, compute

- The probability that X = 20
- The probability that X >= 20

```
>> k = [0:200]';
>> A = 1/10 * (9/10).^(k-1) .* (k>0);
>> B = 2/10 * (8/10).^(k-1) .* (k>0);
>> X = conv(A,B);

>> sum(X)

ans =    1.0000
```

This is a valid pdf: sum of all probabilities is one

```
>> X(21)

ans =    0.0241
```

The probability that X = 20 is 0.0241 (same as z-transforms)

```
>> sum(X(21:400))

ans =    0.2558
```

The probability that X >= 20 is 25.58% (same as z-transforms)

5) Use a Monte-Carlo simulation with 100,000 X's. From your Monte-Carlo simulation, determine

- The probability that X = 20
- The probability that X >= 20

Monte-Carlo Code:

```
Result = zeros(200,1);
for n=1:1e5
    A = 1;
    while(rand > 1/10)
        A = A + 1;
    end
    B = 1;
    while(rand > 2/10)
        B = B + 1;
    end
    X = A + B;
    Result(X) = Result(X) + 1;
end

Result(20)

sum(Result(20:200))
```

	Calculated	Monte-Carlo #1	Monte-Carlo #2
p(X = 20)	2,413	2,333	2,405
p(X >= 20)	25,576	25,342	25,694

(problem 6-8: over)

Pascal Distribution (cont'd)

Let

- A be the number of times you roll a 10-sided die until you roll a 1 (p = 1/10)
- B be the number of times you roll a 8-sided die until you get a 1 (p = 1/8)
- C be the number of times you roll a 6-sided die until you get a 1 (p = 1/6)
- Y = A + B + C

6) Determine the pdf of Y using z-transforms. From this comptue

- The probability that Y = 20
- The probability that Y >= 20

$$A = \left(\frac{1/10}{z-9/10}\right)$$

$$B = \left(\frac{1/8}{z-7/8}\right)$$

$$C = \left(\frac{1/6}{z-5/6}\right)$$

$$Y = ABC = \left(\frac{1/10}{z-9/10}\right)\left(\frac{1/8}{z-7/8}\right)\left(\frac{1/6}{z-5/6}\right)$$

$$zY = ABC = \left(\frac{1/480}{(z-9/10)(z-7/8)(z-5/6)}\right)z$$

$$zY = \left(\left(\frac{1.25}{z-9/10}\right) + \left(\frac{-2}{z-7/8}\right) + \left(\frac{0.75}{z-5/6}\right)\right)z$$

$$zy(k) = \left(1.25\left(\frac{9}{10}\right)^k - 2\left(\frac{7}{8}\right)^k + 0.75\left(\frac{5}{6}\right)^k\right)u(k)$$

$$y(k) = \left(1.25\left(\frac{9}{10}\right)^{k-1} - 2\left(\frac{7}{8}\right)^{k-1} + 0.75\left(\frac{5}{6}\right)^{k-1}\right)u(k-1)$$

Probability that k = 20:

```
>> k = [1:1000]';
>> y = 1.25*(9/10).^(k-1) - 2*(7/8).^(k-1) + 0.75*(5/6).^(k-1);
>> sum(y)

ans =    1.0000
```

This is a valid pdf (probabilities all add to one)

```
>> y(20)

ans =    0.0341

>> sum(y(20:1000))

ans =    0.5639
```

7) Determine the pdf of Y using convolution. From this, compute

- The probability that Y = 20
- The probability that Y >= 20

```
>> k = [0:200]';
>> A = 1/10 * (9/10).^(k-1) .* (k>0);
>> B = 1/8 * (7/8).^(k-1) .* (k>0);
>> C = 1/6 * (5/6).^(k-1) .* (k>0);

>> AB = conv(A,B);
>> ABC = conv(AB,C);
>> ABC(21)

ans =    0.0341

>> sum(ABC(21:600))

ans =    0.5639

>>
```

Same results as z-transform

8) Use a Monte-Carlo simulation with 100,000 Y's. From your Monte-Carlo simulation, determine

- The probability that Y = 20
- The probability that Y >= 20

Matlab Code:

```
Result = zeros(200,1);
for n=1:1e5
    A = 1;
    while(rand > 1/10)
        A = A + 1;
    end
    B = 1;
    while(rand > 1/8)
        B = B + 1;
    end
    C = 1;
    while(rand > 1/6)
        C = C + 1;
    end

    X = A + B + C;
    Result(X) = Result(X) + 1;
end

Result(20)

sum(Result(20:200))
```

	Calculated	Monte-Carlo #1	Monte-Carlo #2
p(X = 20)	3,414	3,344	3,364
p(X >= 20)	56,389	56,289	56,493