ECE 341 - Homework #4

Binomial and Uniform Distributions. Summer 2024

Binomial Distribution

In D&D, a 5th-level fighter needs to roll a 14 or higher on a 20-sided die to save vs. spell (p = 7/20).

$$X(z) = \left(\frac{0.65z+0.35}{z} \right)$$

1) Determine the probability of making six saving throws in 10 rolls

This is a binomial distribution with p = 0.35 (7/20)

$$p(m) = \binom{n}{m} p^m (1-p)^{n-m}$$

$$p(m=6) = \binom{10}{6} (0.35)^6 (0.65)^4$$

$$p(m=6) = 0.06891$$

2) Determine the probability distribution when rolling this die 10 times

Option #1: Use a binomial distribution equation for m=0..10

Option #2: Use convolution

```
>> p1 = [0.65,0.35];
>> p2 = conv(p1,p1);
>> p4 = conv(p2,p2);
>> p8 = conv(p4,p4);
>> p10 = conv(p8,p2);
>> m = [0:10]';
>> [m,p10']

         0    0.0135
    1.0000    0.0725
    2.0000    0.1757
    3.0000    0.2522
    4.0000    0.2377
    5.0000    0.1536
    6.0000    0.0689
    7.0000    0.0212
    8.0000    0.0043
    9.0000    0.0005
   10.0000    0.0000

>>
```

NOAA has been keeping track of world weather for the past 142 years. 27 of last 30 years have been in the 30 hottest years on record.

3a) What is the probability of any given year being one of the 30 hottest on record (i.e. what is p?)

p = 30/142 = 0.21127

3b) What is the probability of 27 of the last 30 years being the hottest on record?

Again, this is a binomial distribution with p = 30/142

$$p(m) = \binom{n}{m} p^m (1-p)^{n-m}$$

$$p(m=27) = \binom{30}{27} (0.21127)^{27} (0.78873)^3$$

$$p(m=27) = 1.174 \cdot 10^{-15}$$

There *is* a chance that nothing is changing and this is just random. The odds against are 8.515e14 : 1 against.

Uniform Distribution

4) The D&D spell *Insect Swarm* does 4-40 damage (the sum of rolling four 10-sided dice).

Y = 4d10

Determine the pdf, mean and standard deviation, and the probability of doing 30 or more damage with an *Insect Swarm* spell.

There are a couple of ways to find the pdf:

- Convolution using Matlab
- Convolution by hand
- z-transforms

z-Transforms actually works out to be the same as convolution: The z-transform for a 10-sided die is:

$$X = \left(\frac{0.1}{z} + \frac{0.1}{z^2} + \frac{0.1}{z^3} + \frac{0.1}{z^4} + \frac{0.1}{z^5} + \frac{0.1}{z^6} + \frac{0.1}{z^7} + \frac{0.1}{z^8} + \frac{0.1}{z^9} + \frac{0.1}{z^{10}} \right)$$

or, putting over a common denominator

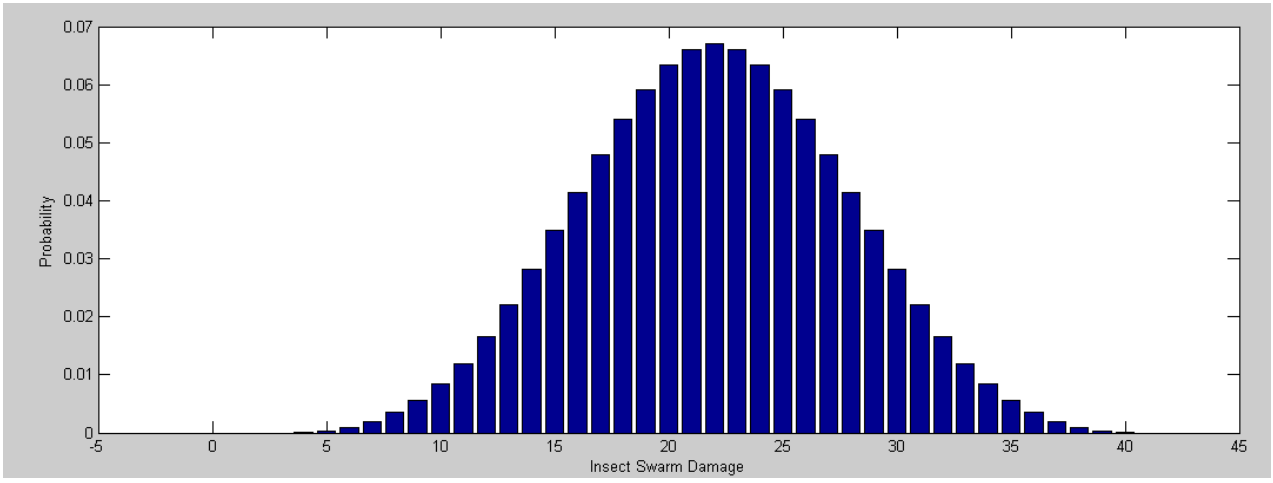
$$X = \left(\frac{1+z+z^2+z^3+z^4+z^5+z^6+z^7+z^8+z^9}{10z^{10}} \right)$$

The sum of four 10-sided dice is then the product of the z-transforms

$$Y = \left(\frac{1+z+z^2+z^3+z^4+z^5+z^6+z^7+z^8+z^9}{10z^{10}} \right)^4$$

Multiplying polynomils is convolution - so I'm stuck using colvolution. Using the first method (seems easiest)

```
>> d10x1 = [0,1,1,1,1,1,1,1,1,1,1]' / 10;
>> d10x2 = conv(d10x1, d10x1);
>> d10x4 = conv(d10x2, d10x2);
>> n = [0:40]';
>> bar(n,d10x4)
>> xlabel('Insect Swarm Damage');
>> ylabel('Probability');
>>
```



sidelight: This bell-shaped curve is called a normal distribution. It'll keep appearing over and over again in this class.

Finding the mean and standard deviation:

$$\mu = \sum p_i \cdot x_i$$

$$\sigma^2 = \sum p_i \cdot (x_i - \mu)^2$$

In Matlab:

```
>> Mean = sum(d10x4 .* n)

Mean =   22.0000

>> Variance = sum( d10x4 .* (n - Mean).^2 )

Variance =   33.0000

>> StDev = sqrt(Variance)

StDev =    5.7446
```

The probability of doing 30 or more damage:

```
>> size(d10x4)

ans =    41     1

>> sum(d10x4(31:41))

ans =    0.0997
```

There is a 9.97% chance of doing 30 or more damage with an *insect swarm* spell.

5) The D&D spell *Ice Storm* does 6-40 damage (the sum of four 6-sided dice and two 8-sided dice).

Y = 4d6 + 2d8

Determine the

- pdf
- mean and standard deviation
- The probability of doing 30 or more damage with an *Ice Storm* spell

Again, use convolution in Matlab

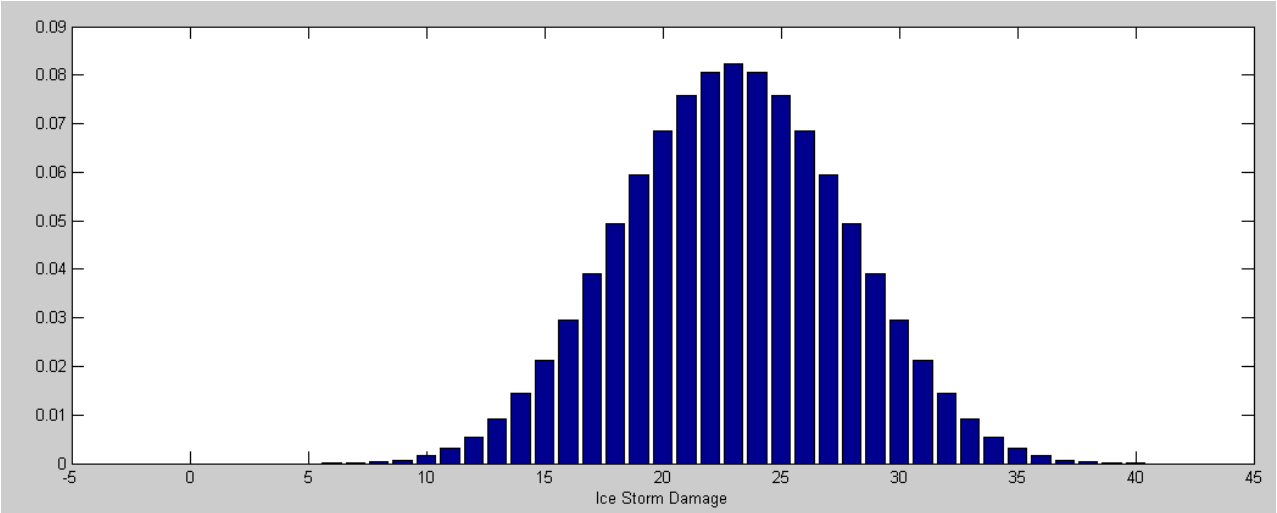
```
>> d6 = [0,1,1,1,1,1,1]' / 6;
>> d8 = [0,1,1,1,1,1,1,1,1]' / 8;
>> d6x2 = conv(d6,d6);
>> d6x4 = conv(d6x2,d6x2);
>> d8x2 = conv(d8,d8);
>> IceStorm = conv(d6x4,d8x2);
>> size(IceStorm)

ans =     41     1

>> n = [0:40]';
>> bar(n,IceStorm)
>> xlabel('Ice Storm Damage');
>> sum(IceStorm)

ans =    1.0000
```

The total probablity is one: this is a valid pdf



Mean and Standard Deviation:

```
>> Mean = sum(IceStorm .* n)

Mean =   23.0000

>> Variance = sum( IceStorm .* (n - Mean).^2 )

Variance =   22.1667

>> StDev = sqrt(Variance)

StDev =    4.7081
```

Probability of doing 30 or more damage

```
>> sum(IceStorm(31:41))

ans =    0.0857
```

6) Assme you roll a 4, 6, 8, 10, and 12 sided die and take the sum

Y = d4 + d6 + d8 + d10 + d12

Determine

- The pdf for Y
- The mean and standard deviation of Y, and
- The probability that the sum is 50 or more

Again, use convolution and Matlab

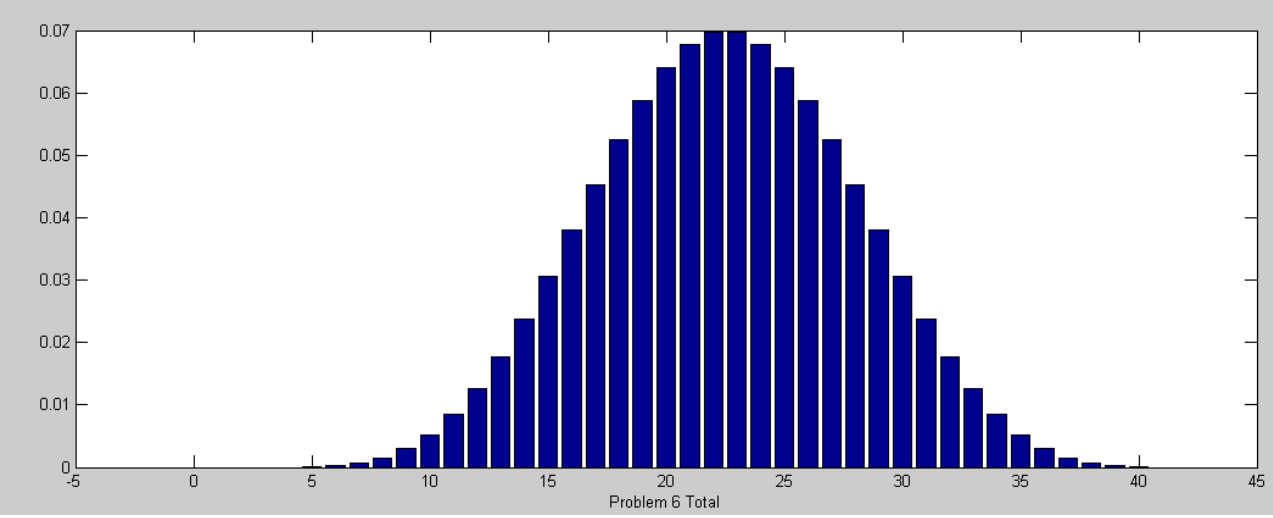
```
>> d4 = [0,1,1,1,1]' / 4;
>> d6 = [0,1,1,1,1,1,1]' / 6;
>> d8 = [0,1,1,1,1,1,1,1,1]' / 8;
>> d10 = [0,1,1,1,1,1,1,1,1,1,1]' / 10;
>> d12 = [0,1,1,1,1,1,1,1,1,1,1,1,1]' / 12;
>> d4d6 = conv(d4,d6);
>> d8d10 = conv(d8,d10);
>> d4d6d8d10 = conv(d4d6,d8d10);
>> Prob6 = conv(d4d6d8d10,d12);
>> size(Prob6)

ans =    41     1

>> n = [0:40]';
>> bar(n,Prob6)
>> xlabel('Problem 6 Total');
>>
>> sum(Prob6)

ans =     1.0000
```

All probabilities add to one: this is a valid pdf



Mean and Standard Deviation:

```
>> Mean = sum(Prob6 .* n)

Mean =   22.5000

>> Variance = sum( Prob6 .* (n - Mean).^2 )

Variance =   29.5833

>> StDev = sqrt(Variance)

StDev =    5.4391
```

Probability the sum is 30 or more:

```
>> sum(Prob6(31:41))

ans =    0.1039
```