ECE 341 - Homework #3

Dice Games and z-Transform - Summer 2023

Farkle (6 dice)

In the game of Farkle, you initially roll six dice

1) Compute using combinatorics the odds of rolling two tripples when rolling 6 dice one time

dice = xxx yyy

M = (6 numbers choose 2)(6 spaces for x choose 3)(3 remain spaces for y, choose 3)

$$M = \binom{6}{2}\binom{6}{3}\binom{3}{3}$$

$$M = 300$$

this is the same result from an exhaustive search

$$N = 6^6$$

$$p = \frac{M}{N} = 0.006430$$

2) Compute using combinatorics the odds of rolling two-pair when rolling 6 dice one time

dice = xx yy ab

M = (6 numbers choose 2 for xy)(6 spots for x choose 2)(4 spots for y choose 2)(4c1 for a)(3c1 for b)

$$M = \binom{6}{2}\binom{6}{2}\binom{4}{2}\binom{4}{1}\binom{3}{1}$$

$$M = 16,200$$

This is the same result as enumeration

3) Compute using combinatorics the odds of rolling three pairs when rolling 6 dice one time

dice = xx yy zz

From enumeration, M = 1800

M = (6 numbers choose 3 for xyz)(6 spots for x choose 2)(4 spots for y choose 2)(2 spots for z choose 2)

$$M = \binom{6}{3}\binom{6}{2}\binom{4}{2}\binom{2}{2}$$

$$M = 1,800$$

Same answer as enumeration

4) Write a Matlab program which computes the number of ways to roll six dice and

- Get two tripples
- Get two pair, and
- Get three pairs

Compare your answers to what you computed using combinatorics

```
6ok     5ok     4ok     3ok       Pair222       Pair42      Pair33    Pair22
6       180     1800    14400        1800         450         300       16200
```

Same answer as combinatorics

```
% Farkle

Pair6 = 0;
Pair5 = 0;
Pair4 = 0;
Pair3 = 0;
Pair222 = 0;
Pair33 = 0;
Pair42 = 0;
Pair22 = 0

for d1 = 1:6
    for d2 = 1:6
        for d3 = 1:6
            for d4 = 1:6
                for d5 = 1:6
                    for d6 = 1:6
                        Dice = [d1,d2,d3,d4,d5,d6];
                        Dice = sort(Dice);
  % check for pairs
                        N = zeros(1,6);
                        for i=1:6
                            N(i) = sum(Dice == i);
                        end
                        [N,b] = sort(N, 'descend');

                        if (N(1) == 6) Pair6 = Pair6 + 1;
                        elseif (N(1) == 5) Pair5 = Pair5 + 1;
                        elseif ((N(1)==4)*(N(2)==2)) Pair42 = Pair42 + 1;
                        elseif (N(1)==4) Pair4 = Pair4 + 1;
                        elseif ((N(1)==3)*(N(2)==3)) Pair33 = Pair33 + 1;
                        elseif (N(1)==3) Pair3 = Pair3 + 1;
                        elseif ((N(1)==2)*(N(2)==2)*(N(3)==2)) Pair222=Pair222+1;
                        elseif ((N(1)==2)*(N(2)==2)*(N(3)<2)) Pair22 = Pair22+1;
                        end
                    end
                end
            end
        end
    end
end

[Pair6,Pair5,Pair4,Pair3,Pair222,Pair42,Pair33, Pair22]
```

z-Transforms

Find the inverse z-transform

5) $$X = \left(\frac{0.001z(z+1)^2}{(z-0.99)(z-0.96)(z-0.95)} \right)$$

Pull out a z

$$X = \left(\frac{0.001(z+1)^2}{(z-0.99)(z-0.96)(z-0.95)} \right) z$$

Do a partial fraction expansion

$$X = \left(\left(\frac{3.3}{z-0.99} \right) + \left(\frac{-12.8053}{z-0.96} \right) + \left(\frac{9.5063}{z-0.95} \right) \right) z$$

Distribute the z

$$X = \left(\frac{3.3z}{z-0.99} \right) + \left(\frac{-12.8053z}{z-0.96} \right) + \left(\frac{9.5063z}{z-0.95} \right)$$

Take the inverse z-transform

$$x(k) = \left(3.3(0.99)^k - 12.8053(0.96)^k + 9.5063(0.95)^k \right) u(k)$$

6) $$X = \left(\frac{0.001z(z+1)^2}{(z-1)(z-0.99)(z-0.96)} \right)$$

Pull out a z

$$X = \left(\frac{0.001(z+1)^2}{(z-1)(z-0.99)(z-0.96)} \right) z$$

Do a partial fraction expansion

$$X = \left(\left(\frac{10}{z-1} \right) + \left(\frac{-13.2}{z-0.99} \right) + \left(\frac{3.2}{z-0.96} \right) \right) z$$

Distribute the z

$$X = \left(\frac{10z}{z-1} \right) + \left(\frac{-13.2z}{z-0.99} \right) + \left(\frac{3.2z}{z-0.96} \right)$$

Take the inverse z transform

$$x(k) = \left(10 - 13.2(0.99)^k + 3.2(0.96)^k \right) u(k)$$

7) A new Volkswagen ID.4 costs $49,441 from Cars.com. If you take out a 60-month loan at 7.66% interest, what is your monthly payment? Solve using z-transforms.

Let x(k) be the amount you owe at month k

The loan value at the start of the next month is

- p = monthly payments starting at month #1
- L = loan amount (lump sum at k=0)

$$x(k+1) = \left(1 + \frac{0.0766}{12}\right) x(k) - p \cdot u(k-1) + L \cdot \delta(k)$$

Take the z-transform

$$zX = (1.006383)X - p\left(\frac{1}{z-1}\right) + L$$

$$(z - 1.006383)X = -p\left(\frac{1}{z-1}\right) + L$$

$$X = -p\left(\frac{1}{(z-1)(z-1.006383)}\right) + L\left(\frac{1}{z-1.006383}\right)$$

Do a patrial fraction expansion

$$X = 156.658p\left(\frac{1}{z-1} - \frac{1}{z-1.006383}\right) + \left(\frac{L}{z-1.006383}\right)$$

Multiply both sides by z

$$zX = -156.658p\left(\frac{z}{z-1.006383} - \frac{z}{z-1}\right) + L\left(\frac{z}{z-1.006383}\right)$$

Take the inverse z transform

$$zx(k) = \left(-156.658p\left((1.006383)^k - 1\right) + L(1.006383)^k\right) u(k)$$

Divide by z (delay by one)

$$x(k) = \left(-156.658p\left((1.006383)^{k-1} - 1\right) + L(1.006383)^{k-1}\right) u(k-1)$$

After 60 payments, the loan is zero

$$x(61) = 0 = -72.82478p + 1.464865L$$

$$p = \frac{1.465865}{72.82478} \cdot L$$

$$p = \$994.50$$

The payments are $994.50/month for 60 months