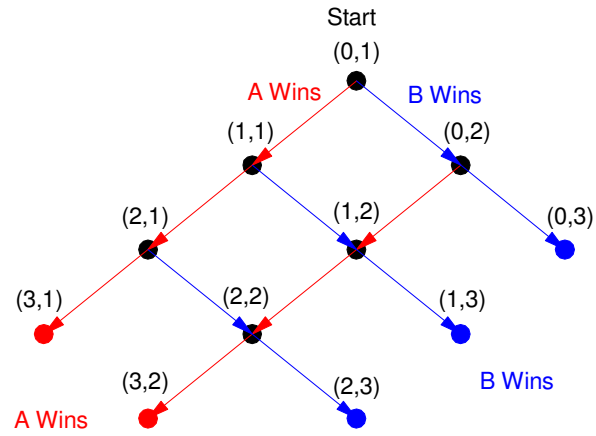# ECE 341 - Homework #1

Tree Diagrams and Enumeration. Summer 2024

1) Two teams, A and B, are playing a best of 5 game series.

- The series is over once one team wins 3 games.
- B starts with +1 points (odds)

Draw the tree diagram for all possible outcomes of the series.

2) List all possible combinations of rolling a 4-sided die and a 6-sided die.

- Also determine the probability of X {0..5} where X is the difference of the die rolls.

	1	2	3	4	5	6
1	(1,1) x=0	(1,2) x=1	(1,3) x=2	(1,4) x=3	(1,5) x=4	(1,6) x=5
2	(2,1) x=1	(2,2) x=0	(2,3) x=1	(2,4) x=2	(2,5) x=3	(2,6) x=4
3	(3,1) x=2	(3,2) x=1	(3,3) x=0	(3,4) x=1	(3,5) x=2	(3,6) x=3
4	(4,1) x=3	(4,2) x=2	(4,3) x=1	(4,4) x=0	(4,5) x=1	(4,6) x=2

The number of results for x is:

- 0: 4 times
- 1: 7 times
- 2: 6 times
- 3: 4 times
- 4: 2 times
- 5: 1 time

The odds for the difference is then

- 0: 4/24
- 1: 7/24
- 2: 6/24
- 3: 4/24
- 4: 2/24
- 5: 1/24

Two players, A and B, are playing a game of dice.

- Player A rolls a 4-sided die and a 6-sided die and takes the difference (i.e. problem #2)
- Player B rolls a 6-sided die and subtracts one.

Player A wins on ties.

The odds for the difference is then

- A=0:   4/24
- A=1:   7/24
- A=2:   6/24
- A=3:   4/24
- A=4:   2/24
- A=5:   1/24

3) What is the conditional probability

- Player A wins given B's score is 3 (B rolled a 4)

In order to win, A has to score 3, 4, or 5

$$p(A = 3) = 4/24$$

$$p(A = 4) = 2/24$$

$$p(A = 5) = 1/24$$

The total is

$$p(A = \{3, 4, 5\}) = 7/24$$

Player A has a 7/24 chance of winning given that B scored 3 points.

4) What is the probability that player A will win any given game?

The odds for the difference is then
- A=0:   4/24
- A=1:   7/24
- A=2:   6/24
- A=3:   4/24
- A=4:   2/24
- A=5:   1/24

This is a conditional probability

$$p(A) = p(A|B=0)p(B=0) + p(A|B=1)p(B=1) + p(A|B=2)p(B=2) + p(A|B=3)p(B=3) + p(A|B=4)p(B=4) + p(A|B=5)p(B=5)$$

Doing each part

$p(A|B=0)p(B=0)$:

$$= (1)*(1/6)$$

$p(A|B=1)p(B=1)$

$$= p(A=\{1,2,3,4,5\})p(B=1)$$

$$= (20/24) * (1/6)$$

$p(A|B=2)p(B=2)$

$$= p(A = \{2,3,4,5\})p(B=2)$$

$$= (13/24) * (1/6)$$

$p(A|B=3)p(B=3)$

$$= p(A = \{3,4,5\}) \, p(B=3)$$

$$= (7/24) * (1/6)$$

$p(A|B=4) \, p(B=4)$

$$= p(A = \{4,5\}) * p(B=4)$$

$$= (3/24) * (1/6)$$

$p(A|B=5) \, p(B=5)$

$$= p(A = 5) * p(B=5)$$

$$= (1/24) * (1/6)$$

Adding them all up

$$p(A) = 68 / 144$$

$$p(A) = 0.4722$$

A has a 30.55% chance of winning any given game

Enumeration & Farkle

Write a Matlab program to go through every combination of 6d6 and determine...

5) The odds of rolling two tripples

      dice = xxx yyy   x, y different values

Odds are 300 / 46,656

6) The odds of rolling two pair

      dice = xx yy ab   x, y, a, b different

Odds are 16,200 / 46,656

matlab Code:

```
% Farkle

Pair33 = 0;
Pair22 = 0

for d1 = 1:6
    for d2 = 1:6
        for d3 = 1:6
            for d4 = 1:6
                for d5 = 1:6
                    for d6 = 1:6
                        Dice = [d1,d2,d3,d4,d5,d6];
                        Dice = sort(Dice);
    % check for pairs
                        N = zeros(1,6);
                        for i=1:6
                            N(i) = sum(Dice == i);
                        end
                        [N,b] = sort(N, 'descend');

                        if (N(1) == 6) Pair6 = Pair6 + 1;
                        elseif (N(1) == 5) Pair5 = Pair5 + 1;
                        elseif ((N(1)==4)*(N(2)==2)) Pair42 = Pair42 + 1;
                        elseif (N(1)==4) Pair4 = Pair4 + 1;
                        elseif ((N(1)==3)*(N(2)==3)) Pair33 = Pair33 + 1;
                        elseif (N(1)==3) Pair3 = Pair3 + 1;
                        elseif ((N(1)==2)*(N(2)==2)*(N(3)==2)) Pair222=Pair222+1;
                        elseif ((N(1)==2)*(N(2)==2)*(N(3)<2)) Pair22 = Pair22+1;
                        end
                    end
                end
            end
        end
    end
end


[Pair33, Pair22]
```

Enumeration in 6-card Poker

warning: Enumeration took 31 minutes to run on my computer....

7) In 6-card poker, you're dealt 6 cards and keep the best 5. Determine using enumeration the odds of being dealt a full-house

      hand = (xxx yy a) or (xxx yyy)   x, y, a different values

The probability of a full-house is 0.00815304

      122.65 : 1 against

8) Determine using enumeration the odds of being dealt three of a kind

      hand = xxx a b c   a, b, c, x different values

The probability of a 3-of-a-kind is 0.035963321498812

      27.806 : 1 against

Matlab Results:

```
ans =
    Hands          Full-House   3-of-a-kind
   20358520           165984         732160

Elapsed time is 3354.129381 seconds.
>>
>> format long
>> Pair32 / H

ans =

   0.008153048453424

>> Pair3 / H

ans =

   0.035963321498812
```

Code:

```
% 6-Card Stud
% Probability of 2-pair & 2 of a kind

tic
Pair32 = 0; % full-house
Pair3 = 0;  % 3 of a kind
H = 0;      % total number of hands

for c1=1:47
  for c2 = c1+1:48
    for c3 = c2+1:49
        clc
        disp([c1,c2, c3])
        for c4 = c3+1:50
          for c5 = c4+1:51
              for c6 = c5+1:52
                  H = H + 1
                  Hand = [c1,c2,c3,c4,c5,c6];
                  Value = mod(Hand,13) + 1;
                  Suit = floor(Hand/13) + 1;

                  N = zeros(1,13);
                  for n=1:13
                      N(n) = sum(Value == n);
                  end

                  [N,a] = sort(N, 'descend');
                  if ((N(1) == 3)*(N(2) >= 2)) Pair32 = Pair32 + 1; end
                  if ((N(1) == 3)*(N(2) == 1)) Pair3 = Pair3 + 1; end
              end
          end
        end

    end
  end
end

[H, Pair32, Pair3]
toc
```