ECE 111 - Homework #15

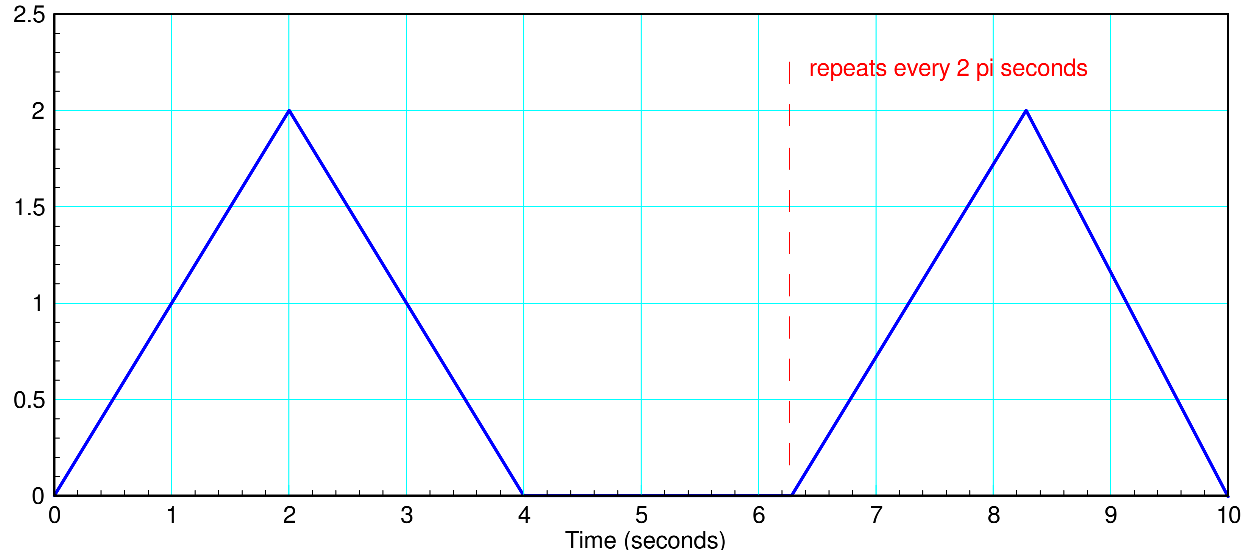
Week #11 - Signals & Frequency Content of a Signal
Due Monday, May 5th. Please submit via email or on BlackBoard

Problem 1-5) Let x(t) be a function which is periodic in 2π as shown below

$$x(t) = x(t + 2\pi)$$

or in Matlab:

```
t = [0:0.001:2*pi]' + 1e-9;
x = t .* (t<2)   +   (4-t).*(t>2).*(t<4);
plot(t,x)
```



x(t)   Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

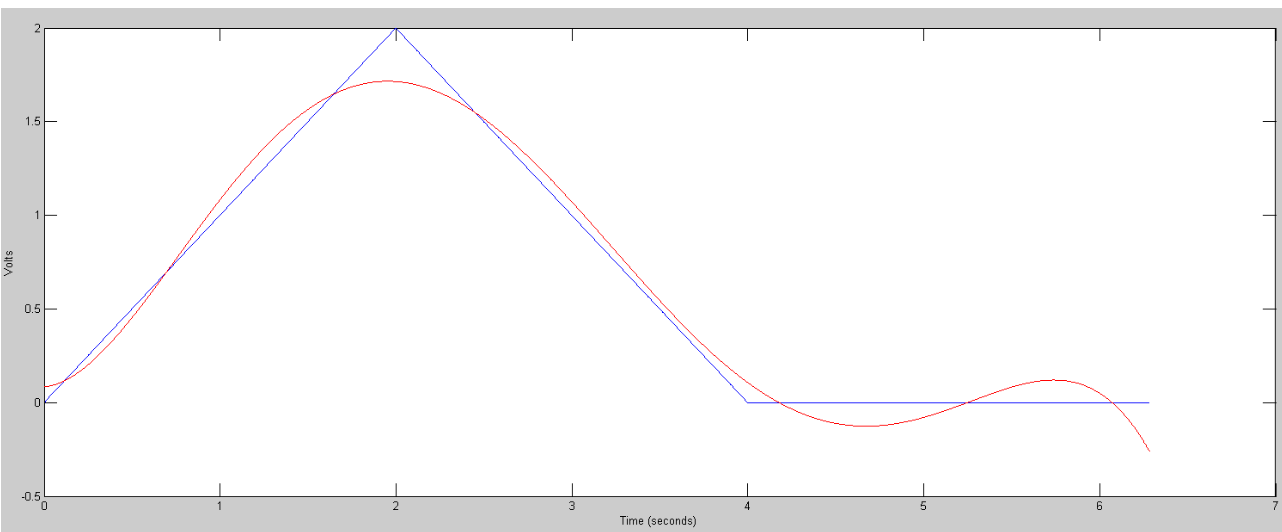
Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]' + 1e-9;
>> x = t .* (t<2)  +  (4-t).*(t>2).*(t<4);

>> B = [t.^0, t, t.^2, t.^3, t.^4, t.^5];
>> A = inv(B'*B)*B'*x

a0    0.0840
a1    0.0767
a2    1.8299
a3   -1.1126
a4    0.2196
a5   -0.0142

>> plot(t,x,'b',t,B*A,'r')
>> xlabel('Time (seconds)')
>> ylabel('Volts')
```



Comments

- *This is a reasonably good approximation for x(t)*
- *The result isn't helpful for finding y(t)*

Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

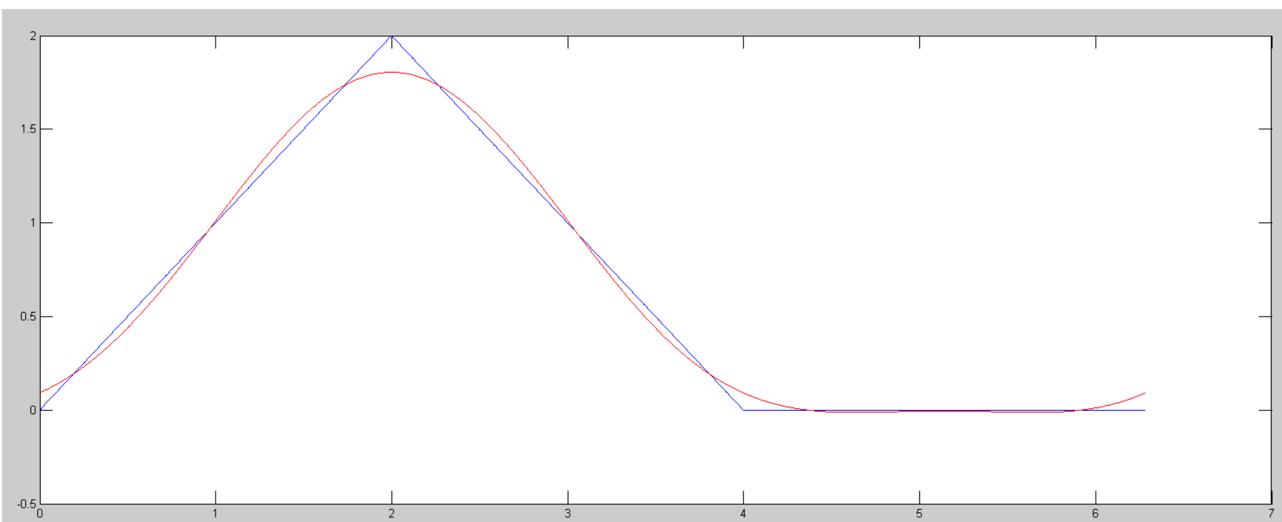
$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]' + 1e-9;
>> x = t .* (t<2)  +  (4-t).*(t>2).*(t<4);
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x

a0    0.6366
a1   -0.3752
b1    0.8198
a2   -0.1721
b2   -0.1992
a3    0.0027
b3   -0.0008

>> plot(t,x,'b',t,B*A,'r')
>>
```



Comments

- *This is also a fairly accurate approximation for x(t)*
- *In this case, the result is useful. Since x(t) is now expressed in terms of sine waves, you can find y(t) using phasors and superposition.*

Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{0.5}{s^2+s+0.5}\right)X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

Fourier Transforms is just another way to find an appoximation for x(t) in terms of sine waves. It's no different than a least squares curve fit like problem #2. It's a little more efficient to compute - but it's really no different.

```
>> a0 = mean(x)

a0 =    0.6365

>> a1 = 2*mean(x .* cos(t))

a1 =   -0.3751

>> b1 = 2*mean(x .* sin(t))

b1 =    0.8197

>> a2 = 2*mean(x .* cos(2*t))

a2 =   -0.1720

>> b2 = 2*mean(x .* sin(2*t))

b2 =   -0.1992

>> a3 = 2*mean(x .* cos(3*t))

a3 =    0.0027

>> b3 = 2*mean(x .* sin(3*t))

b3 = -7.8712e-004
```

Note the results are the same as problem #2

- *This is just another way to compute the coefficients of the sine waves*

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

same as problem #2

4) Determine the output, y(t), at DC (w = 0)

```
>> s = 0;
>> X0 = a0

X0 =    0.6365

>> Y0 = ( 0.5 / (s^2 + s + 0.5) ) * X0

Y0 =    0.6365
```

$$y_0(t) = 0.6365$$

5) Determine the output, y(t), at 1 rad/sec

```
>> s = 1i;
>> X1 = a1 - j*b1

X1 =  -0.3751 - 0.8197i

>> Y1 = ( 0.5 / (s^2 + s + 0.5) ) * X1

Y1 =  -0.2528 + 0.3140i
```

$$y_1(t) = -0.2528\cos(t) - 0.3140\sin(t)$$

6) Determine the output, y(t), at 2 rad/sec

```
>> s = 2i;
>> X2 = a2 - j*b2

X2 =  -0.1720 + 0.1992i

>> Y2 = ( 0.5 / (s^2 + s + 0.5) ) * X2

Y2 =   0.0308 - 0.0109i
```

$$y_2(t) = 0.0308\cos(2t) + 0.0109\sin(2t)$$

7) Determine the output, y(t), at 3 rad/sec

```
>> s = 3i;
>> X3 = a3 - j*b3

X3 =   0.0027 + 0.0008i

>> Y3 = ( 0.5 / (s^2 + s + 0.5) ) * X3

Y3 = -1.2695e-004 -9.1107e-005i
```

$$y_3(t) = -0.000126\cos(3t) + 0.000091\sin(3t)$$

8) Determine the total answer, y(t)

This is a linear system, meaning

$$f(a+b+c+d) = f(a)+f(b)+f(c)+d(c)$$

Sum up the previous answers to get the total y(t)

$$y(t) = y_0 + y_1 + y_2 + y_3$$

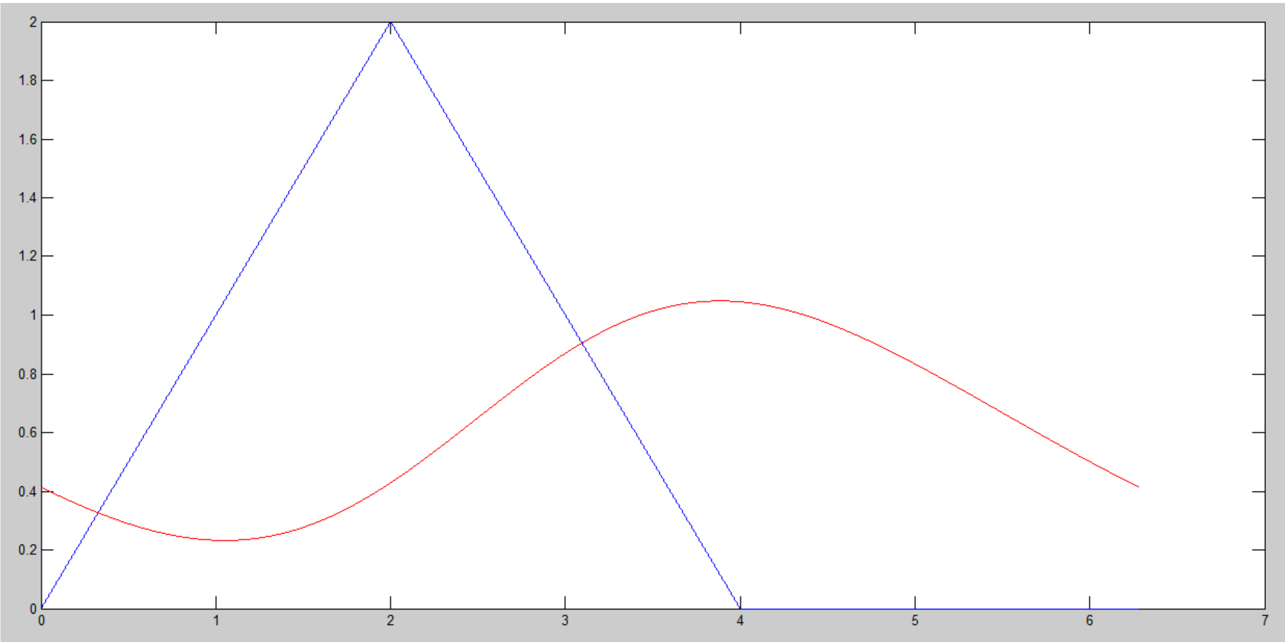
$y(t) =$	0.6365	*DC term*
	$-0.2528\cos(t) - 0.3140\sin(t)$	*1st harmonic*
	$+0.0308\cos(2t) + 0.0109\sin(2t)$	*2nd harmonic*
	$-0.000126\cos(3t) + 0.000091\sin(3t)$	*3rd harmonic*
	$\vdots$	*and so on...*

Comment

- In theory, you have to go out to infinity
- In practice, just a few terms dominate the response provide a good approximation

Plot x(t) and y(t)

```
>> y0 = a0;
>> y1 = a1*cos(t) - b1*sin(t);
>> y2 = a2*cos(2*t) - b2*sin(2*t);
>> y3 = a3*cos(3*t) - b3*sin(3*t);
>> y = y0+y1+y2+y3;
>> plot(t,x,'b',t,y,'r');
>> xlabel('Time (seconds)')
```



x(t) (blue) & y(t) (red)