ECE 111 - Homework #14

ECE 343 Signals & Systems & Filters

Filter Analysis

1) A filter has the following transfer function

$$Y = \left(\frac{100(s+4)}{(s+1)(s+3)(s+5)} \right) X$$

1a) What is the differential equation relating X and Y?

Cross multiply and multuply out the polynomials

$$(s+1)(s+3)(s+5)Y = 100(s+4)X$$

$$(s^3 + 9s^2 + 23s + 15)Y = 100(s+4)X$$

'sY' means 'the derivative of y(t)'

$$y''' + 9y'' + 23y' + 15y = 100x' + 400x$$

1b) Find y(t) assuming $x(t) = 4$

Evaluate using phasors.

$$X = 4$$

$$s = 0$$

$$Y = \left(\frac{100(s+4)}{(s+1)(s+3)(s+5)} \right)_{s=0} \cdot (4)$$

In Matlab

```
>> s = 0;
>> X = 4;
>> Y = 100*(s+4) / ( (s+1)*(s+3)*(s+5) ) * X

Y =  106.6667
```

meaning

$$y(t) = 106.667$$

1c) Find y(t) assuming $x(t) = 5\sin(6t)$

Evaluate using phasors

$X = 0 - j5$

$s = j6$

$$Y = \left(\frac{100(s+4)}{(s+1)(s+3)(s+5)} \right)_{s=j6} \cdot (0 - j5)$$

```
>> s = 6i;
>> X = 0 - 5i;
>> Y = 100*(s+4) / ( (s+1)*(s+3)*(s+5) ) * X

Y =  -7.5912 + 8.3887i
```

meaning

$$y(t) = -7.5912\cos(6t) - 8.3887\sin(6t)$$

or if you prefer polar form

```
>> abs(Y)

ans =   11.3136

>> angle(Y)

ans =    2.3063
```

$$y(t) = 11.3136\cos(6t + 2.3063)$$

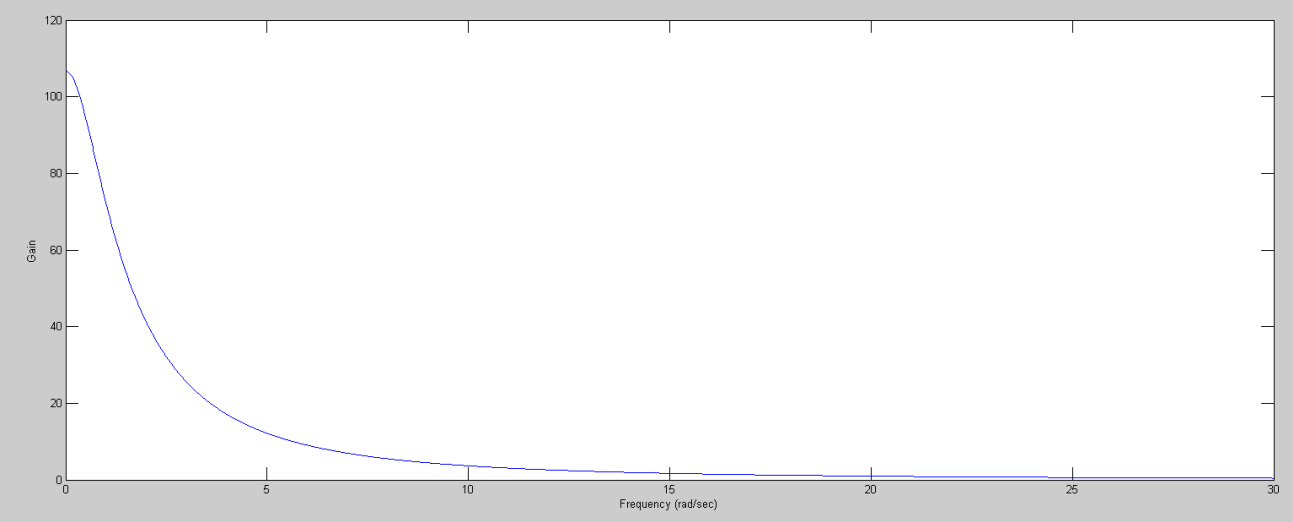
Either answer is correct

2) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{400(s+4)}{(s+1)(s+3)(s+5)} \right) X$$

```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 400 * (s+4) ./ ( (s+1) .* (s+3) .* (s+5) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)')
>> ylabel('Gain')
>>
```

This is a low-pass filter



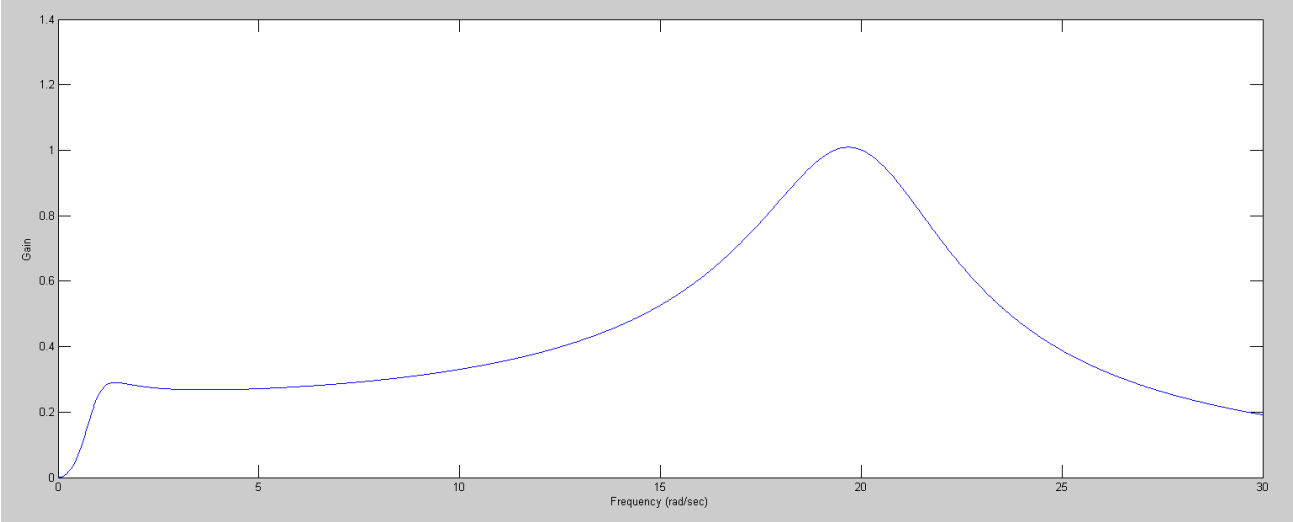
3) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{100s^2}{\left(s^2+s+1\right)\left(s^2+5s+400\right)} \right) X$$

```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 100*(s.^2) ./ ( (s.^2 + s + 1).*(s.^2 + 5*s + 400) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)')
>> ylabel('Gain')
>>
```

Comment

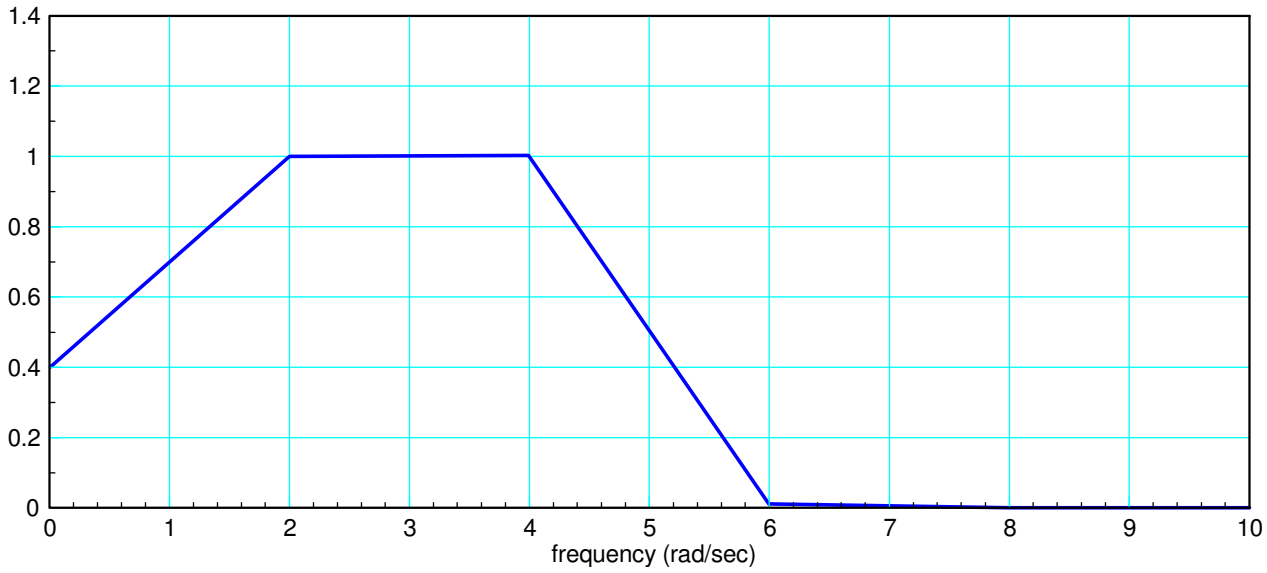
- This is kind of a band-pass filter
- Filter analysis is pretty straight-forward



Filter Design

Problem 4-6) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$



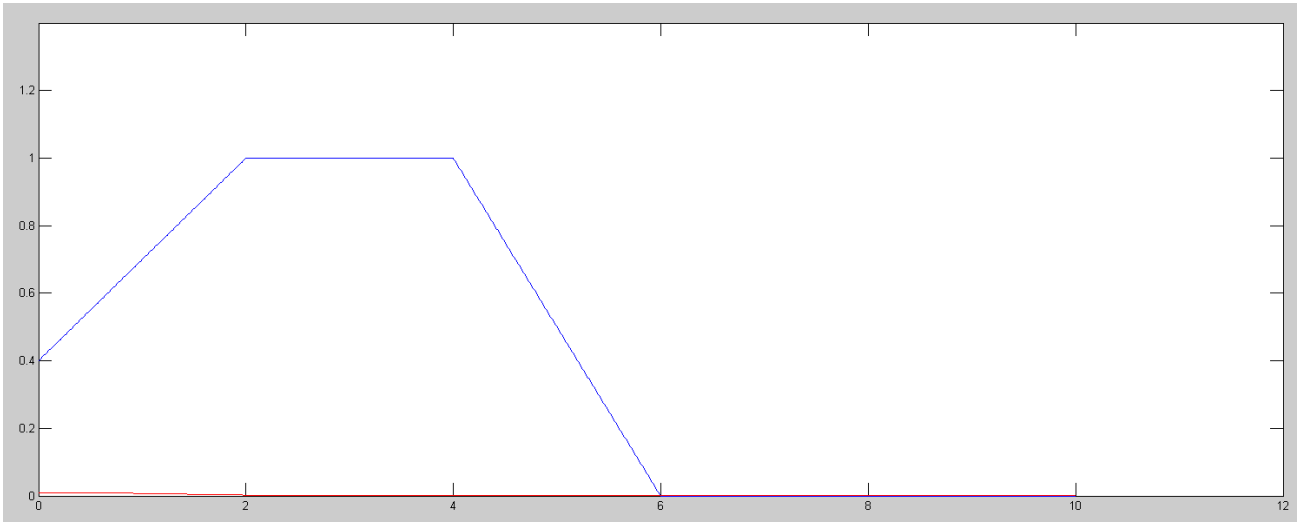
4) Write an m-file, cost.m, which

- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

```
function [ J ] = costf( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 g = z(7);

 w = [0:0.01:10]' + 1e-6;
 s = j*w;

 g1 = (0.3*w + 0.4) .* (w < 2);
 g2 = (1)* (w>2).*(w<4);
 g3 = (-0.5*w + 3) .* (w>4).*(w<6);

 Gideal = g1 + g2 + g3;

 G = a ./ ( (s.^2 + b*s + c) .* (s.^2 + d*s + e ).*(s.^2 + f*s + g) );
 G = abs(G);
 E = abs(Gideal) - abs(G);

 J = sum(E .^ 2);

 plot(w,Gideal,w,abs(G),'r');
 ylim([0,1.4]);
 pause(0.01);
end
```

Check:

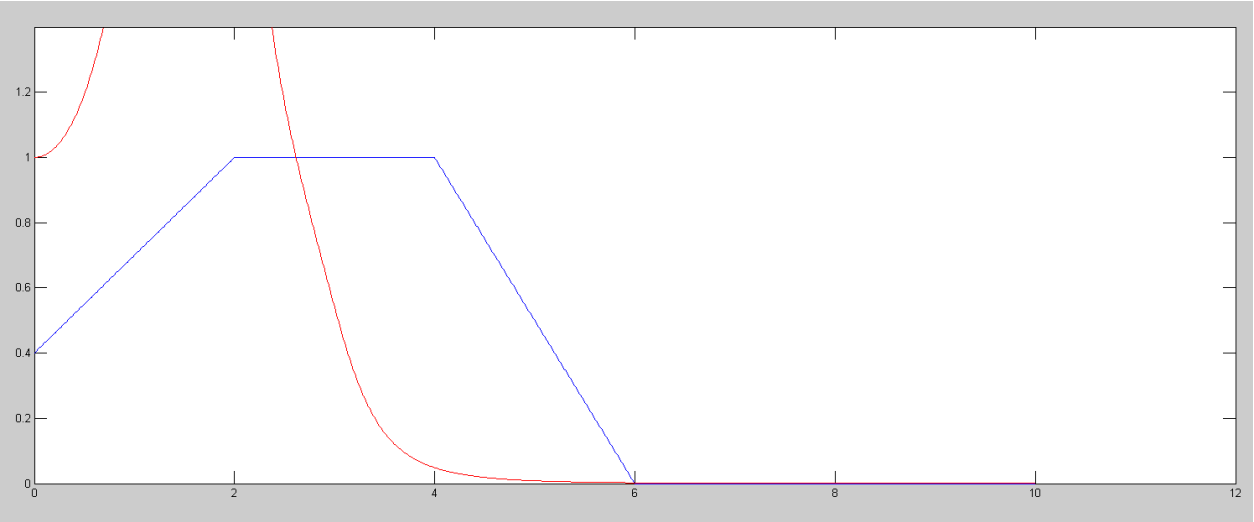
```
>> Costf([1,2,3,4,5,6,7])
ans =  368.6847
```



5) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right) = \left(\frac{72}{(s^2+s+2)(s^2+s+4)(s^2+s+9)} \right)$$

```
>> Costf([72,1,2,1,4,1,9])

ans =  566.4615
```



6) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a}{\left(s^2+bs+c\right)\left(s^2+ds+e\right)\left(s^2+fs+g\right)} \right)$$

a) Give the resulting (a, b, c, d, e, f, g)

```
>> Costf([72,1,2,1,4,1,9])
ans =  566.4615

>> [Z,e] = fminsearch('Costf',[72,1,2,1,4,1,9])

        a          b         c         d        e          f         g
Z =  877.7658    1.7573    4.4774    1.6202   15.1010    1.2201   25.1484

e =    0.7965
```

b) Give the resulting filter, and

$$G(s) = \left(\frac{877.76}{\left(s^2+1.757s+4.477\right)\left(s^2+1.62s+15.10\right)\left(s^2+1.22s+25.14\right)} \right)$$

c) Plot the 'optimal' filter's gain vs. frequency

note:

- *With Matlab, you can design some pretty good filters, even if you know nothing about filter design*
- *You'll cover how to design filters and different types of filters when you get to Circuits II*
- *Matlab is pretty useful*