ECE 111 - Homework #13

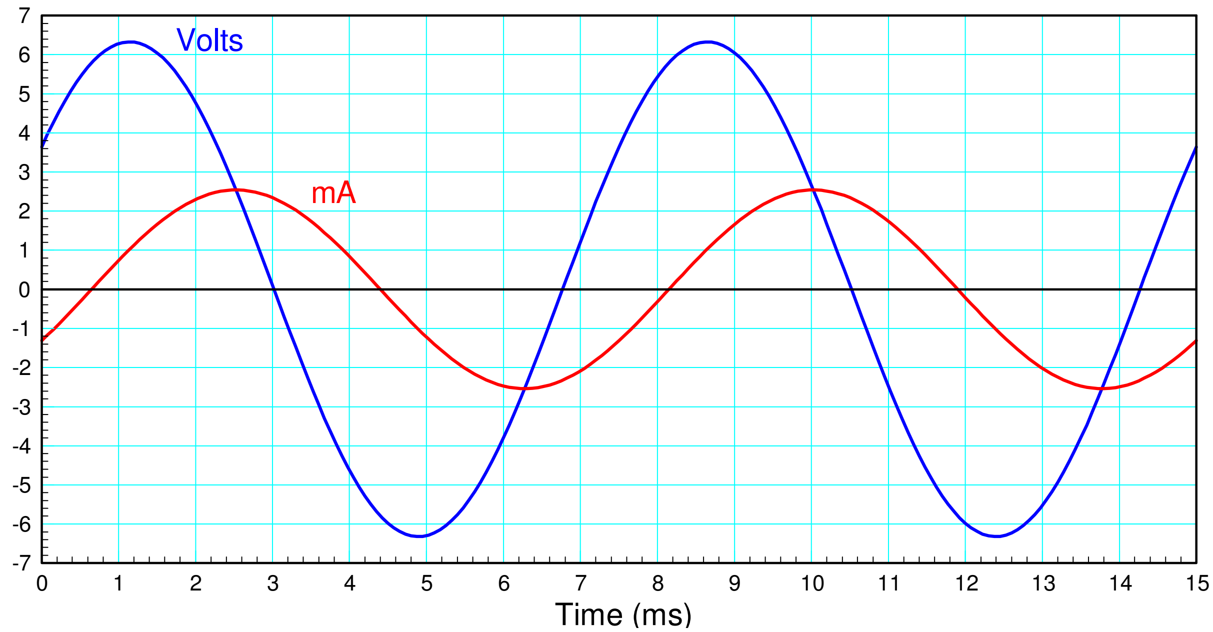
ECE 311 Circuits II - Phasors

Due Monday, April 21st. Please submit via email or on BlackBoard

Phasor Voltages

1) Express V and I as phasors (i.e. as complex numbers)

- From this, determine the impedance, Z = V/I



Voltage:

- Peak = 6.2
- Delay = 1.2ms
- Period = 7.5ms

$$\theta = -\left(\frac{1.2ms}{7.5ms}\right)360^0 = -57.6^0$$

$$V = 6.2\angle -57.6^0 \text{ Volts}$$

Current

- Peak = 2.5mA
- Delay = 2.5ms
- Period = 7.5ms

$$\theta = -\left(\frac{2.5ms}{7.5ms}\right)360^0 = -120^0$$

$$I = 2.5\angle -120^0 mA$$

The impedance is then

$$Z = \frac{V}{I}$$

$$Z = \left(\frac{6.2\angle -57.6^0}{0.0025\angle -120^0}\right)$$

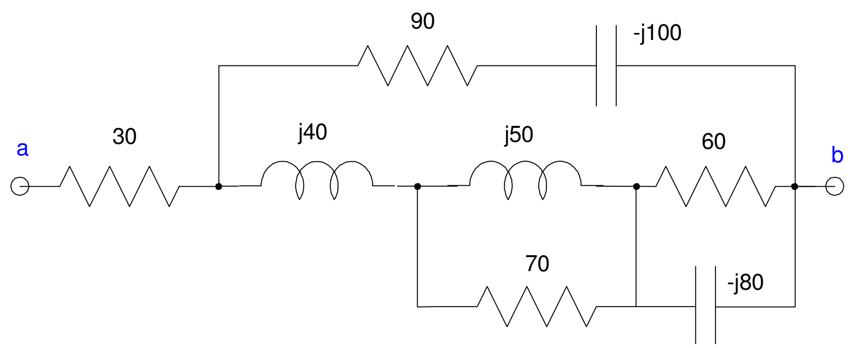
$$Z = 2680\angle 62.4^0$$

or if you prefer rectangular form

$$Z = 1241.63 + j2375.02$$

Phasor Impedances

2) Determine the impedance, Zab



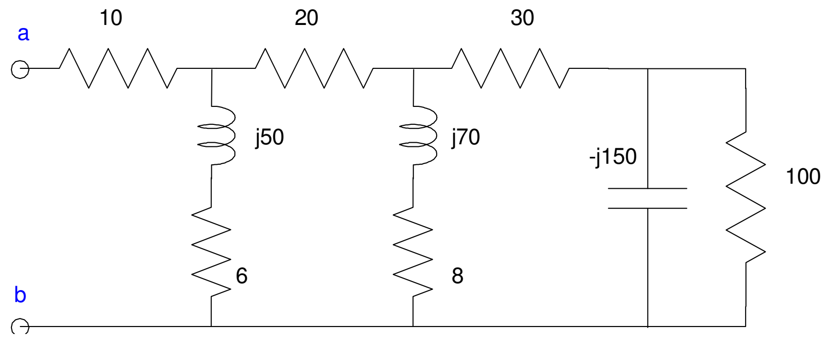
In Matlab

```
>> Z1 = 1 / (1/60 + 1/(-80i))
Z1 =  38.4000 -28.8000i

>> Z2 = 1 / (1/(50i) + 1/70)
Z2 =  23.6486 +33.1081i

>> Z3 = Z1 + Z2
Z3 =  62.0486 + 4.3081i

>> Z4 = Z3 + 40i
Z4 =  62.0486 +44.3081i

>> Z5 = 1 / (1/Z4 + 1/(90-100i))
Z5 =  62.7860 + 8.4153i

>> Z6 = Z5 + 30
Z6 =  92.7860 + 8.4153i
```

With a Free42 calculator

```
60
1/x
0
enter
-80
complex
1/x
+
1/x
70
1/x
0
enter
50
complex
1/x
+
1/x
0
enter
40
complex
+
1/x
90
enter
-100
complex
1/x
+
1/x
30
+
```

3) Determine the impedance, Zab



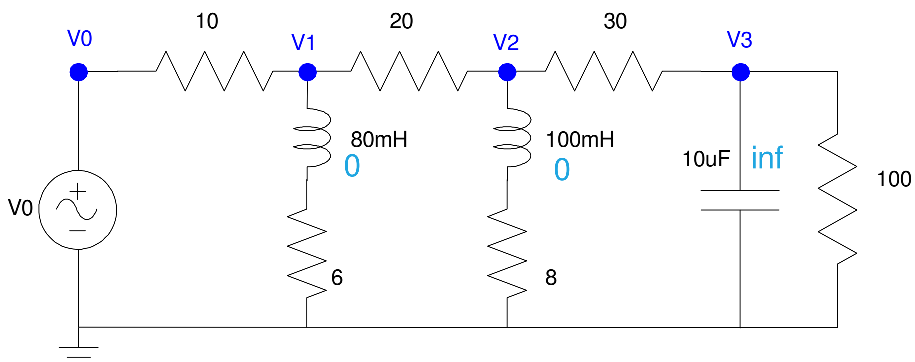
In Matlab

```
>> Za = 1 / (1/100+ 1/(-150i))
Za =  69.2308 -46.1538i

>> Zb = Za + 30
Zb =  99.2308 -46.1538i

>> Zc = 1 / ( 1/Zb + 1/(8+70i) )
Zc =  48.7605 +50.4908i

>> Zd = 20 + Zc
Zd =  68.7605 +50.4908i

>> Ze = 1 / ( 1/Zd + 1/(6+50i) )
Ze =  13.8990 +31.3568i

>> Zab = Ze + 10
Zab =  23.8990 +31.3568i
```

With a Free42 Calculator

```
100
1/x
0
enter
-150
complex
1/x
+
1/x
30
+
1/x
8
enter
70
complex
1/x
+
1/x
20
+
1/x
6+50i
1/x
+
1/x
10
+
```

Voltage Nodes with Phasors



4) Assume V0 = 10.

a) Determine the impedances of each element at 0 rad/sec

- Inductors are zero Ohms at DC
- Capacitors are infinte Ohms at DC

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{6}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{8}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{\infty}\right) + \left(\frac{V_3}{100}\right) = 0$$

you can leave out the term that divides by infinity. This current is zero.

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{10}\right)V_0 + \left(\frac{1}{10} + \frac{1}{6} + \frac{1}{20}\right)V_1 - \left(\frac{1}{20}\right)V_2 = 0$$

$$-\left(\frac{1}{20}\right)V_1 + \left(\frac{1}{20} + \frac{1}{8} + \frac{1}{30}\right)V_2 - \left(\frac{1}{30}\right)V_3 = 0$$

$$-\left(\frac{1}{30}\right)V_2 + \left(\frac{1}{30} + \frac{1}{100}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10}+\frac{1}{6}+\frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 \\ 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20}+\frac{1}{8}+\frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{100}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

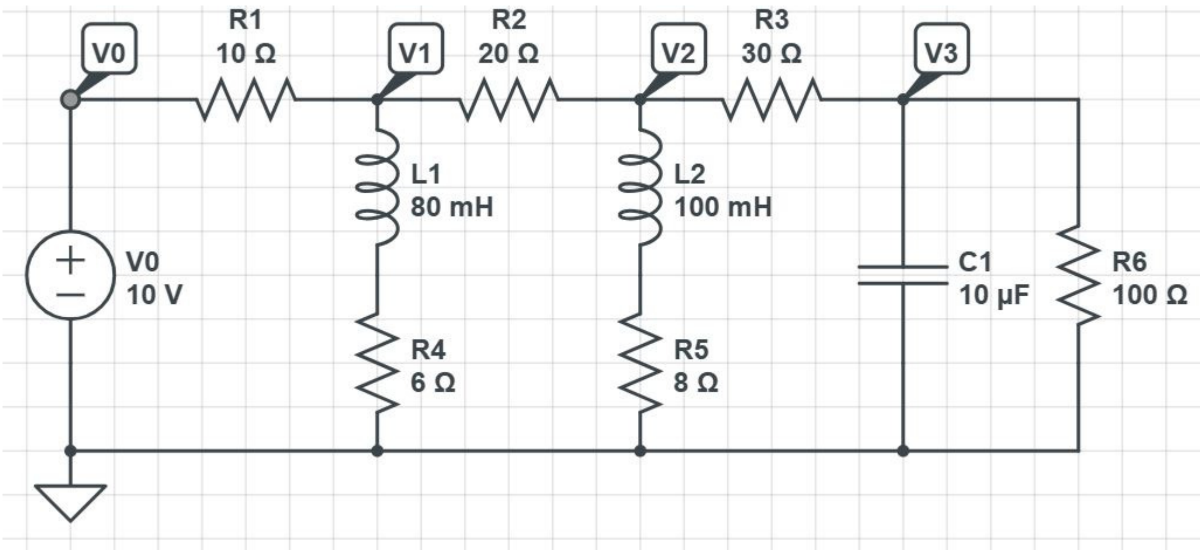
Solve using Matlab

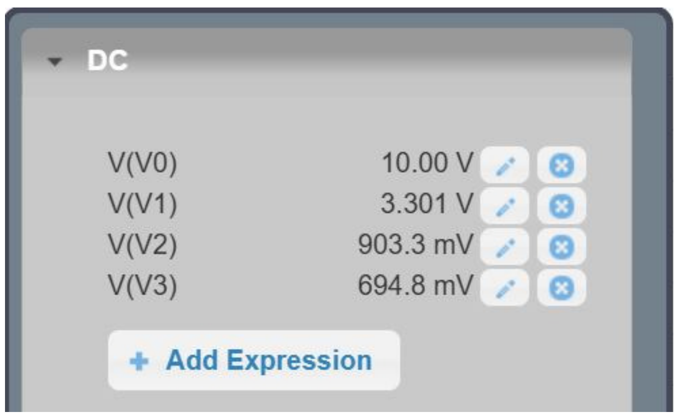
```
>> a1= [1,0,0,0];
>> a2 = [-1/10, 1/10+1/6+1/20,-1/20,0];
>> a3 = [0,-1/20,1/20+1/8+1/30,-1/30];
>> a4 = [0,0,-1/30,1/30+1/100];
>> A = [a1;a2;a3;a4]

    1.0000         0         0         0
   -0.1000    0.3167   -0.0500         0
         0   -0.0500    0.2083   -0.0333
         0         0   -0.0333    0.0433

>> B = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(A)*B
```

```
V0   10.0000
V1    3.3005
V2    0.9033
V3    0.6948
```

Note: Hand calculators work pretty well when dealing with one equation and one unknown. They really struggle when dealing with N equations and N unknowns. That's where Matlab shines.

5) Check your results in CircuitLab

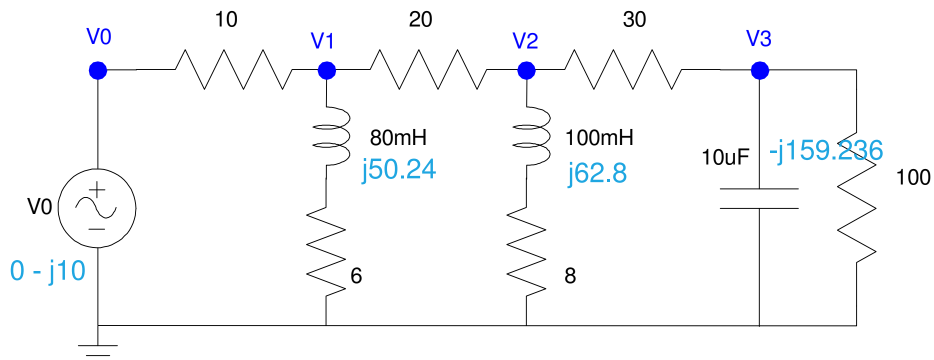




This matches previous calculations

```
>> V = inv(A)*B

V0   10.0000
V1    3.3005
V2    0.9033
V3    0.6948
```

6) Assume $V_0 = 10\sin(628t)$ *10V, 628 rad/sec sine wave (100Hz)*



a) Determine the impedances of each element at 628 rad/sec

80mH:

$$Z = j\omega L = j \cdot 628 \cdot 0.08$$

$$Z = j50.24\Omega$$

100mH

$$Z = j\omega L = j62.8\Omega$$

10uF

$$Z = \frac{1}{j\omega C} = -j159.236\Omega$$

V0

V = 0 - j10

real = cosine

-imag =sine

b) Write the votlage node equations

same as we had at DC, only now with complex numbers

$$V_0 = 0 - j10$$

$$\left(\frac{V_1 - V_0}{10}\right) + \left(\frac{V_1}{6+j50.24}\right) + \left(\frac{V_1 - V_2}{20}\right) = 0$$

$$\left(\frac{V_2 - V_1}{20}\right) + \left(\frac{V_2}{8+j62.8}\right) + \left(\frac{V_2 - V_3}{30}\right) = 0$$

$$\left(\frac{V_3 - V_2}{30}\right) + \left(\frac{V_3}{-j159.236}\right) + \left(\frac{V_3}{100}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

Group terms

$$V_0 = -j10$$

$$-\left(\frac{1}{10}\right)V_0 + \left(\frac{1}{10} + \frac{1}{6+j50.24} + \frac{1}{20}\right)V_1 - \left(\frac{1}{20}\right)V_2 = 0$$

$$-\left(\frac{1}{20}\right)V_1 + \left(\frac{1}{20} + \frac{1}{8+j62.8} + \frac{1}{30}\right)V_2 - \left(\frac{1}{30}\right)V_3 = 0$$

$$-\left(\frac{1}{30}\right)V_2 + \left(\frac{1}{30} + \frac{1}{100} + \frac{1}{-j159.236}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{10}\right) & \left(\frac{1}{10} + \frac{1}{6+j50.24} + \frac{1}{20}\right) & \left(\frac{-1}{20}\right) & 0 \\ 0 & \left(\frac{-1}{20}\right) & \left(\frac{1}{20} + \frac{1}{8+j62.8} + \frac{1}{30}\right) & \left(\frac{-1}{30}\right) \\ 0 & 0 & \left(\frac{-1}{30}\right) & \left(\frac{1}{30} + \frac{1}{100} + \frac{1}{-j159.236}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -j10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
a1= [1,0,0,0];
a2 = [-1/10, 1/10+1/(6+50.24i)+1/20,-1/20,0];
a3 = [0,-1/20,1/20+1/(8+62.8i)+1/30,-1/30];
a4 = [0,0,-1/30,1/30+1/100+1/(-159.236i)];

A = [a1;a2;a3;a4]

   1.0000                 0                  0                   0
  -0.1000            0.1523 - 0.0196i  -0.0500                   0
        0           -0.0500              0.0853 - 0.0157i  -0.0333
        0                 0            -0.0333               0.0433 + 0.0063i

>> B = [-10i;0;0;0]

        0 -10.0000i
        0
        0
        0

>> V = inv(A)*B
```

V0 0 -10.0000i
V1 2.0597 - 8.3895i
V2 2.9829 - 6.3703i
V3 1.5518 - 5.1251i

d) Express V1, V2, and V3 in terms of sine and cosine function:

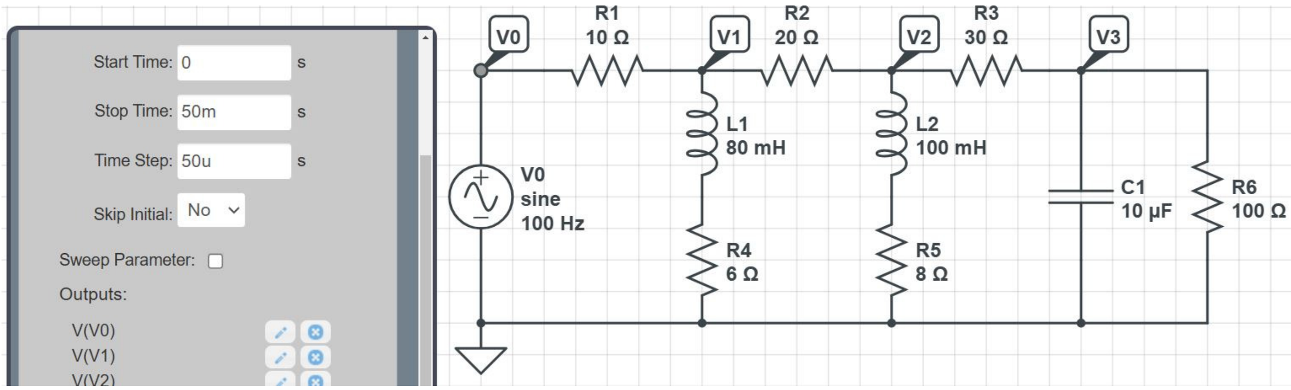
```
V0 =       0 cos(628t) + 10.0000 sin(628t)
V1 =  2.0597 cos(628t) +  8.3895 sin(628t)
V2 =  2.9829 cos(628t) +  6.3703 sin(628t)
V3 =  1.5518 cos(628t) +  5.1251 sin(628t)
```

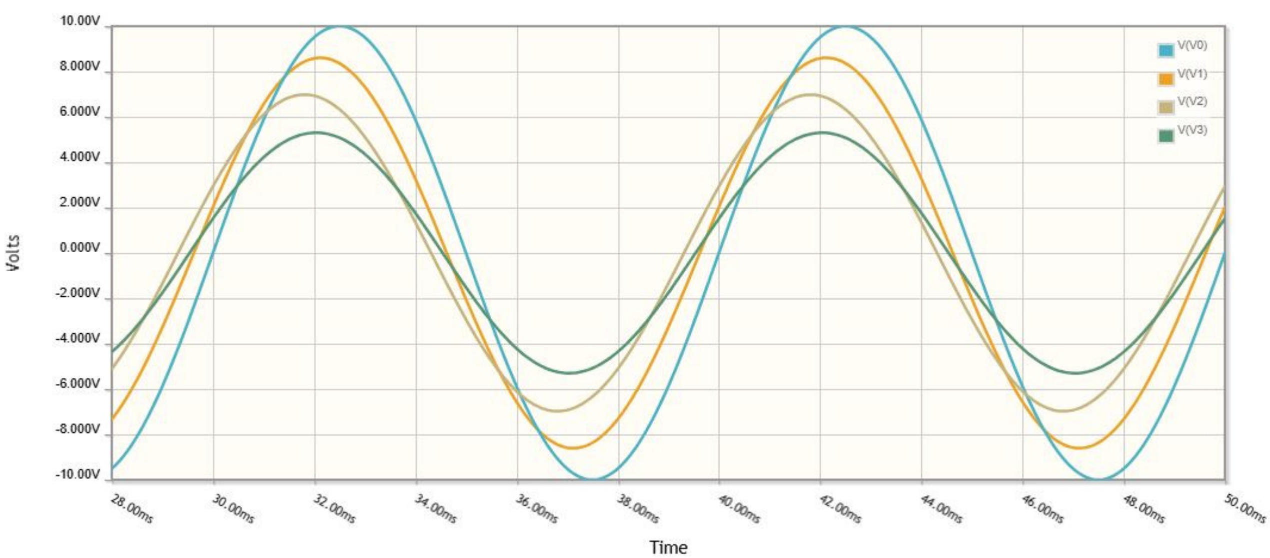
Note: Hand calculators struggle when dealing with N equations and N unknowns with real numbers. They *really* struggle when dealing with N equations and N unknowns with complex numbers. This is where Matlab *really* shines. Matlab has no problem dealing with complex numbers. Matlab has no problem dealing with N equations and N unknowns.

7) Check your results in CircuitLab using a transient simulation for 50ms (time step = 50us).

The amplitude matches calculations

```
>> abs(V)
V0    10.0000
V1     8.6387
V2     7.0341
V3     5.3549
```





The CircuitLab results match calculations from Matlab

Peak Voltage	V0	V1	V2	V3
Matlab	10.000	8.6387	7.0341	5.3549
CircuitLab	10.000	8.545	6.922	5.326