ECE 111 - Homework #12

Complex Numbers
Due Monday, April 14th. Please submit via email or on BlackBoard

Complex Numbers

1) Determine the rectangular or polar form of each complex number

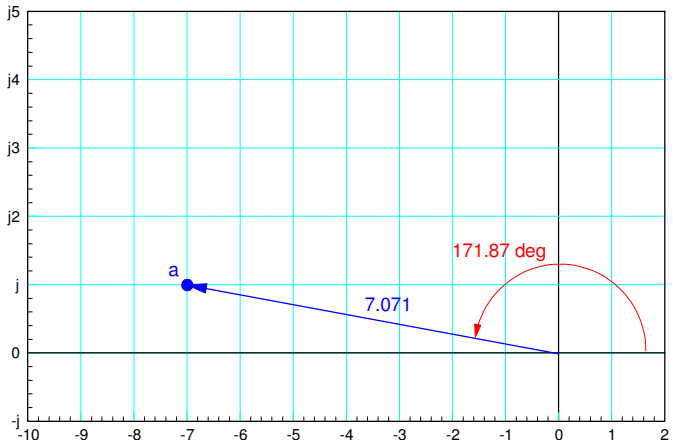
$a = -7 + j$

```
>> a = -7 + j
a =  -7.0000 + 1.0000i

>> abs(a)
ans =   7.0711

>> angle(a)*180/pi
ans =  171.8699
```

$$-7 + j = 7.0711 \angle 171.8699^0$$



With Free42:

```
modes
rect
-7
enter
1
complex
modes
polar
```

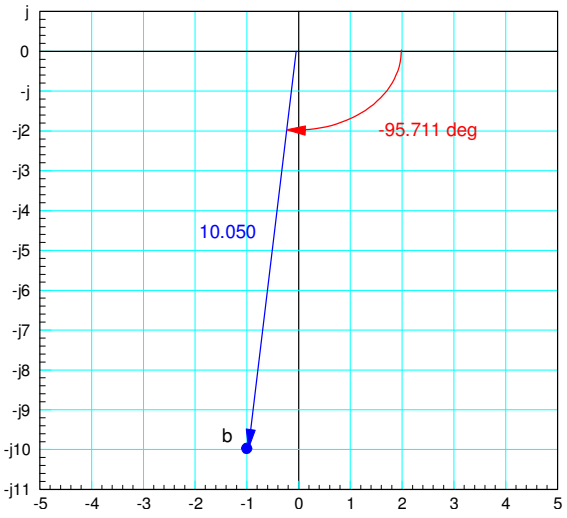
$b = -1 - j10$

```
>> b = -1 - 10i
b =  -1.0000 -10.0000i

>> abs(b)
ans =   10.0499

>> angle(b)*180/pi
ans =  -95.7106
```

$$-1 - j10 = 10.0499\angle -95.7016^0$$



With Free42

```
modes
rect
-1
enter
-10
complex
modes
polar
```

$c = 7\angle -125^0$

```
>> c = 7 * exp(-125*pi/180 * j)
c =  -4.0150 - 5.7341i
```

Free42:

```
modes
polar
7
enter
-125
polar
modes
rect
```

$d = 8\angle -37^0$

```
>> d = 8 * exp(-37*pi/180 * j)
d =   6.3891 - 4.8145i
```

Free42

```
modes
polar
8
enter
-37
polar
modes
rect
```

2) Determine y as a complex number

$$y = (1 - j7) + (5 - j) + (-8 - j5)$$

```
>> y = (1-7i) + (5-1i) + (-8-5i)

y =  -2.0000 -13.0000i
```

On an Free42 calculator

```
modes
rect
1
enter
-7
complex
5
enter
-1
complex
+
-8
enter
-5
complex
+
```

$$y = (10\angle -84^0) + (5\angle 79^0) + (9\angle -99^0)$$

```
>> ya = 10*exp(-84*pi/180 * j)
ya =   1.0453 - 9.9452i

>> yb = 5*exp(79*pi/180 * j)
yb =   0.9540 + 4.9081i

>> yc = 9 * exp(-99*pi/180 * j)
yc =  -1.4079 - 8.8892i

>> y = ya + yb + yc
y =   0.5914 -13.9263i
```

On an Free42 calculator

```
modes
polar
10
enter
-84
complex
5
enter
79
complex
+
9
enter
-99
complex
+
modes
rect
```

3) Determine y as a complex number

$$y = \left(\frac{(1-j7)(5-j)}{(-8-j5)} \right)$$

```
>> y = ( (1-7i) * (5-i) ) / (-8-5i)

y =   2.2022 + 3.1236i
```

On an Free42

```
modes
rect
1
enter
-7
complex
5
enter
-1
complex
*
-8
enter
-5
complex
/
```

$$y = \left(\left(\frac{7-j8}{-20-j5} \right) + \left(\frac{6-j}{8-j6} \right) \right) \left(\frac{-5-j7}{-7+j7} \right)$$

```
>> y = ( (7-8i)/(-20-5i) + (6-1i)/(8-6i) ) * ( (-5-7i)/(-7+7i) )

y =   -0.6768 + 0.1556i
```

On a Free42

```
7
enter
-8
complex
-20
enter
-5
complex
/
6
enter
-1
complex
8
enter
-6
complex
/
+
-5
enter
-7
complex
-7
enter
7
complex
/
*
```

4) Determine y as a complex number

$y = e^{(2+j)}$

```
>> exp(2+1i)

ans =   3.9923 + 6.2177i
```

$y = \ln(-7 + j7)$

```
>> log(-7+7i)

ans =   2.2925 + 2.3562i
```

$y = (2 - j3)^{(3-j2)}$

```
>> (2-3i) ^ (3-2i)

ans =   4.7141 + 4.5698i
```

Free42

```
2
enter
-3
complex
3
enter
-2
complex
y^x
```

Comments:

- Functions of complex numbers produce a complex number.
- Matlab has no problem with these functions
- Some calculators (HP) can do complex operations. Others will give you an error message

Partial Fractions with Complex Numbers

5) Determine the partial fraction expansion

$$\left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+1-j4)(x+5)}\right) = \left(\frac{a}{x+1+j4}\right) + \left(\frac{b}{x+1-j4}\right) + \left(\frac{c}{x+5}\right)$$

Use the cover-up method

$$a = \left(\frac{3(x+1)(x+2)}{(x+1-j4)(x+5)}\right)_{x=-1-j4}$$

In Matlab

```
>> x = -1-4i;
>> a = 3*(x+1)*(x+2) / ( (x+1-4i)*(x+5) )
```

```
a =   0.9375 - 0.5625i
```

$$b = \left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+5)}\right)_{x=-1+j4}$$

```
>> x = -1 + 4i;
>> b = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+5) )
```

```
b =   0.9375 + 0.5625i
```

$$c = \left(\frac{3(x+1)(x+2)}{(x+1+j4)(x+1-j4)}\right)_{x=-5}$$

```
>> x = -5;
>> c = 3*(x+1)*(x+2) / ( (x+1+4i)*(x+1-4i) )
```

```
c =    1.1250
```

Free42: Find a:

$$a = \left(\frac{3(x+1)(x+2)}{(x+1-j4)(x+5)} \right)_{x=-1-j4}$$

Store -1-j4 in variable X

```
-1
enter
-4
sto
Alpha
X
Alpha
```

Now find 'a'

```
3
RCL
X
1
+
*
RCL
X
2
+
*
RCL
X
-1
enter
-4
complex
+
/
RCL
X
5
+
/
```

a = 0.9375 - 0.5625i

6) Determine the partial fraction expansion

$$\left(\frac{8(x+1+j2)(x+1-j2)}{(x+1)(x+3)(x+5+j2)(x+5-j2)}\right) = \left(\frac{a}{x+1}\right) + \left(\frac{b}{x+3}\right) + \left(\frac{c}{x+5+j2}\right) + \left(\frac{d}{x+5-j2}\right)$$

Using the cover-up method

$$a = \left(\frac{8(x+1+j2)(x+1-j2)}{(x+3)(x+5+j2)(x+5-j2)}\right)_{x=-1}$$

```
>> x = -1;
>> a = 8*(x+1+2i)*(x+1-2i) / ( (x+3)*(x+5+2i)*(x+5-2i) )

a =    0.8000
```

$$b = \left(\frac{8(x+1+j2)(x+1-j2)}{(x+1)(x+5+j2)(x+5-j2)}\right)_{x=-3}$$

```
>> x = -3;
>> b = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+5+2i)*(x+5-2i) ) * (x+3)

b =   -4.0000
```

$$c = \left(\frac{8(x+1+j2)(x+1-j2)}{(x+1)(x+3)(x+5-j2)}\right)_{x=-5-j2}$$

```
>> x = -5-2i;
>> c = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5-2i) )

c =   1.6000 + 3.2000i
```

$$d = \left(\frac{8(x+1+j2)(x+1-j2)}{(x+1)(x+3)(x+5+j2)}\right)_{x=-5+j2}$$

```
>> x = -5+2i;
>> d = 8*(x+1+2i)*(x+1-2i) / ( (x+1)*(x+3)*(x+5+2i) )

d =   1.6000 - 3.2000i

>>
```