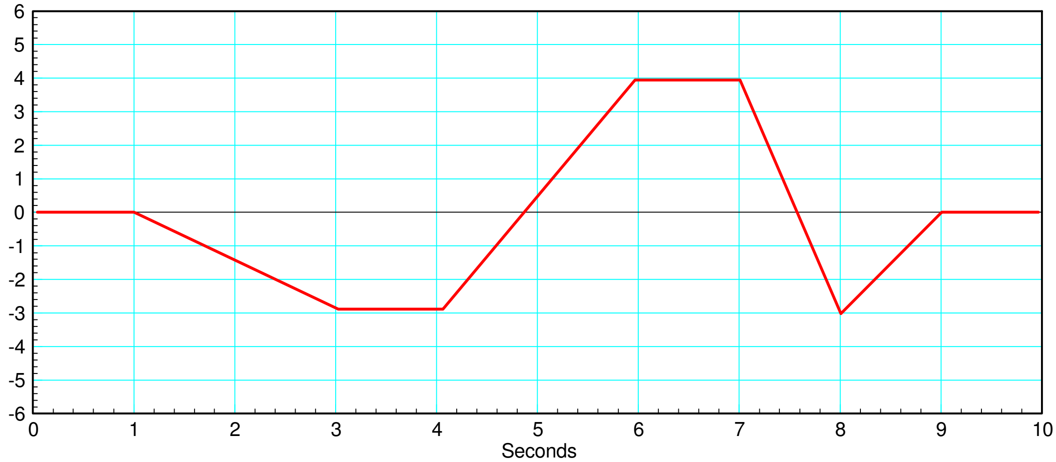
ECE 111 - Homework #11

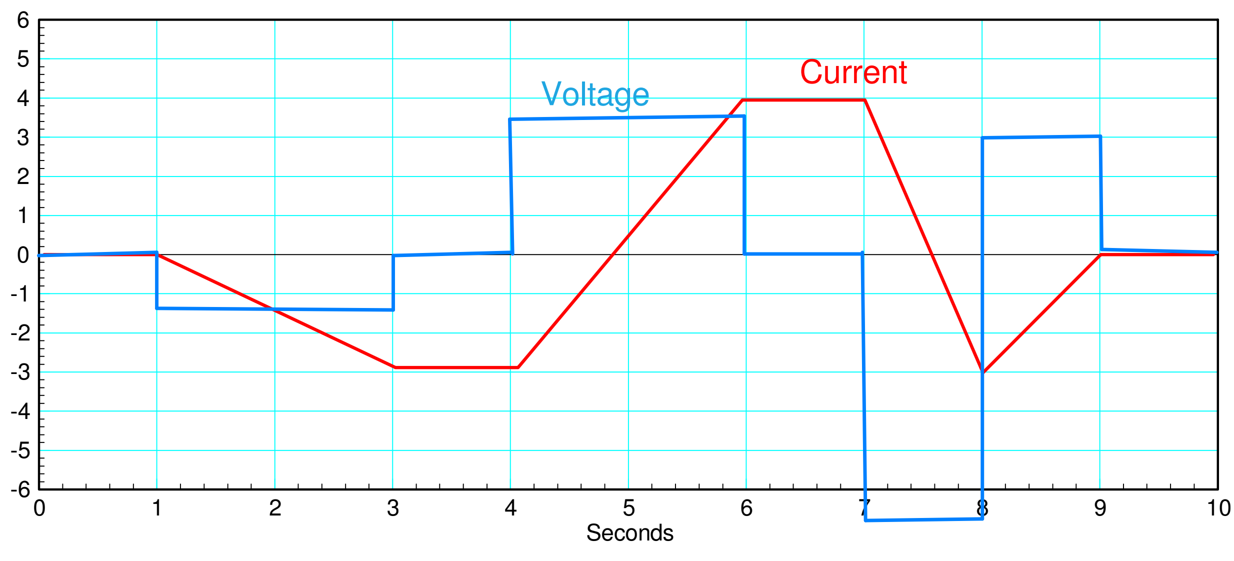
ECE 351 Electromagnetics & The Wave Equation
Due April 7th. Please submit via email or on BlackBoard

1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

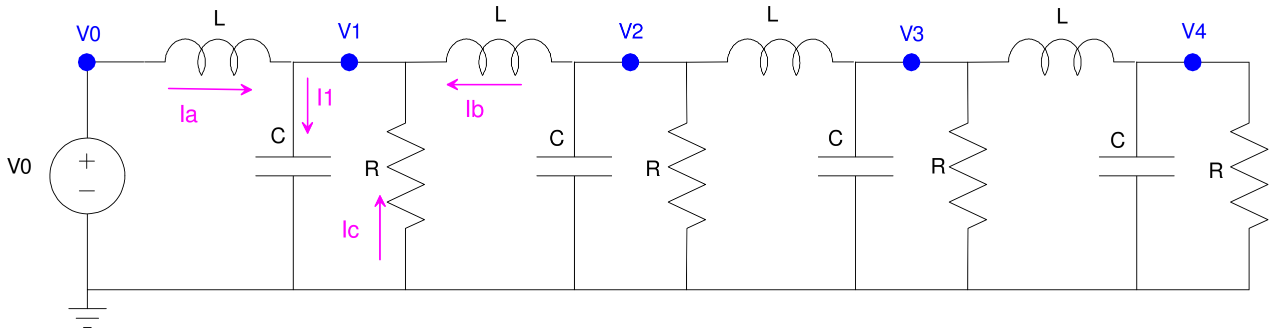
$$V = L\frac{dI}{dt}$$



Capacitors act as integrators (last lecture). Inductors act as differentiators (this lecture). Note that this os the same problem as we saw in homework #6. The solution is the same as well:



4-Node RLC Circuit



R = 330Ω, C = 0.1F, L = 0.12H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

This is a little bit tricky. One way to write the dynamics is to use conservation of current

$$I_1 = I_a + I_b + I_c$$

Using dot notation for differentiation

$$\frac{dV}{dt} \equiv \dot{V}$$

$$I_1 = C\dot{V}_1 = I_a + I_b - \left(\frac{1}{R}\right)V_1$$

Differentiate both sides

$$C\ddot{V}_1 = \dot{I}_a + \dot{I}_b - \left(\frac{1}{R}\right)\dot{V}_1$$

Note that the inductor equations are:

$$V_0 - V_1 = L\dot{I}_a$$

$$V_2 - V_1 = L\dot{I}_b$$

Substituting for Ia and Ib

$$C\ddot{V}_1 = \left(\frac{V_0 - V_1}{L}\right) + \left(\frac{V_2 - V_1}{L}\right) - \left(\frac{1}{R}\right)\dot{V}_1$$

Simplifying

$$\ddot{V}_1 = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)\dot{V}_1$$

This repeats for nodes 2 and 3

$$\ddot{V}_2 = \left(\frac{1}{LC}\right)V_1 - \left(\frac{2}{LC}\right)V_2 + \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{RC}\right)\dot{V}_2$$

$$\ddot{V}_3 = \left(\frac{1}{LC}\right)V_2 - \left(\frac{2}{LC}\right)V_3 + \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_3$$

Node #4 is missing the Ib term, resulting in

$$\ddot{V}_4 = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)\dot{V}_4$$

Plugging in numbers for R, L, and C:

$$\ddot{V}_1 = 83.33V_0 - 166.66V_1 + 83.33V_2 - 0.03\dot{V}_1$$

$$\ddot{V}_2 = 83.33V_1 - 166.66V_2 + 83.33V_3 - 0.03\dot{V}_2$$

$$\ddot{V}_3 = 83.33V_2 - 166.66V_3 + 83.33V_4 - 0.03\dot{V}_3$$

$$\ddot{V}_4 = 83.33V_3 - 83.33V_4 - 0.03\dot{V}_4$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*

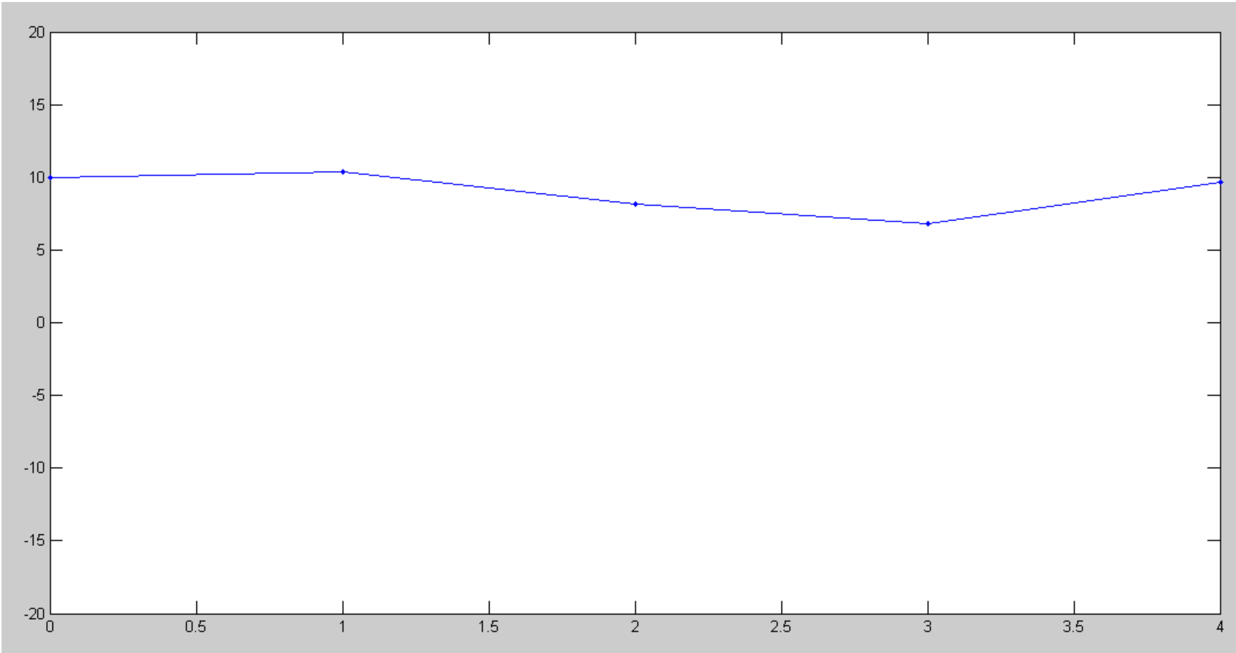
Matlab Code: At any given time point

- Compute the second derivative of {V1, V2, V3, V4}
- Integrate to get the first derivative
- Integrate again to get the voltages

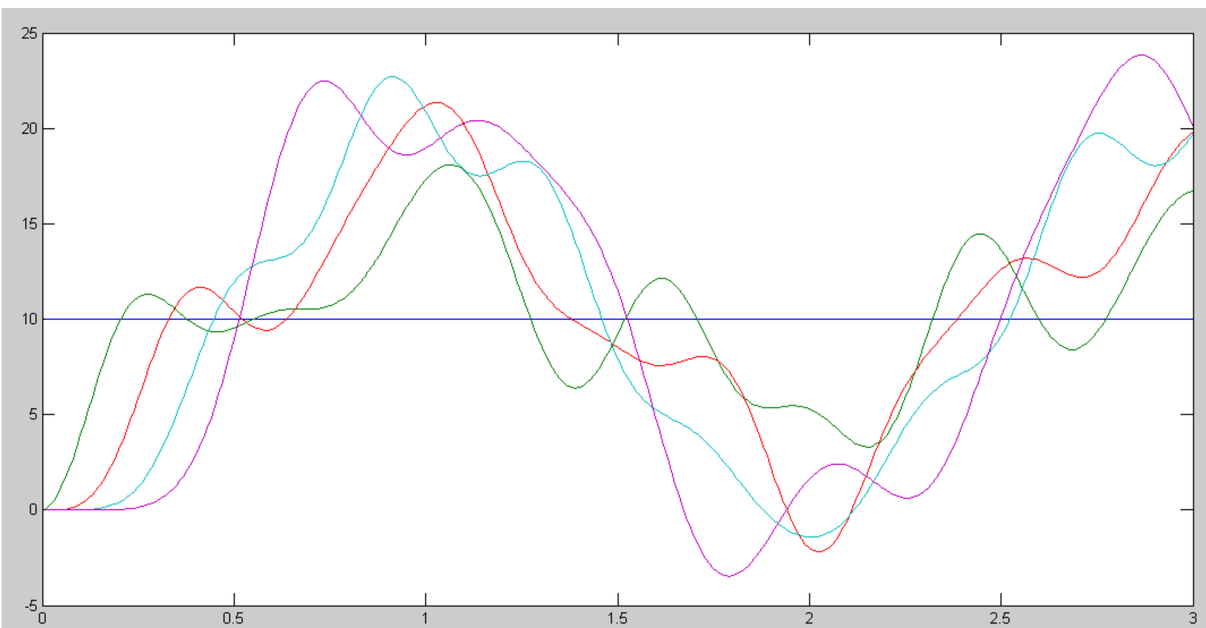
```
V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.01;

while(t < 2.99)

   ddV1 = 83.33*V0 - 166.66*V1 + 83.33*V2 - 0.03*dV1;
   ddV2 = 83.33*V1 - 166.66*V2 + 83.33*V3 - 0.03*dV2;
   ddV3 = 83.33*V2 - 166.66*V3 + 83.33*V4 - 0.03*dV3;
   ddV4 = 83.33*V3 - 83.33*V4            - 0.03*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   ylim([-20,20]);
   pause(0.01);

   V = [V ; [V0,V1,V2,V3,V4]];

end

pause(2)
t = [1:length(V)]' * dt;
plot(t,V)
```

This results in the voltages bouncing up and down

note that the wave equation behaves very differently than the heat equation



Node voltages at t = 2



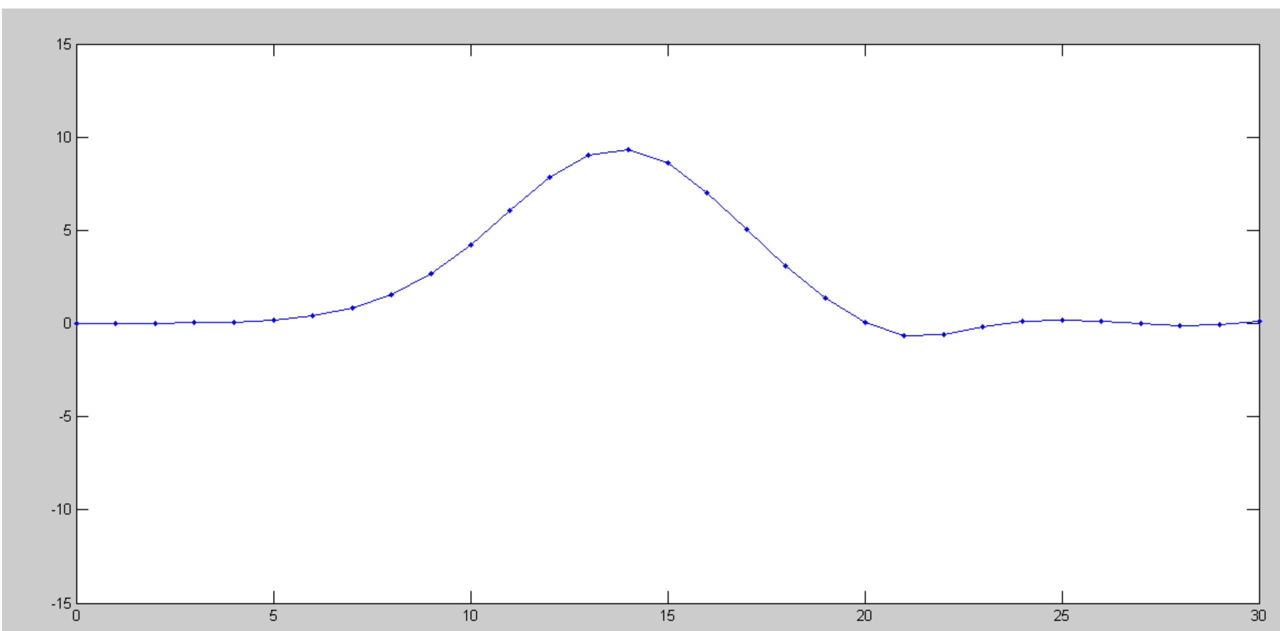
Voltages vs. Time

30-Node RLC Circuit (hint: modify the program Wave.m)

4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 6 seconds (just after the reflection) for $1 / R_{30}C = 0.01$

Matlab Code:

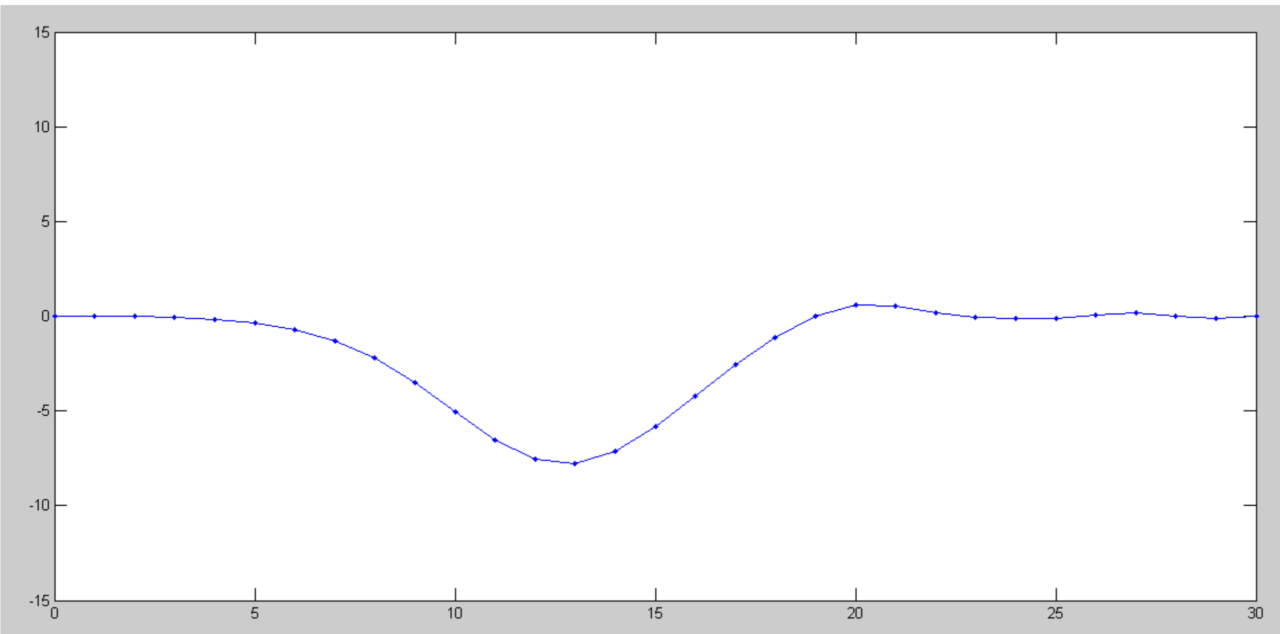
```
V = zeros(30,1);
dV = zeros(30,1);
t = 0;
dt = 0.01;

while(t < 6)
   if (t < 1.5) V0 = 10* ( sin(2*t).^2);
      else V0 = 0;
      end
   ddV(1) = 83.33*V0 - 166.66*V(1) + 83.33*V(2) - 0.03*dV(1);
   for i=2:29
      ddV(i) = 83.33*V(i-1) - 166.66*V(i) + 83.33*V(i+1) - 0.03*dV(i);
      end
   ddV(30) = 83.33*V(29) - 83.33*V(30) - 0.01*dV(30);
   for i=1:30
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) = V(i) + dV(i)*dt;
      end
   t = t + dt;

 plot([0:30],[V0;V],'.-');
 ylim([-15,15]);
 pause(0.01);
 end
```



5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$

Matlab Code:

```
V = zeros(30,1);
dV = zeros(30,1);
t = 0;
dt = 0.01;

while(t < 6)
   if (t < 1.5) V0 = 10* ( sin(2*t).^2);
      else V0 = 0;
      end
   ddV(1) = 83.33*V0 - 166.66*V(1) + 83.33*V(2) - 0.03*dV(1);
   for i=2:29
      ddV(i) = 83.33*V(i-1) - 166.66*V(i) + 83.33*V(i+1) - 0.03*dV(i);
      end
   ddV(30) = 83.33*V(29) - 83.33*V(30) - 100*dV(30);
   for i=1:30
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) = V(i) + dV(i)*dt;
      end
   t = t + dt;

 plot([0:30],[V0;V],'.-');
 ylim([-15,15]);
 pause(0.01);
 end
```



6) Determine experimentally R_{30} so that the reflection is almost zero

- 10 works pretty well
- When you take ECE 351, you'll compute what this value should be. It' should be $\sqrt{83.33}$
- Note that you can come pretty close just by using Matlab and itterating

```
V = zeros(30,1);
dV = zeros(30,1);
t = 0;
dt = 0.01;

while(t < 6)
   if (t < 1.5) V0 = 10* ( sin(2*t).^2);
      else V0 = 0;
      end
   ddV(1) = 83.33*V0 - 166.66*V(1) + 83.33*V(2) - 0.03*dV(1);
   for i=2:29
      ddV(i) = 83.33*V(i-1) - 166.66*V(i) + 83.33*V(i+1) - 0.03*dV(i);
      end
   ddV(30) = 83.33*V(29) - 83.33*V(30) - 10*dV(30);
   for i=1:30
      dV(i) = dV(i) + ddV(i)*dt;
      V(i) = V(i) + dV(i)*dt;
      end
   t = t + dt;

 plot([0:30],[V0;V],'.-');
 ylim([-15,15]);
 pause(0.01);
 end
```