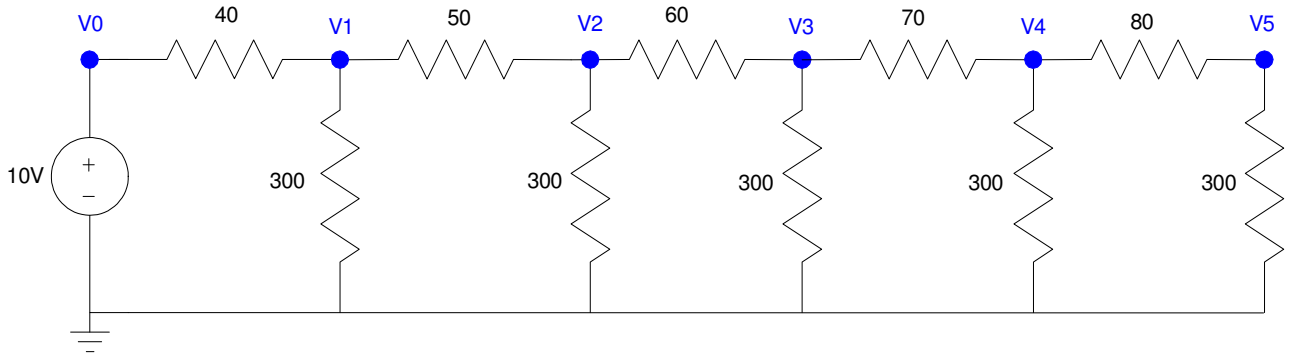
ECE 111 - Homework #9

EE 206 Circuts I - Voltage Nodes & Current Loops
Due Monday, March 24th. Please submit via email or on BlackBoard

Voltage Nodes

1) Use Voltage Nodes write N equations for N unknowns for the following circuit.



Node V0:

$$V_0 = 10$$

Node V1: The sum of the currents from the node must be zero

$$\left(\frac{V_1 - V_0}{40}\right) + \left(\frac{V_1}{300}\right) + \left(\frac{V_1 - V_2}{50}\right) = 0$$

Node V2:

$$\left(\frac{V_2 - V_1}{50}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2 - V_3}{60}\right) = 0$$

Node V3:

$$\left(\frac{V_3 - V_2}{60}\right) + \left(\frac{V_3}{300}\right) + \left(\frac{V_3 - V_4}{70}\right) = 0$$

Node V4:

$$\left(\frac{V_4 - V_3}{70}\right) + \left(\frac{V_4}{300}\right) + \left(\frac{V_4 - V_5}{80}\right) = 0$$

Node V5:

$$\left(\frac{V_5 - V_4}{80}\right) + \left(\frac{V_5}{300}\right) = 0$$

2) Solve for the node voltages in Matlab.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{40}\right)V_0 + \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{50}\right)V_1 - \left(\frac{1}{50}\right)V_2 = 0$$

$$-\left(\frac{1}{50}\right)V_1 + \left(\frac{1}{50}+\frac{1}{300}+\frac{1}{60}\right)V_2 - \left(\frac{1}{60}\right)V_3 = 0$$

$$-\left(\frac{1}{60}\right)V_2 + \left(\frac{1}{60}+\frac{1}{300}+\frac{1}{70}\right)V_3 - \left(\frac{1}{70}\right)V_4 = 0$$

$$-\left(\frac{1}{70}\right)V_3 + \left(\frac{1}{70}+\frac{1}{300}+\frac{1}{80}\right)V_4 - \left(\frac{1}{80}\right)V_5 = 0$$

$$-\left(\frac{1}{80}\right)V_4 + \left(\frac{1}{80}+\frac{1}{300}\right)V_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{50}\right) & \left(\frac{-1}{50}\right) & 0 & 0 & 0 \\ 0 & \left(\frac{-1}{50}\right) & \left(\frac{1}{50}+\frac{1}{300}+\frac{1}{60}\right) & \left(\frac{-1}{60}\right) & 0 & 0 \\ 0 & 0 & \left(\frac{-1}{60}\right) & \left(\frac{1}{60}+\frac{1}{300}+\frac{1}{70}\right) & \left(\frac{-1}{70}\right) & 0 \\ 0 & 0 & 0 & \left(\frac{-1}{70}\right) & \left(\frac{1}{70}+\frac{1}{300}+\frac{1}{80}\right) & \left(\frac{-1}{80}\right) \\ 0 & 0 & 0 & 0 & \left(\frac{-1}{80}\right) & \left(\frac{1}{80}+\frac{1}{300}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

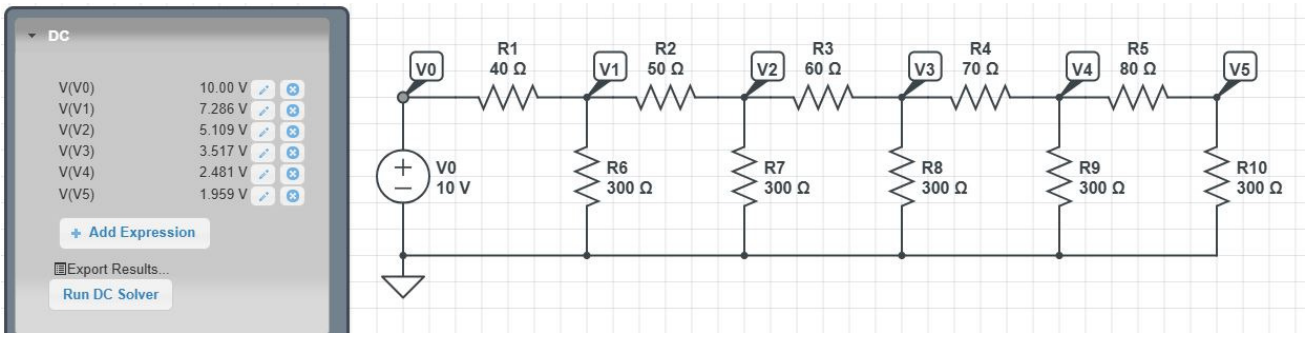
Solve using Matlab

```
>> a1 = [1,0,0,0,0,0];
>> a2 = [-1/40,1/40+1/300+1/50,-1/50,0,0,0];
>> a3 = [0,-1/50,1/50+1/300+1/60,-1/60,0,0];
>> a4 = [0,0,-1/60,1/60+1/300+1/70,-1/70,0];
>> a5 = [0,0,0,-1/70,1/70+1/300+1/80,-1/80];
>> a6 = [0,0,0,0,-1/80,1/80+1/300];
>> A = [a1;a2;a3;a4;a5;a6]

    1.0000         0         0         0         0         0
   -0.0250    0.0483   -0.0200         0         0         0
         0   -0.0200    0.0400   -0.0167         0         0
         0         0   -0.0167    0.0343   -0.0143         0
         0         0         0   -0.0143    0.0301   -0.0125
         0         0         0         0   -0.0125    0.0158

>> B = [10;0;0;0;0;0]

    10
     0
     0
     0
     0
     0
```

```
>> V = inv(A)*B

V0   10.0000
V1    7.2864
V2    5.1087
V3    3.5172
V4    2.4812
V5    1.9589
```

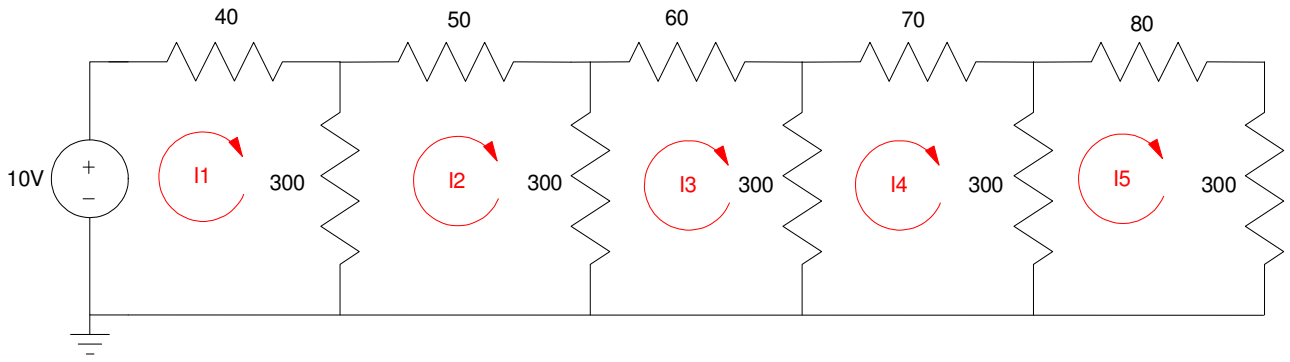
3) Check your answers in CircuitLab

Answers match



Current Loops

4) Use Current Loops to write N equations for N unknowns for the following circuit.



The sum of the volages around any closed loop must be zero

Loop I1

$$-10 + 40I_1 + 300(I_1 - I_2) = 0$$

Loop I2

$$300(I_2 - I_1) + 50I_2 + 300(I_2 - I_3) = 0$$

Loop I3:

$$300(I_3 - I_2) + 60I_3 + 300(I_3 - I_4) = 0$$

Loop I4:

$$300(I_4 - I_3) + 70I_4 + 300(I_4 - I_5) = 0$$

Loop I5:

$$300(I_5 - I_4) + 80I_5 + 300(I_5) = 0$$

5) Solve for the currents in Matlab

Group terms

$$340I_1 - 300I_2 = 10$$

$$-300I_1 + 650I_2 - 300I_3 = 0$$

$$-300I_2 + 660I_3 - 300I_4 = 0$$

$$-300I_3 + 670I_4 - 300I_5 = 0$$

$$-300I_4 + 680I_5 = 0$$

Place in matrix form

$$\begin{bmatrix} 340 & -300 & 0 & 0 & 0 \\ -300 & 650 & -300 & 0 & 0 \\ 0 & -300 & 660 & -300 & 0 \\ 0 & 0 & -300 & 670 & -300 \\ 0 & 0 & 0 & -300 & 680 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve using Matlab

```
>> A = [340,-300,0,0,0 ; -300,650,-300,0,0 ; 0,-300,660,-300,0];
>> A = [A ; 0,0,-300,670,-300 ; 0,0,0,-300,680]

   340  -300     0     0     0
  -300   650  -300     0     0
     0  -300   660  -300     0
     0     0  -300   670  -300
     0     0     0  -300   680

>> B = [10;0;0;0;0]

    10
     0
     0
     0
     0

>> I = inv(A)*B

I1    0.0678 A
I2    0.0436 A
I3    0.0265 A
I4    0.0148 A
I5    0.0065 A

>> mA = inv(A)*B * 1000

I1   67.8411 mA
I2   43.5533 mA
I3   26.5243 mA
I4   14.8002 mA
I5    6.5295 mA
```

6) Check your answers in CircuitLab.

```
>> mA = inv(A)*B * 1000

I1    67.8411 mA
I2    43.5533 mA
I3    26.5243 mA
I4    14.8002 mA
I5     6.5295 mA
```

answers match