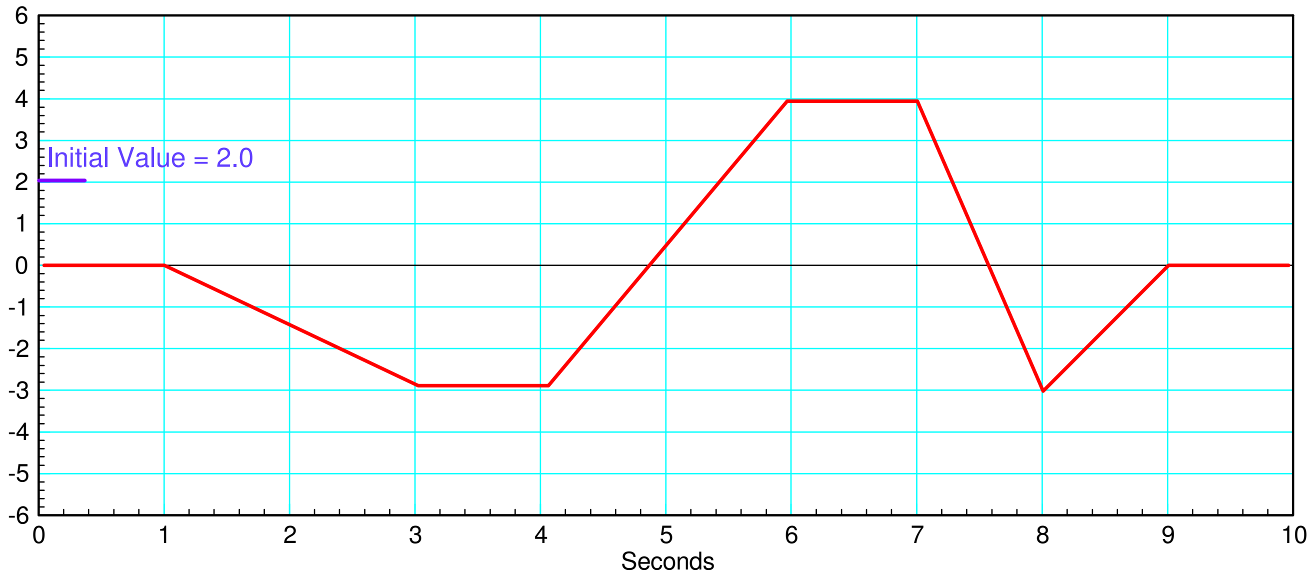
ECE 111 - Homework #7:

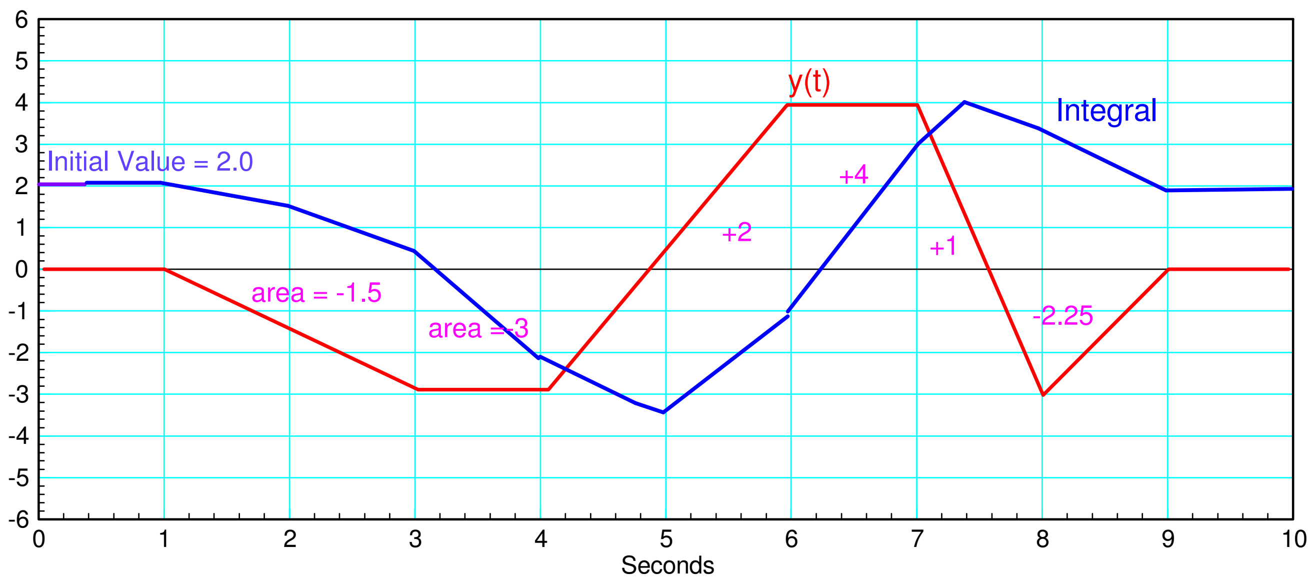
Math 166: Integration
Due Monday, Mrch 3rd. Please submit via email or on BlackBoard

1) Sketch the integral of the following funciton. Assume its initial value is +2.



If this is how much money you are depositing (positive) or withdrawing (negative) from your checking account, what is the balance at each instance?

The integral is the area under the curve



Numerical Integration

2) Use numerical methods to determine the integral of y

$$y = \exp(-x^2) \cdot \sin(x)$$

$$z = \int y \cdot dx$$

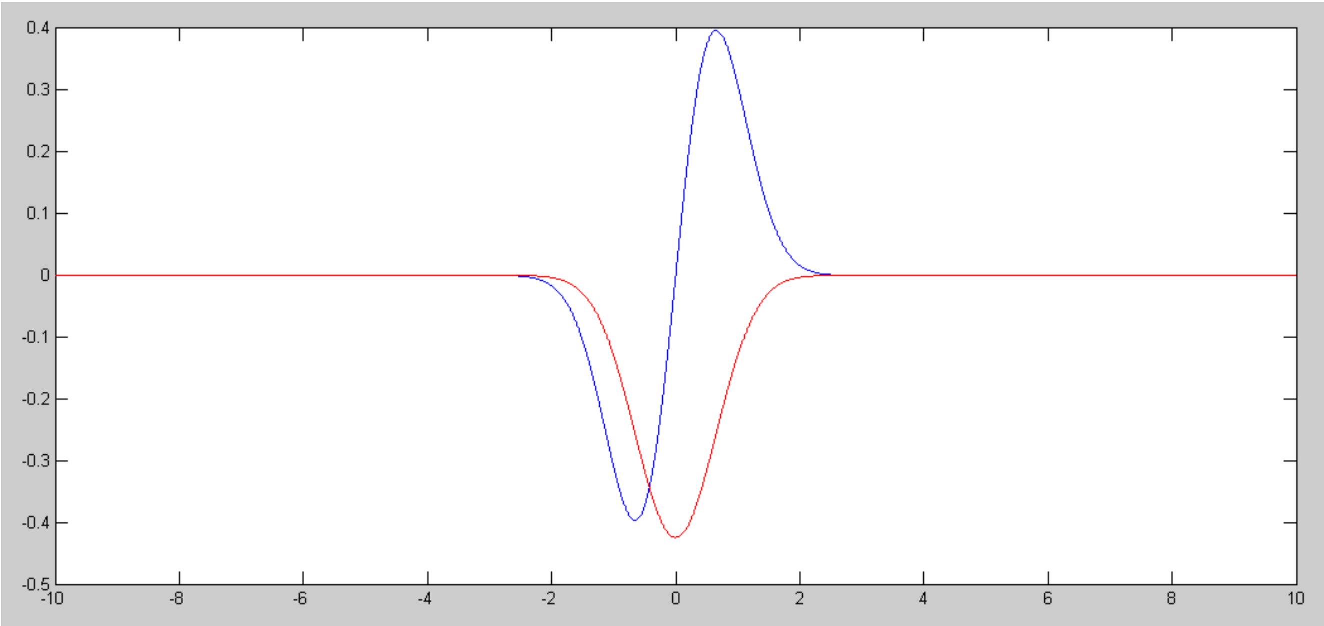
for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

Start with a Matlab funciton to integrate

```
function [y ] = Integrate( x, dy )
npt = length(x);
y = 0*dy;
for i=2:npt
    y(i) = y(i-1) + 0.5*(dy(i) + dy(i-1)) * (x(i) - x(i-1));
    end
end
```

In the command window in Matlab

```
>> x = [-10:0.001:10]';
>> y = exp(-x.^2) .* sin(x);
>> z = Integrate(x,y);
>> plot(x,y,'b',x,z,'r')
```



y(x) (blue) & it's integral (red)

Note: With numerical methods, if you can get the funciton into Matlab, you can find the integral using numerical methods. Even if you can't do so by hand.

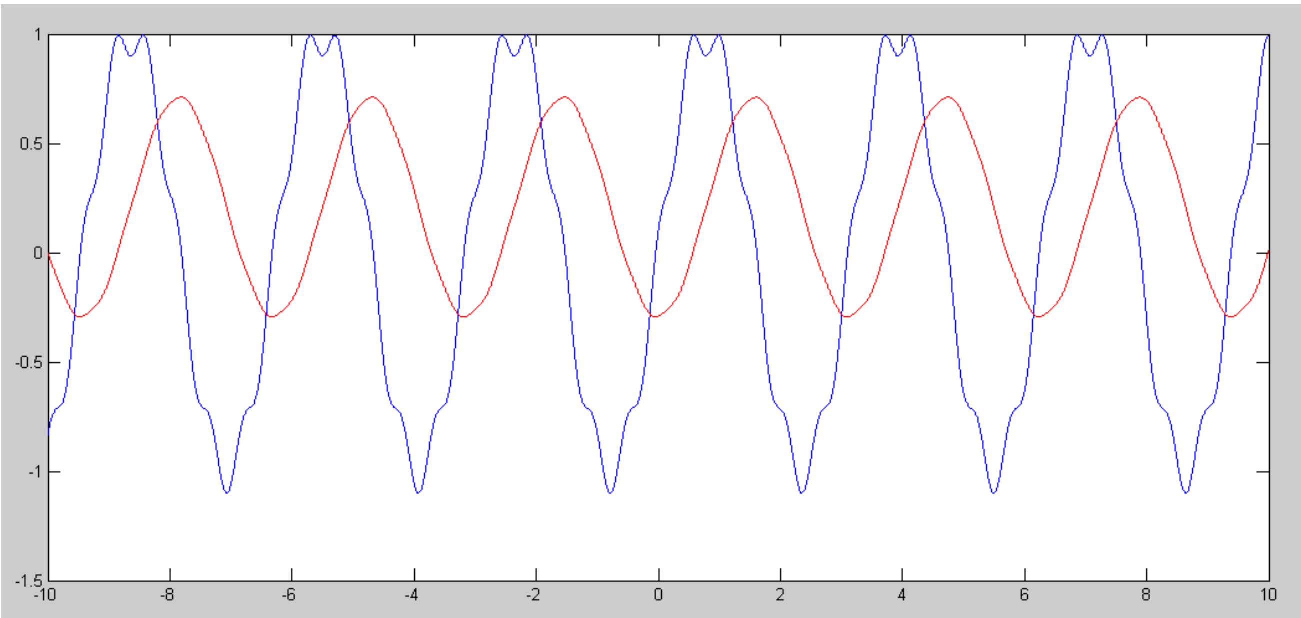
3) Use numerical methods to determine the integral of y

$$y = \sin(2x) + 0.1\cos(12x)$$

$$z = \int y \cdot dx$$

for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

```
>> x = [-10:0.001:10]';
>> y = sin(2*x) + 0.1*cos(12*x);
>> z = Integrate(x,y);
>> plot(x,y,'b',x,z,'r')
>>
```



y(x) (blue) & it's integral (red)

Animation in Matlab with Numerical Integration

4) Calculate the position of a bouncing ball in freefall:

- The acceleration is y'' = -3.72 m/s2 (gravity on Mars)
- If the ball hits the ground (y<0) the velocity becomes positive: y' = |y'|
- The initial position is (x = 0, y = 3)
- The initial velocity is (x' = +2, y' = +2)

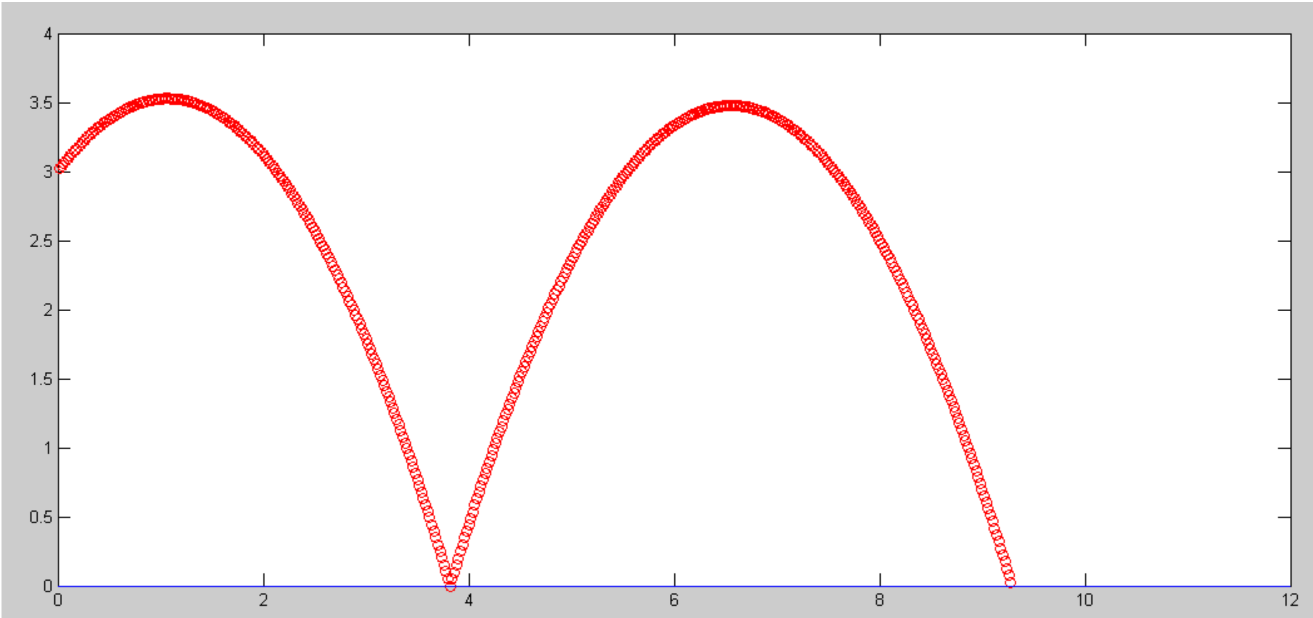
Plot the path of the ball for two bounces

- i.e. find the x position of the ball at its 2nd bouce

Matlab Code:

```
x = 0;
y = 3;

dx = 2;
dy = 2;

t = 0;
dt = 0.01;
g = -3.72;


Bounce = 0;

while(Bounce < 2)
    ddx = 0;
    ddy = g;

    dx = dx + ddx * dt;
    dy = dy + ddy * dt;

    x = x + dx * dt;
    y = y + dy * dt;

    t = t + dt;

    if(y < 0)
        dy = abs(dy);
        Bounce = Bounce + 1;
        end

    plot(x,y,'ro',[0,12],[0,0],'b',0,10,'b+');
    xlim([0,12]);
    ylim([0,4]);

    pause(0.01);
end

> disp(x)
    9.3000
```

The ball hits a second time at 9.30

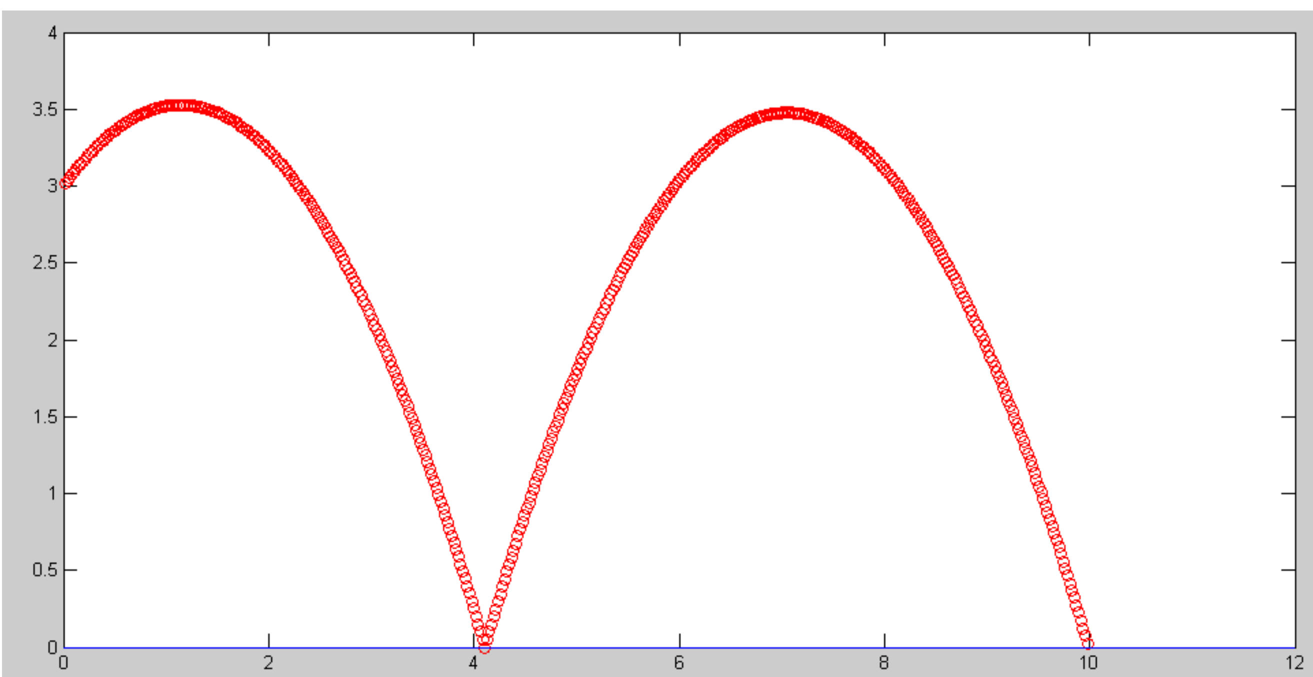


5) Determine the initial velocity on x' so that the ball hits a target at (x=10, y=0) on the second bounce

- note: this is a f(x) = 0 problem

This should be linear. Scaled as

$$dx = 2 \cdot \left(\frac{10}{9.3}\right)$$

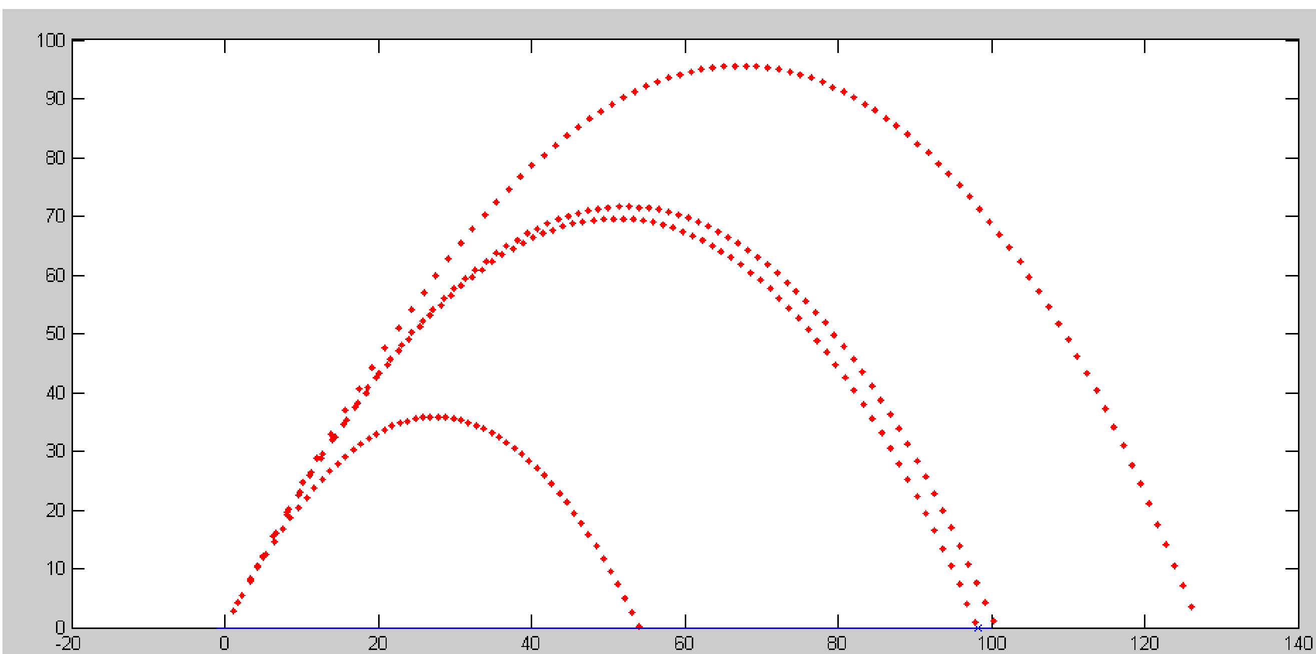


f(x) = 0: Shoot Game

- Pick a random number from 50 to 100 for your target.
- Pick a random number from 30 to 70 for your firing angle

6) Use trial and error to find the initial velocity (X) to fire a tennis ball to hit the target (result is zero)

```
Target =   98.2444

>> Angle = 50*rand + 20

Angle =   68.5296

>> Shoot(30, Angle, Target)

ans =  -44.1856

>> Shoot(60, Angle, Target)

ans =   28.7245

>> Shoot(47, Angle, Target)

ans =    2.2787

>> Shoot(46, Angle, Target)

ans =   -0.1223

>> ylim([0,100])
>>
```



7) Repeat using California (or Newton's) method to find the initial velocity (X) to fire the tenis ball to hit the target

```
>> X0 = 0;
>> Y0 = -Target;

>> X1 = 30;
>> Y1 = Shoot(X1, Angle, Target)
Y1 =  -44.1856

>> X2 = X1 - (X1-X0)/(Y1-Y0)*Y1
X2 =   54.5209

>> Y2 = Shoot(X2, Angle, Target)
Y2 =   18.6355

>> X3 = X2 - (X2-X1)/(Y2-Y1)*Y2
X3 =   47.2469

>> Y3 = Shoot(X3, Angle, Target)
Y3 =    2.8634

>> X4 = X3 - (X3-X2)/(Y3-Y2)*Y3
X4 =   45.9263

>> Y4 = Shoot(X4, Angle, Target)
Y4 =   -0.3012

>> X5 = X4 - (X4-X3)/(Y4-Y3)*Y4
X5 =   46.0520

>> Y5 = Shoot(X5, Angle, Target)
Y5 =    0.0040
```

If you're willing to do some calculations, you can find the answer pretty quickly.