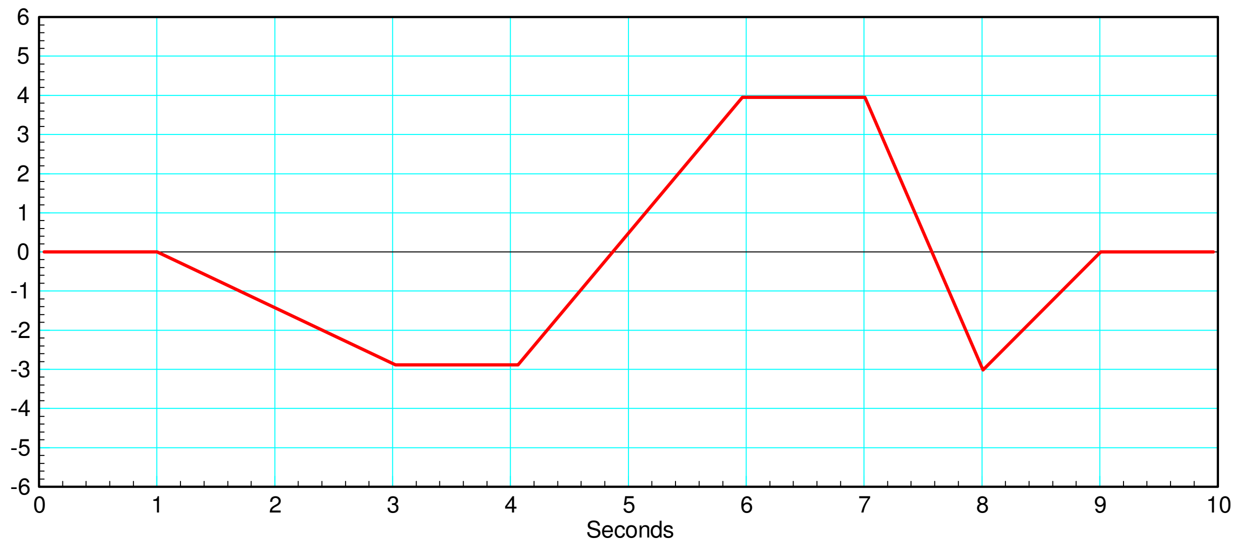
ECE 111 - Homework #6:

Math 165: Differentiation

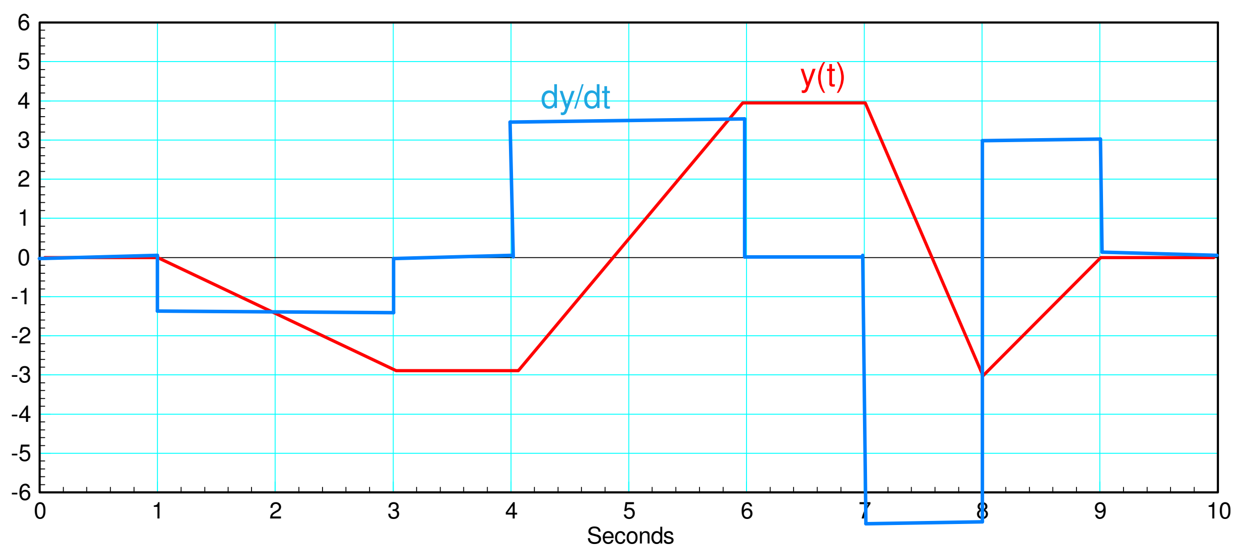
Due Monday, February 24th. Please submit via email or on BlackBoard

1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?



The derivative is the slope



Numerical Differentiation:

2) Use numerical methods to determine the derivative of y:

$$y = \exp(-x^2) \cdot \sin(x)$$

$$z = \frac{d}{dx}(y)$$

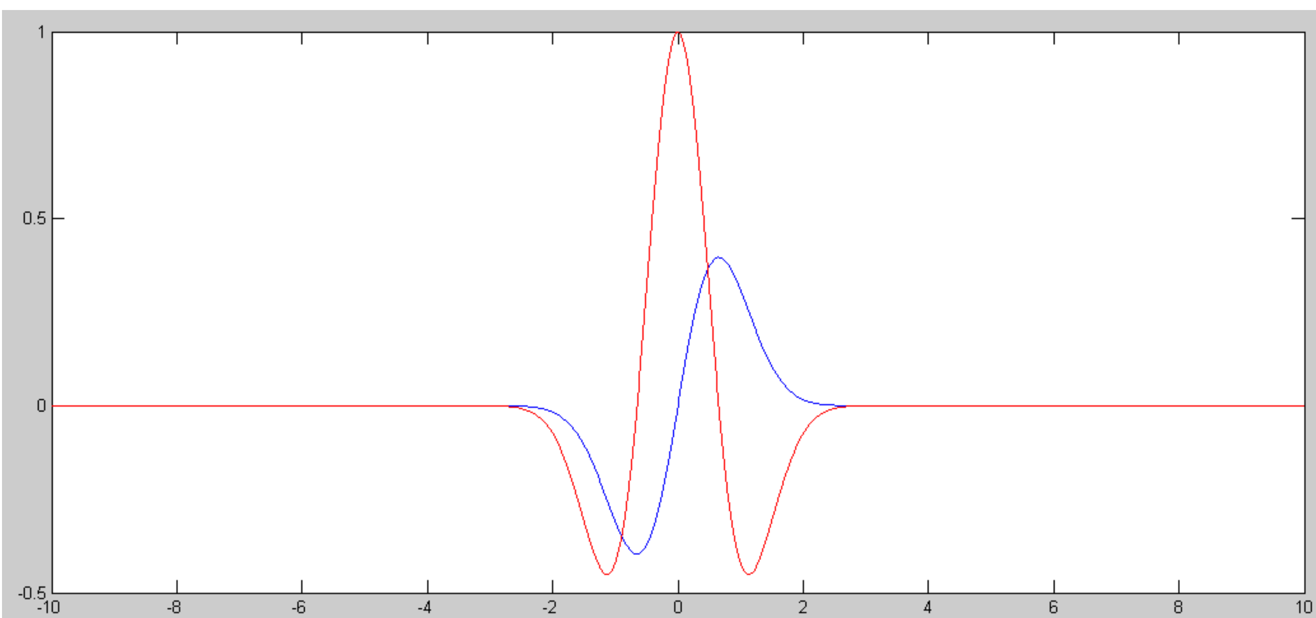
for -10 < x < 10. (a plot is sufficient).

Create a funciton in Matlab to compute the derivative (slope) using numerical methods

```
function [ dy ] = derivative( x, y )

dy = 0*y;
n = length(y);
for i=2:n-1
  dy(i) = ( y(i+1) - y(i-1) ) / ( x(i+1) - x(i-1) );
  end

dy(1) = 2*dy(2) - dy(3);
dy(n) = 2*dy(n-1) - dy(n-2);

end
```

Using this...

```
>> x = [-10:0.01:10]';
>> y = exp(-x.^2) .* sin(x);
>> z = derivative(x,y);
>> plot(x,y,'b',x,z,'r')
```



y(x) (blue) and the derivative dy/dx (red)

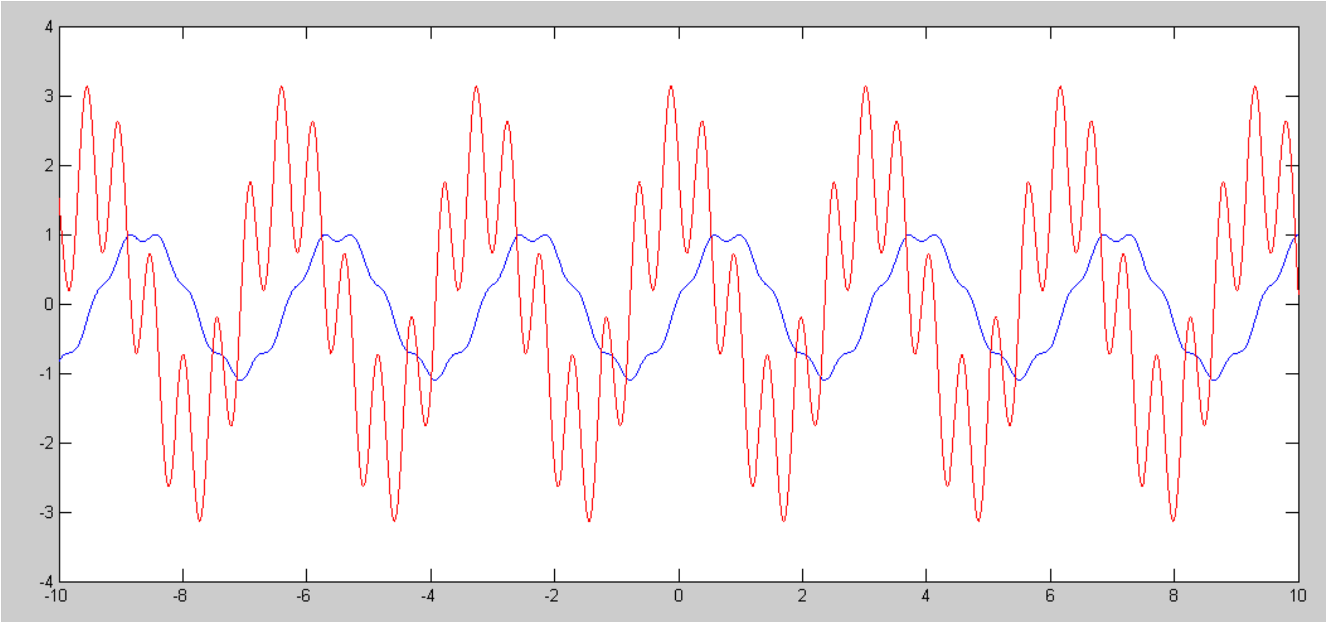
3) Use numerical methods to determine the derivative of y:

$$y = \sin(2x) + 0.1\cos(12x)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> x = [-10:0.01:10]';
>> y = sin(2*x) + 0.1*cos(12*x);
>> z = derivative(x,y);
>> plot(x,y,'b',x,z,'r')
>>
```



y(x) (blue) and the derivative dy/dx (red)

Note:

- This is the power or Matlab
- If you can get the funciton into Matlab, you can find the derivative using numerical techniques
- Even if you havn't had calculus yet or don't know how to compute the derivative by hand

Sidelight:

- When you get to Calculus, this is a good way to check your answers
- Compare the funciton you computes with the numerical solution found by Matlab
- The two solutions should match

Path Planning

4) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ t/5 & 0 < t < 5 \\ 1 & t > 5 \end{cases}$$

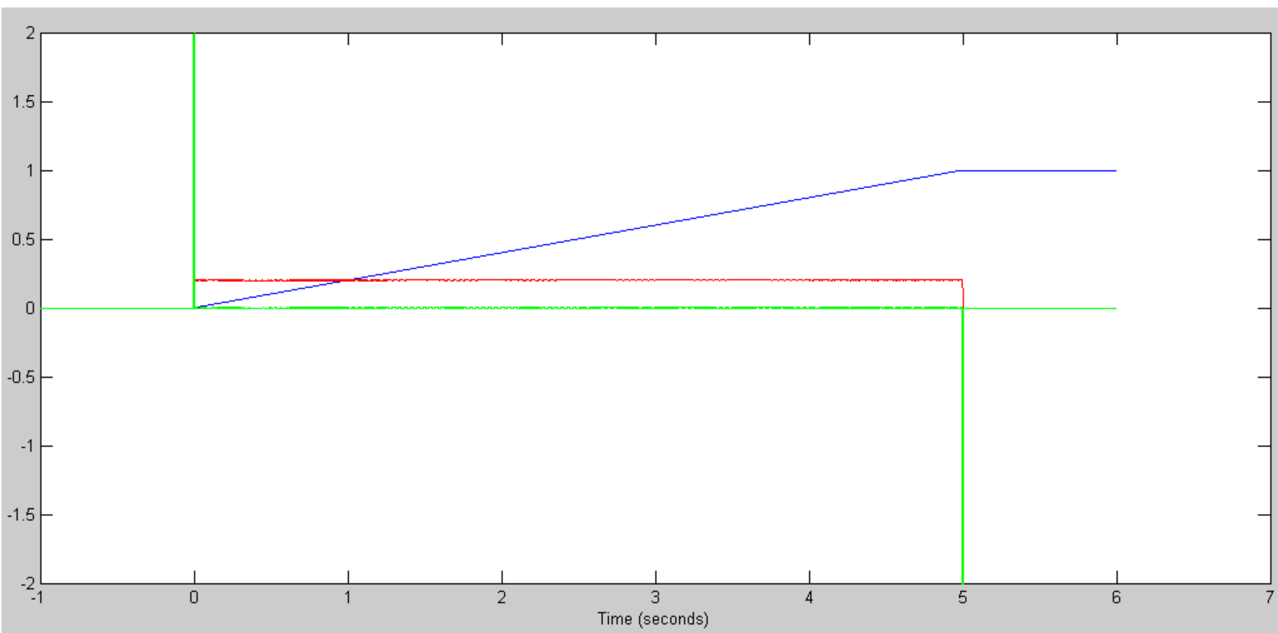
Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

Matlab Code (time is offset by 1e-6 to avoid probems with equals)

```
t = [-1:0.001:6]' + 1e-6;
q = (t/5) .* (t>0) .* (t<5)  +  1*(t>5);

dq = derivative(t,q);
ddq = derivative(t,dq);

plot(t,q,'b',t,dq,'r',t,ddq,'g')
xlabel('Time (seconds)')
ylim([-2,2])
```

Results:



Motor Position (blue), Velocity (red), and Acceleration (green)

Comment:

- Speed = Voltage (red)
- Acceleration = Current (green)
- The current goes to +/- infinity - which is a problem

5) Assume a motor's angle is as follows:

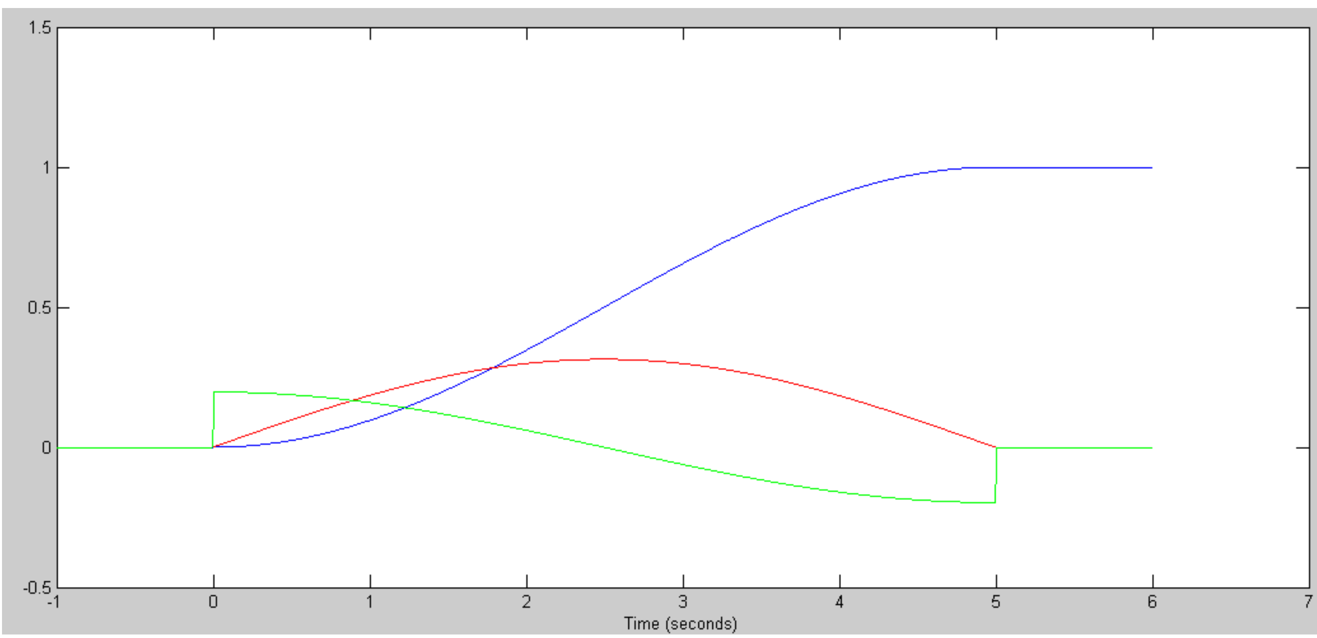
$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{5}\pi t\right) & 0 < t < 5 \\ 1 & t > 5 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

In Matlab

```
t = [-1:0.001:6]' + 1e-6;
q = (0.5 - 0.5*cos(t*pi/5)) .* (t>0) .* (t<5)   +   1*(t>5);

dq = derivative(t,q);
ddq = derivative(t,dq);

plot(t,q,'b',t,dq,'r',t,ddq,'g')
xlabel('Time (seconds)')
ylim([-0.5,1.5])
```



Motor Position (blue), Velocity (red), and Acceleration (green)

Comment:

- Not a lot different from problem #4 in terms of position, but
- Motor current (green) is now finite
- This is a reasonable path for going from 0 to 1 in 5 seconds

6) Assume a motor's angle is as follows:

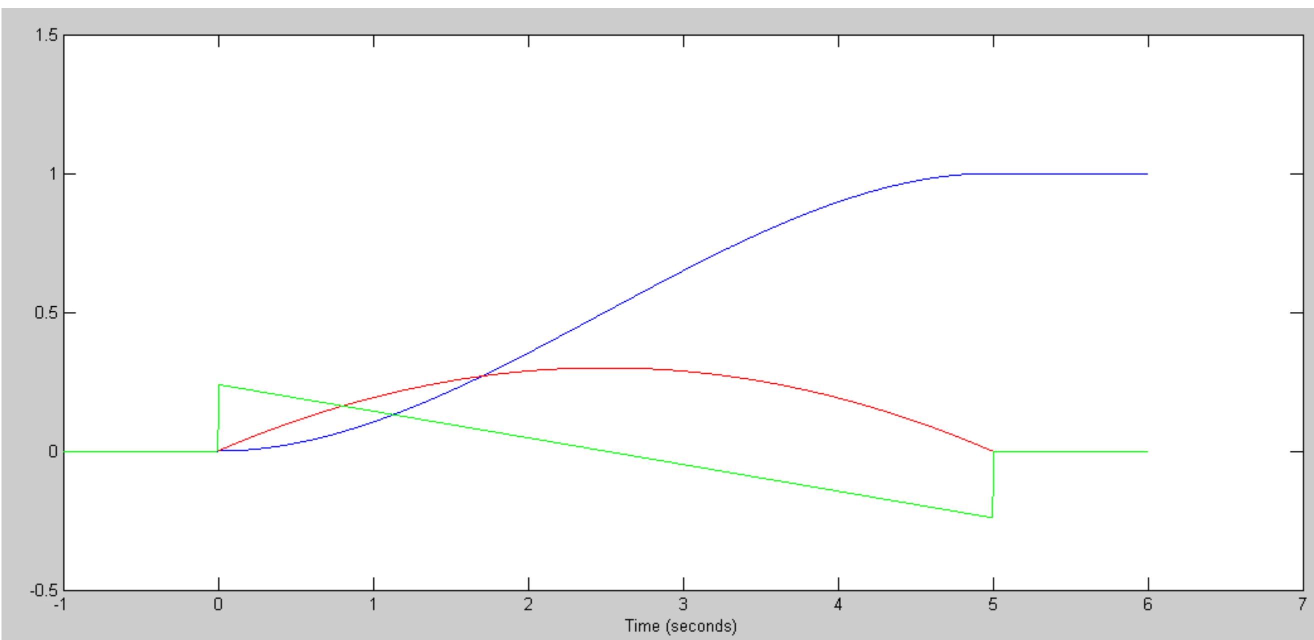
$$\theta = \begin{cases} 0 & t < 0 \\ -0.016t^3 + 0.12t^2 & 0 < t < 5 \\ 1 & t > 5 \end{cases}$$

Calculate using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

In Matlab:

```
t = [-1:0.001:6]' + 1e-6;
q = (-0.016*(t.^3) + 0.12*(t.^2)) .* (t>0) .* (t<5)  +  1*(t>5);

dq = derivative(t,q);
ddq = derivative(t,dq);

plot(t,q,'b',t,dq,'r',t,ddq,'g')
xlabel('Time (seconds)')
ylim([-0.5,1.5])
```



Motor Position (blue), Velocity (red), and Acceleration (green)

Comments:

- Not a lot different from problem #6
- The motor voltage (red) and current (green) are reasonably well behaved
- This is a reasonable path for going from 0 to 1 in 5 seconds