ECE 111 - Homework #4

Math 129 Linear Algebra.
Due Monday, Febryary 10th. Please submit via email or on BlackBoard

N equations & N unknowns

1) Solve for {x, y}

$$4x - 4y = 3$$

$$5x + 4y = -7$$

Solution: Express in matrix form

$$\begin{bmatrix} 4 & -4 \\ 5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -7 \end{bmatrix}$$

$$BX = A$$

The solution is then

$$X = B^{-1}A$$

In Matlab

```
>> B = [4,-4 ; 5,4]

     4    -4
     5     4

>> A = [3;-7]

     3
    -7

>> X = inv(B)*A

x   -0.4444
y   -1.1944
```

2) Solve for {x, y, z}

$$8x + 3y - 5z = 4$$

$$0x + 2y - 5z = 8$$

$$9x - 6y - 0z = 9$$

Express in matrix form

$$\begin{bmatrix} 8 & 3 & -5 \\ 0 & 2 & -5 \\ 9 & -6 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 9 \end{bmatrix}$$

$$BX = A$$

Solving

$$X = B^{-1}A$$

In Matlab

```
>> B = [8,3,-5 ; 0,2,-5 ; 9,-6,0]

     8     3    -5
     0     2    -5
     9    -6     0

>> A = [4;8;9]

     4
     8
     9

>> X = inv(B)*A
```

x -0.2632
y -1.8947
z -2.3579

3) Solve for {a, b, c, d}

$$1a + 7b + 9c + 2d = 2$$

$$7a + 5b + 3c + d = -1$$

$$7a - 6b + 6c + 3d = -8$$

$$5a + 5b + 5c - 7d = 4$$

Express in matrix form

$$\begin{bmatrix} 1 & 7 & 9 & 2 \\ 7 & 5 & 3 & 1 \\ 7 & -6 & 6 & 3 \\ 5 & 5 & 5 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -8 \\ 4 \end{bmatrix}$$

Solving in Matlab

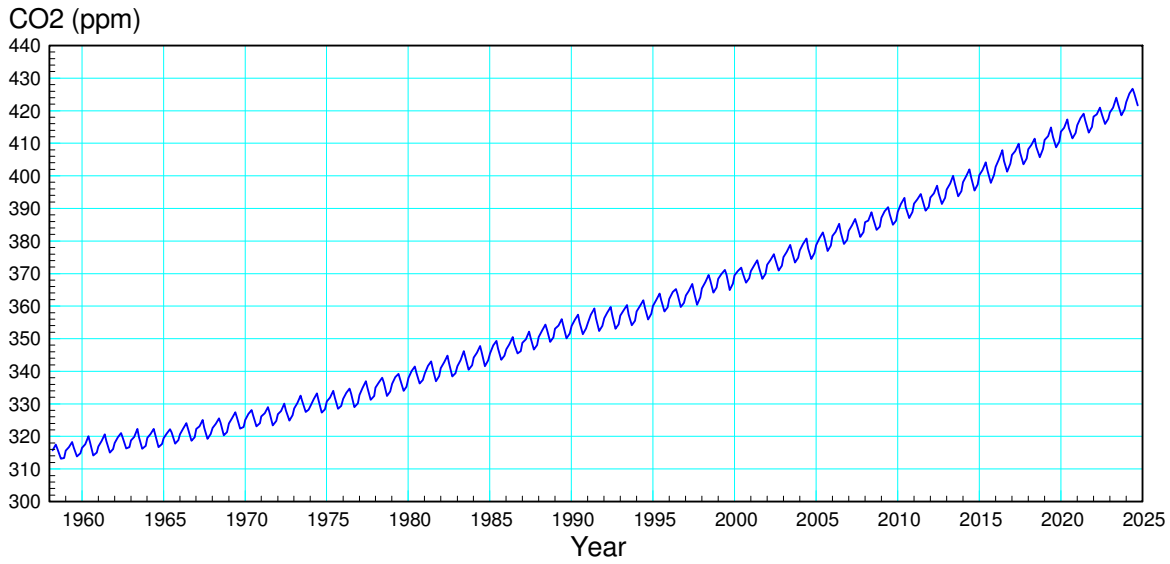
```
>> B = [1,7,9,2 ; 7,5,3,1 ; 7,-6,6,3 ; 5,5,5,-7]

     1     7     9     2
     7     5     3     1
     7    -6     6     3
     5     5     5    -7

>> A = [2;-1;-8;4]

     2
    -1
    -8
     4

>> X = inv(B)*A

a   -0.4373
b    0.5337
c   -0.0281
d   -0.5227

>>
```

Comments

- With Matlab, solving N equations for N unknowns is pretty easy
- This trick is going to be used over and over in courses you take in ECE. Whenever you see a problem, think about how you can set it up as solving N equations for N unknowns. If you can do this, you can solve the problem using Matlab.

Global CO2 Levels

The CO2 levels measured at Mauna Loa observatory for the past 56 years are:



https://gml.noaa.gov/webdata/ccgg/trends/co2/co2_mm_mlo.txt
http://www.bisonacademy.com/ECE111/Code/CO2%20Levels.txt

Problem 4) Determine a parabolic curve fit for this data in the form of

$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year.

Note: This is similar to the previous problems except that you have more equations than unknowns. In this casem you can solve for the unknowns using least squares.

Step 1: Express the solution in matrix form

$$\begin{bmatrix} CO2_1 \\ CO2_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} y_1^2 & y_1 & 1 \\ y_2^2 & y_2 & 1 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$Y = BA$$

Multiply both sides by B transpose

$$B^T Y = B^T BA$$

Multiply on the left by the inverse of B'B

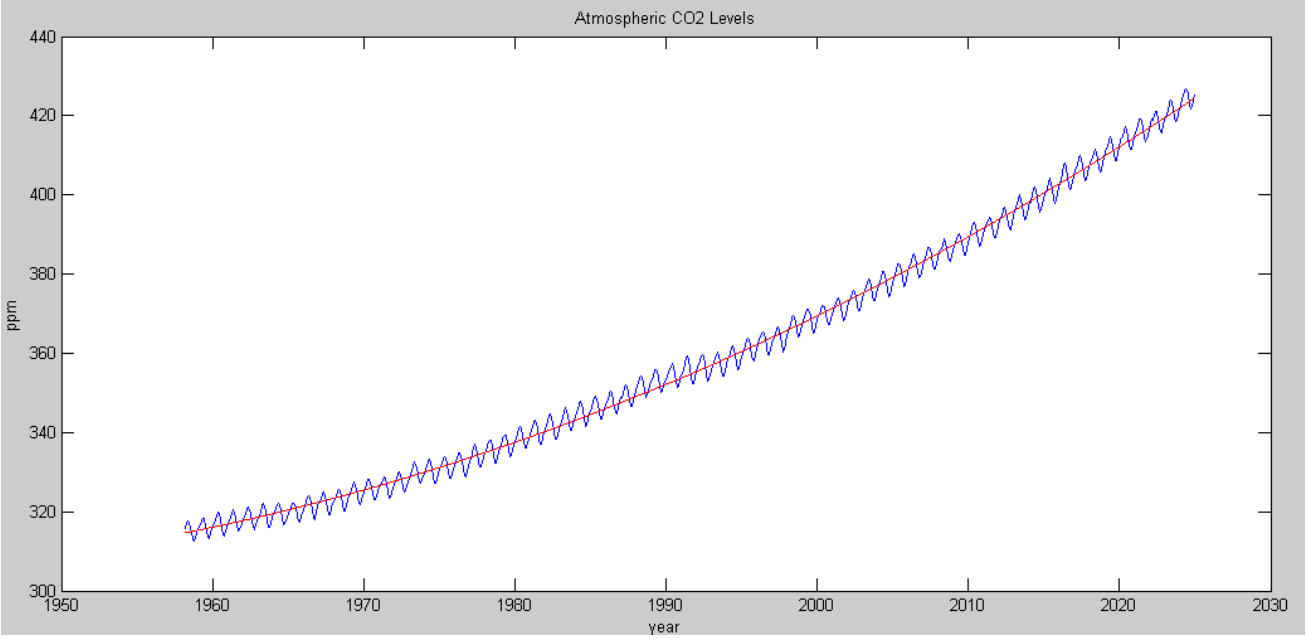
$$A = (B^T B)^{-1} B^T Y$$

The result is the least-squares solution

- also known as a least-squares curve fit

In Matlab - copy the CO2 data and paste into Matlab

```
>> Data = [
< paste in data >
];
>> year = Data(:,3);
>> CO2 = Data(:,4);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*CO2

a  1.3300e-002
b -5.1333e+001
c  4.9835e+004

>> plot(year, CO2, 'b', year, B*A, 'r')
>> xlabel('year');
>> ylabel('ppm');
>> title('Atmospheric CO2 Levels')
>>
```



When will global CO2 levels reach 400ppm?

```
>> roots(A - [0;0;400])
  2014.8       the year it reaches 400ppm (matches the graph)
  1844.8       stray soluion
```

When will it reach 600ppm?

```
>> roots(A - [0;0;600])
  2079.0       the year CO2 levels reach 600ppm  (off the graph - a prediction)
  1780.6
```

When will it reach 2000ppm?

```
>> roots(A - [0;0;2000])
  2286.9       the year it reaches 2000ppm  (off the graph - a prediction)
  1572.7       stray solution
```

Fargo Temperatures

note: Column #1 of the data set is year, column #14 is yearly average temeperature in degrees F

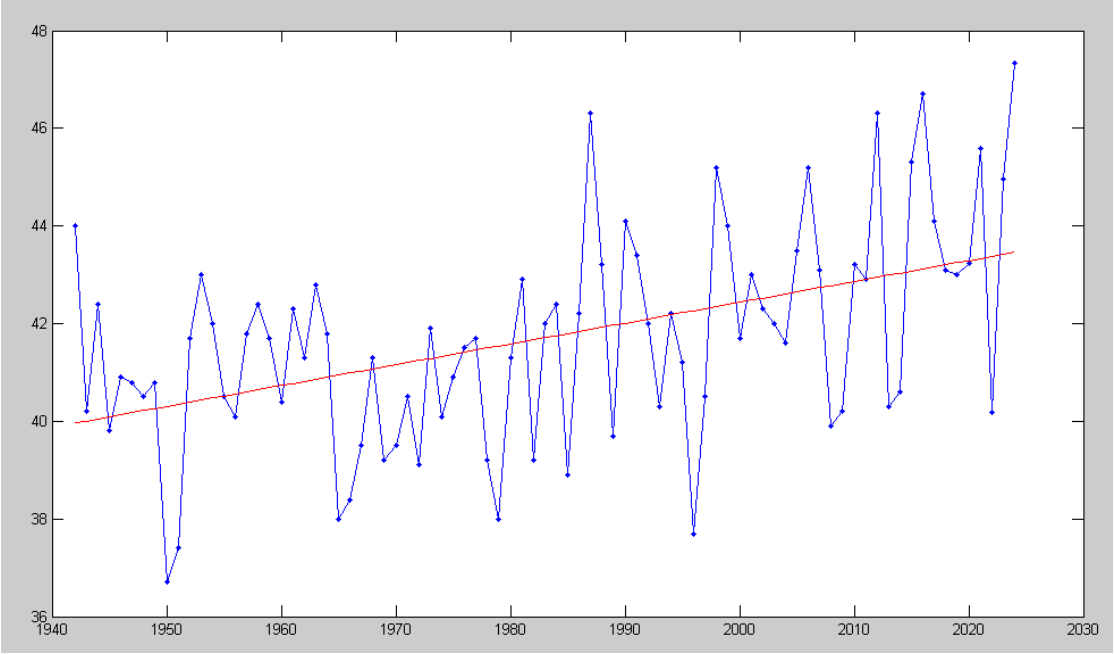
```
year = DATA(:,1);
T = DATA(:,14);
```

5) Using the average temperature in Fargo from 1942 to 2022:

5a) Determine a curve fit of the form of $T = ay + b$

In Matlab

```
>> year = Data(:,1);
>> T = Data(:,14);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*T

a   4.2649e-002
b  -4.2861e+001

>> plot(year,T,'b.-',year,B*A,'r')
```



5b) How much has Fargo warmed up over the past 80 years?

This is the slope (a) time years

```
>> a = A(1);
>> b = A(2);
>> dT = a*80

dT =    3.4119
```

Fargo has gotten 3.4119 degrees warmer over the past 80 years

5c) What will the average temperature in Fargo be

- In the year 2050?
- In the year 2100?

This is just plugging in numbers. From least squares,

$$T \approx ay + b$$

$$T \approx 0.0426 \cdot y - 42.8605$$

Plug in the year 2050:

```
>> a*2050 + b

ans =   44.5698
```

If the trend continues, Fargo's average temperature in 2050 wlil be 44.57 degrees F

Plug in the year 2100

```
>> a*2100 + b

ans =  46.7022
```

If the trend continues, Fargo's average temperature in 2050 wlil be 46.72 degrees F

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

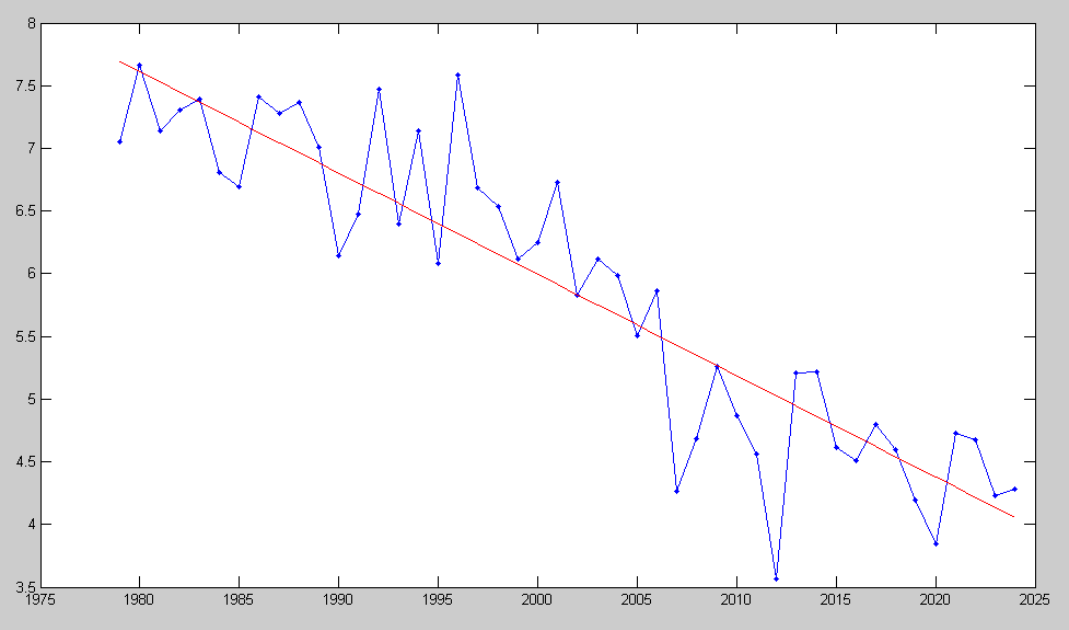
6) Approximate this data from the years 1979 - 2024 with a line

$$Area \approx ay + b$$

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

In Matlab

```
>> year = Data(:,1);
>> Ice = Data(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*Ice

a   -0.0808
b  167.5683

>> plot(year,Ice,'b.-',year,B*A,'r')
```



To find the zero crossing, use the Matlab function *roots()*

```
>> roots(A)

ans =  2074.2
```

If this trend continues, and assuming a constant rate of ice loss, the Arctic will be ice free in the year 2074 (49 years from now).

7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

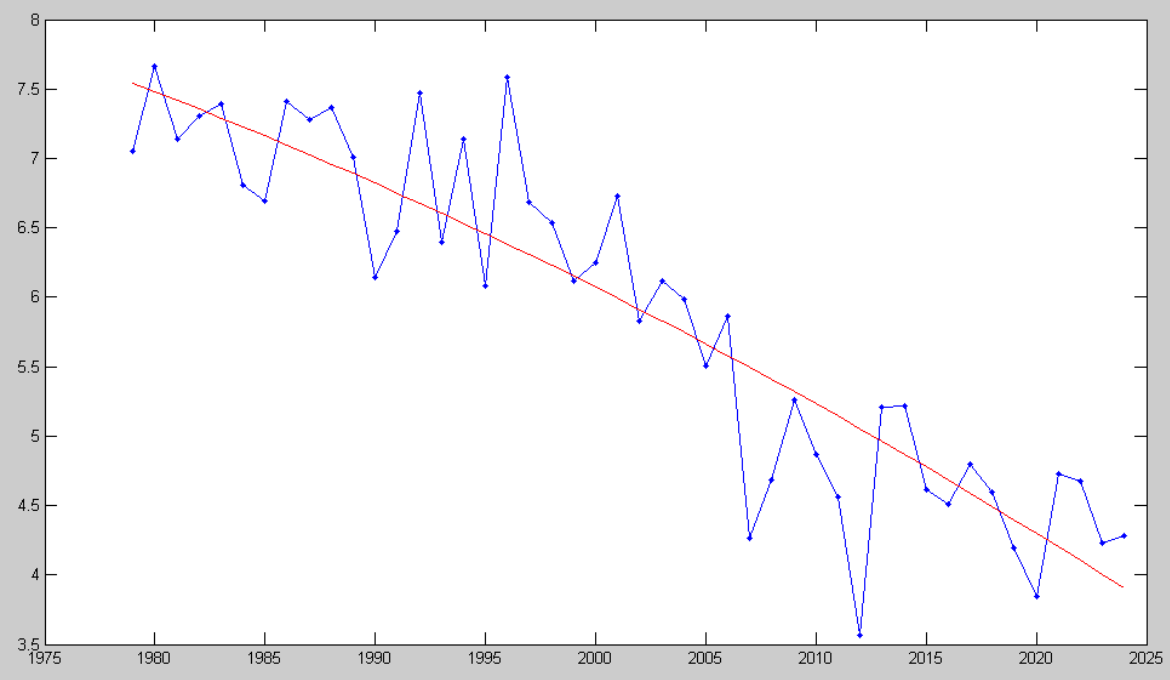
```
>> B = [year.^2, year, year.^0];
```

Change the basis function (B) and you have a different curve fit. In Matlab:

```
>> year = Data(:,1);
>> Ice = Data(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*Ice
```

```
a -4.4980e-004
b  1.7197e+000
c -1.6342e+003
```

```
>> plot(year,Ice,'b.-',year,B*A,'r')
```



To find when the ice level is zero, use the *roots()* command in Matlab:

```
>> roots(A)

  2057.6
  1765.8        stray solution
```

Assuming the rate of ice loss increases by the same amount each year, the data suggests the Arctic will be ice free in the year 2057 (32 years from now)

Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (144 years of data).

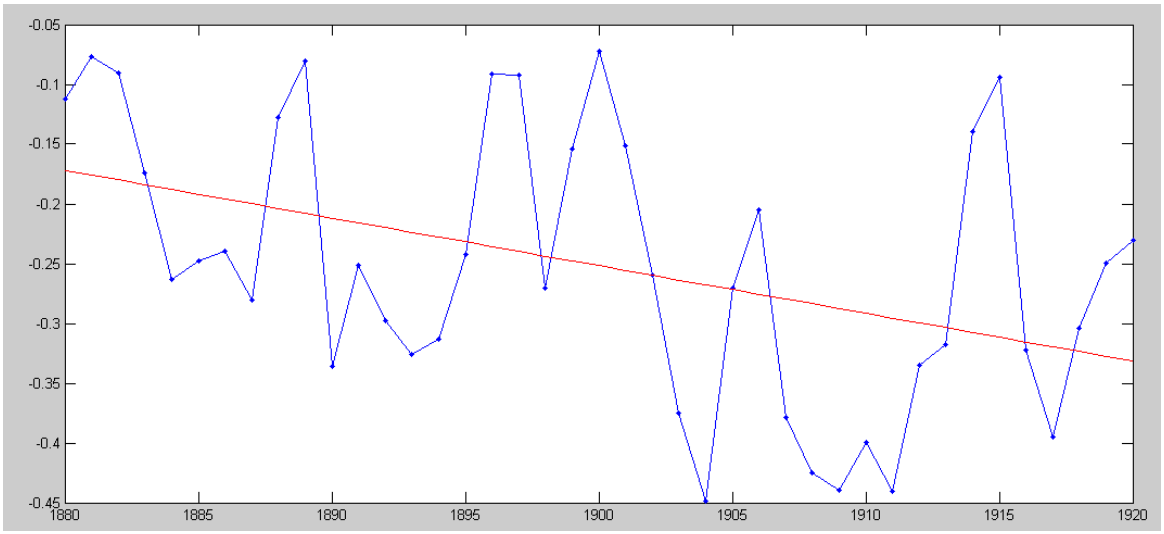
8) Determine a least-squares curve fit for this data from the year 1880 - 1920 in the form of

$$\delta T = ay + b$$

Based upon this data, what *should* the temperature deviation be in the year 2024?

In Matlab, copy and paste the data from 1880 - 1920

```
>> Data = [  <paste in the data>  ];
>> year = Data(:,1);
>> dT = Data(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

a -3.9853e-003
b  7.3204e+000

>> plot(year,dT,'b.-',year,B*A,'r')
>>
```



If this trend continued, the year 2024 should see a temperature deviation of -0.749 degrees C

```
>> a = A(1);
>> b = A(2);
>> a*2024 + b

   -0.7459 degrees C
```

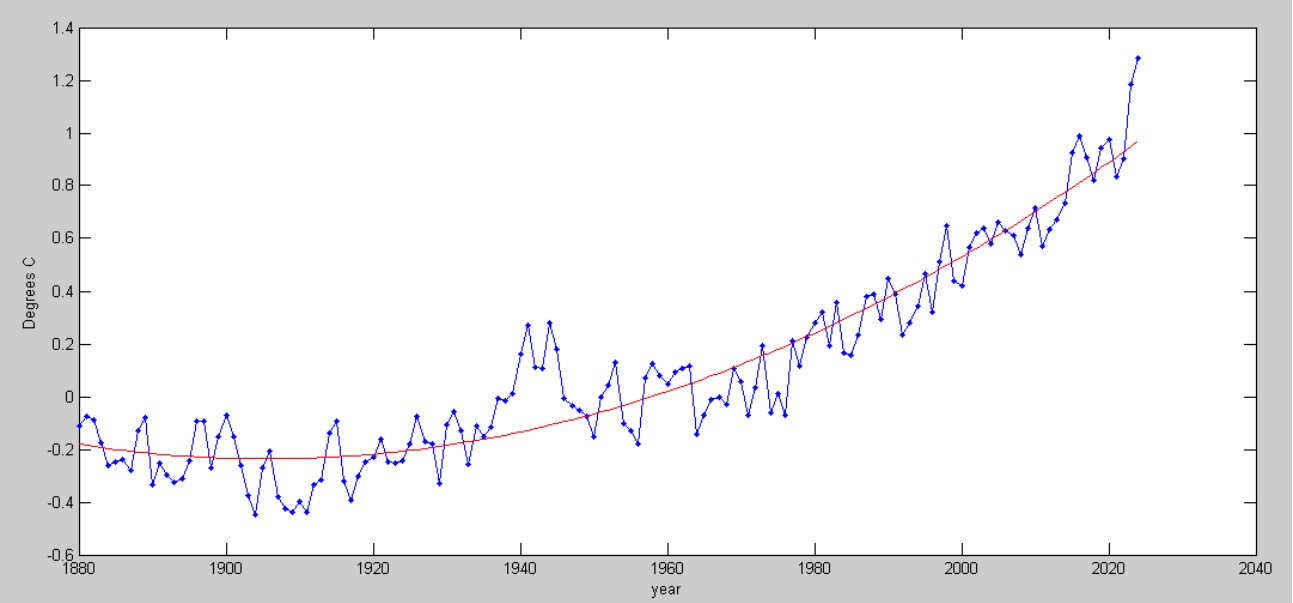
Note: Back in 1900, it looked like we were entering another ice age.

9) Determine a least-squares curve fit for this data from the year 1880 - 2024 in the form of

$$\delta T \approx ay^2 + by + c$$

In Matlab, copy and paste the data from 1880 - 2024

```
>> year = Data(:,1);
>> dT = Data(:,2);
>> B = [year.^2, year, year.^0];
>> format short e
>> A = inv(B'*B)*B'*dT

a  8.5737e-005
b -3.2674e-001
c  3.1106e+002

>> format short
>> plot(year,dT,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees C');
>>
```



What will the temperature deviation be in the year 2100?

```
>> a = A(1);
>> b = A(2);
>> c = A(3);
>> a*2100^2 + b*2100 + c

ans =    3.0085
```

Assuming a parabolic curve fit, the temperate deviation by the year 2100 will be +3.00 degees C

When will the temperature deviation reach +10C?

Use the Matlab command *roots()* to find this

```
>> roots(A - [0;0;10])

    2251.0
    1559.9    stray solution
```

If nothing changes, we should see a +10 degree temperature rise by the year 2251 (226 years from now)

10) What does a temperature rise of 10 degrees mean for the planet?

not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2025 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])

    2025.5
    1785.4
```

Two Degrees: 2067. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
>> roots(A - [0;0;2])

    2067.0
    1744.0
```

Three Degrees: 2099. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
>> roots(A - [0;0;3])

    2099.7
    1711.2
```

Four Degrees: 2127. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
>> roots(A - [0;0;4])

    2127.7
    1683.2
```

Six Degrees: 2175. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
>> roots(A - [0;0;6])

    2175.2
    1635.8
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.