ECE 111 - Homework #3

Math 105: Trigonometry.

Due Monday, February 3rd. Please submit via email or on BlackBoard

Polar to Rectangular Conversions

1) Determine the final position of A: (x,y)

$$A = (6\angle -93^{\circ}) + (11\angle 70^{\circ}) + (8\angle 87^{\circ})$$

In Matlab

```
>> x1 = 6*cos(-93*pi/180)
x1 = -0.3140
>> y1 = 6*sin(-93*pi/180)
y1 = -5.9918
>> x2 = 11*cos(70*pi/180)
x2 = 3.7622
>> y2 = 11*sin(70*pi/180)
y2 = 10.3366
>> x3 = 8*cos(87*pi/180)
x3 = 0.4187
>> y3 = 8*sin(87*pi/180)
y3 = 7.9890
>> Ax = x1 + x2 + x3
Ax = 3.8669
>> Ay = y1 + y1 + y3
Ay = -3.9945
```

On an HP Prime

• Setting - Entry - RPN

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11 angle 70
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8 angle 87
+
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```

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2) Determine final position of B: (x,y)

$$B = (5\angle -22^{0}) + (22\angle 31^{0}) + (20\angle -66^{0})$$

In Matlab

>>			
>>	Ву	= y1	+ y2 + y3
By	=	-8.8	131
>>	Bx	= x1	+ x2 + x3
Bx	=	31.6	283
>>	уЗ	= 20*	sin(-66*pi/180)
уЗ	=	-18.2	709
>>	x3	= 20*	cos(-66*pi/180)
x3	=	8.1	347
>>	y2	= 22*	sin(31*pi/180)
y2	=	11.3	308
>>	x2	= 22*	cos(31*pi/180)
x2	=	18.8	577
>>	x =	= 22*c	os(31*pi/180)
X =	=	18.85	77
>>	y1	= 5*s	in(-22*pi/180)
y1	=	-1.8	730
>>	x1	= 5 *	cos(-22*pi/180)
x1	=	4.6	359

On an HP-Prime calculator

5 angle -22 enter 22 angle 31 + 20 angle -66 + angle

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3) Where is B relative to A (i.e. what is C = B - A?)

- In (x,y) coordinates
- In polar coordinates

In Matlab

```
>> Cx = Bx - Ax
Cx = 27.7614
>> Cy = By - Ay
Cy = -4.8186
>> Cr = sqrt(Cx^2 + Cy^2)
Cr = 28.1765
>> Cq = atan2(Cy, Cx) * 180/pi
Cq = -9.8468 (degrees)
>>
```

Plotting Polar Functions

4) Plot the following functions in Matlab for $-2\pi < \theta < 2\pi$

- Note: plot() plots in cartesian coordinates. Each function needs to be converted from polar to rectangular.
- a) $r = 2\sin(\theta) + 3\cos(\theta)$

Matlab Script & Figure:

not surprisingly, this plots as a circle. Trig functions are all about circles.



b) $r = (2\pi - \theta)(2\pi + \theta)$

sort of a heart - happy Valentine's day!





another variation of a heart



Robot Tip Position (Forward Kinematics)

A 2D robot has three arms with lengths of {3.0, 2.0, 1.0} meters. The final tip positionis

$$\begin{aligned} x_1 &= 3\cos{(\theta_1)} & y_1 &= 3\sin{(\theta_1)} \\ x_2 &= x_1 + 2\cos{(\theta_1 + \theta_2)} & y_2 &= y_1 + 2\sin{(\theta_1 + \theta_2)} \\ x_3 &= x_2 + \cos{(\theta_1 + \theta_2 + \theta_3)} & y_3 &= y_2 + \sin{(\theta_1 + \theta_2 + \theta_3)} \end{aligned}$$

5) Plot the tip position (x3, y3) for

 $\theta_1 = 41^0$ $\theta_2 = -94^0$ $\theta_3 = -45^0$

Matlab program:

- Pass the angles in degrees
- Return the tip position (x3, y3)
- Just for fun, also plot the position of the robot (not required but fun to see)

File RRR.m

```
function [x3, y3] = RRR(q1, q2, q3)
   % convert to radians
   q1 = q1 * pi / 180;
q2 = q2 * pi / 180;
q3 = q3 * pi / 180;
   % compute the joint positions
   x0 = 0;
   v0 = 0;
   x1 = x0 + 3 \cos(q1);
   y1 = y0 + 3*\sin(q1);
   x^2 = x^1 + 2 \cos(q^1+q^2);
   y^2 = y^1 + 2 \sin(q^1+q^2);
   x3 = x2 + 1 \cos(q1+q2+q3);
   y3 = y2 + 1*sin(q1+q2+q3);
   % just for fun, plot the resulting robot position
   plot([x0,x1,x2,x3],[y0,y1,y2,y3],'b.-');
   ylim([-1,4]);
   xlim([-1,4]);
   pause(0.01);
   end
```

From the command window, call the subroutine

>>> [x3, y3] = RRR(41, -94, -45) x3 = 3.3286 y3 = -0.6194



Tip Position 3.3286 -0.6194

6) Plot the tip position (x3, y3) for

$$\theta_1 = -91^0$$
 $\theta_2 = -81^0$ $\theta_3 = 65^0$
>> [x3, y3] = RRR(-91, -81, 65)
x3 = -2.3253
y3 = -4.2342



Robot Tip Position (Inverse Kinematics & fminsearch())

7) Write a Matlab function which

- Is passed the angles $(\theta_1, \theta_2, \theta_3)$,
- Computes the tip position, and
- Returns the distance from the tip position and point (x = 2.0, y = 0.0)

Comment: Part of the power of Matlab is once you write a function, that function becomes part of the Matlab library of functions you can use. That lets you build up a rather powerful set of instructions.

For example, use the previous function from problem #5 (RRR.m) to create a new function; RRR_Cost

Matlab Function: RRR_Cost.m

```
• uses RRR.m (problem #5)
```

```
function [J] = RRR_Cost(z)

q1 = z(1);
q2 = z(2);
q3 = z(3);

[x3, y3] = RRR(q1, q2, q3);
J = sqrt( (x3-2)^2 + (y3-0)^2);
end
```

Checking: The tip is 1.4659 meters from the point (2,0)

```
>> J = RRR_Cost([41, -94, -45])
J = 1.4659
```



8) Use the fminsearch() to determine the joint angles which place the robot at $(x_3 = 2.0, y_3 = 1.0)$

From the command window:

```
>> [Q,e] = fminsearch('RRR_Cost', [41, -95, -45])
Q = 68.5320 -132.1667 -25.5707
e = 8.5387e-008
```

One solution is

- q1 = 68.5320 degrees
- q2 = -132.1667 degrees
- q3 = -25.5707 degrees



This solution isn't unique. With 3 degrees of freedom and two constraints, there are an infinite number of solutions. Change the initial guess and you converge to another valid solution

>> [Q,e] = fminsearch('RRR_Cost', [21, -195, -85])
Q = 7.4057 -153.5350 -167.1550
e = 3.7470e-009

