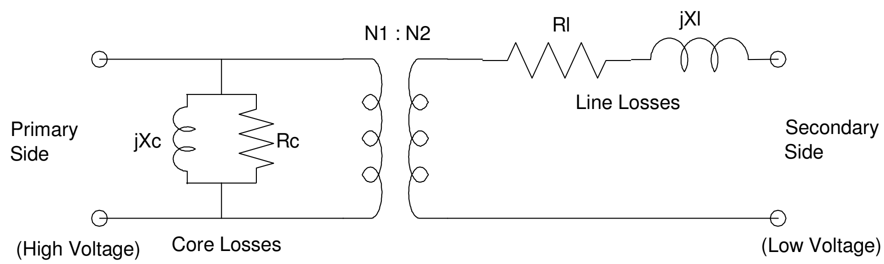
ECE 111: Homework 16

ECE 331 Energy Conversion

1) Determine the circuit model for a 13.2kV : 240V transformer is tested with the following test results:



Transformer Model

	V	Power	pf
Open-Circuit Test	V1 = 13.2kV	60 W	0.02
Short-Circuit Test	V2 = 40V	15 W	0.99

Equations: Open-Circuit Test (code model)

$$P = V \cdot I \cdot pf$$

$$60W = 13.2kV \cdot I \cdot 0.02$$

$$I = 0.2273$$

$$Z = \left(\frac{V}{I}\right)\angle\theta$$

$$\theta = \arccos(0.02) = 88.854^0$$

$$Z = \left(\frac{13.2kV}{0.2273A}\right)\angle 88.854^0$$

$$Z = 1161.6 + j58068$$

To get the parallel model

$$\frac{1}{Z} = \frac{1}{R_c} + \frac{1}{jXc} = 3.4435e-007 - 1.7214e-005i$$

$$R_c = \frac{1}{3.4435e{-}007} = 2.9040e{+}006$$

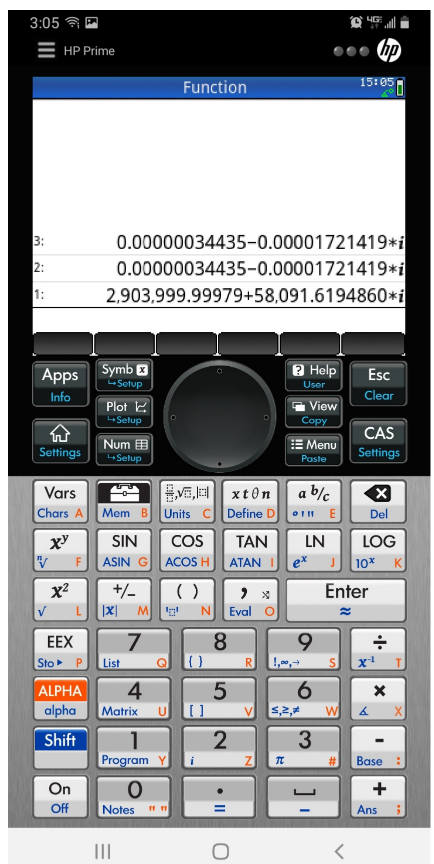
$$X_c = \frac{1}{1.7214e{-}005i} = 5.8092e{+}004$$

In Matlab

```
>> V = 13.2e3;
>> P = 60;
>> pf = 0.02;
>> I = P / (V*pf)

I =    0.2273

>> q = acos(0.02)

q =    1.5508    radians

>> Z = (V/I) * exp(j*q)

Z =  1.1616e+003 +5.8068e+004i                    series model

>> 1/Z

ans =  3.4435e-007 -1.7214e-005i

>> Rc = 1/real(1/Z)

Rc =  2.9040e+006

>> Xc = -1/imag(1/Z)

Xc =  5.8092e+004
```

HP Prime

```
60
enter
13200
/
0.02
/            the current, I
x-1
13200
*            impedance, Z
0.02
ACOS         angle in degrees
180
/
pi
*            angle in radians
0i1
*
e^x
*            series model
x^-1
enter
enter
Mem - Arithmetic - complex - Real Part
x^-1         Rc
(grab the stack for 1/series model)
Mem - Arithmetic - complex - Imaginary Part
x^-1         Xc
+/-
0i1
*
+            Rc  + jXc
```



Line Model

$$P = V \cdot I \cdot pf$$

$$I = \left(\frac{15W}{40V \cdot 0.99}\right) = 0.3788A$$

$$\theta = \arccos(0.99) = 8.11^0$$

$$Z = \left(\frac{V}{I}\right) \angle\theta$$

$$Z = \left(\frac{40V}{0.3788A}\right) \angle 8.11^0$$

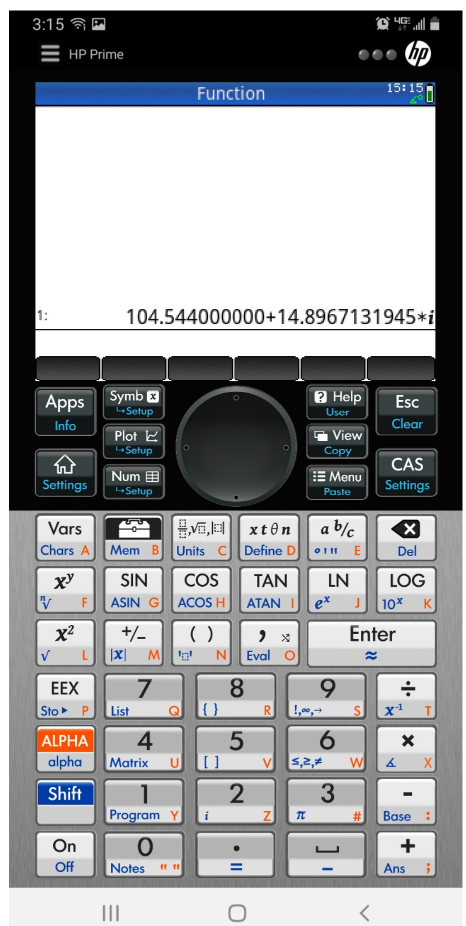
$$Z = R_L + jX_L = 104.54 + j14.897$$

In Matlab

```
>> P = 15;
>> V = 40;
>> pf = 0.99;
>> I = P / (V*pf)

I =     0.3788          amps

>> q = acos(pf)

q =     0.1415          radians

>> Z = (V/I) * exp(j*q)

Z =   1.0454e+002 +1.4897e+001i

>> RL = real(Z)
RL =   104.5440

>> XL = imag(Z)
XL =    14.8967
```

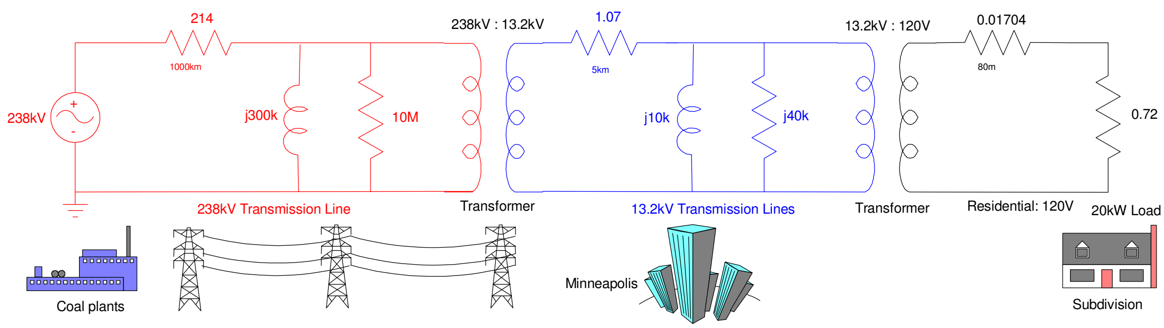
HP Prime

```
15
enter
40
/
0.99
/           current, I
x^-1
40
*           impedance Z
0.99
ACOS
180
/
pi
*
0i1
*
e^x
*
```



For the utility grid on the back of the page....

2) Convert the voltages and impeances to the 120V node (right side)



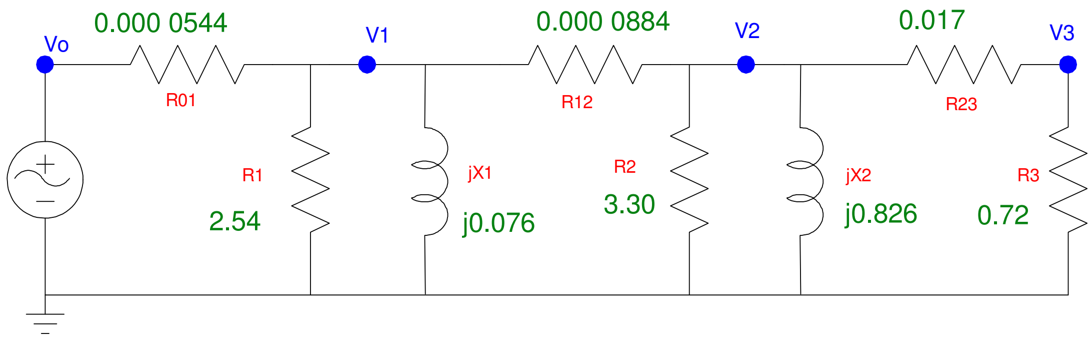
Voltages convert as the turns-ratio

$$238kV\left(\frac{13.2kV}{238kV}\right)\left(\frac{120V}{13.2kV}\right) = 120V$$

Impedances convert as the turn-ratio squared

```
>> R01 = 214 * (120/238e3)^2

R01 =   5.4403e-005

>> R1 = 10e6 * (120/238e3)^2

R1 =    2.5422

>> X1 = j*300e3 * (120/238e3)^2

X1 =         0 + 0.0763i

>> R12 = 1.07 * (120/13.2e3)^2

R12 =   8.8430e-005

>> R2 = 40e3 * (120/13.2e3)^2

R2 =    3.3058

>> X2 = j*10e3 * (120/13.2e3)^2

X2 =         0 + 0.8264i

>> R23 = 0.01704;
>> R3 = 0.72;
```

3) Write the voltage node equations for this circuit (with transformers removed)



$$V_0 = 120$$

$$\left(\frac{V_1 - V_0}{R_{01}}\right) + \left(\frac{V_1}{R_1}\right) + \left(\frac{V_1}{jX_1}\right) + \left(\frac{V_1 - V_2}{R_{12}}\right) = 0$$

$$\left(\frac{V_2 - V_1}{R_{12}}\right) + \left(\frac{V_2}{R_2}\right) + \left(\frac{V_2}{jX_2}\right) + \left(\frac{V_2 - V_3}{R_{12}}\right) = 0$$

$$\left(\frac{V_3 - V_2}{R_{23}}\right) + \left(\frac{V_3}{R_3}\right) = 0$$

4) Determine the voltages at each node

Group terms

$$V_0 = 120$$

$$-\left(\frac{1}{R_{01}}\right)V_0 + \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right)V_1 - \left(\frac{1}{R_{12}}\right)V_2 = 0$$

$$-\left(\frac{1}{R_{12}}\right)V_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right)V_2 - \left(\frac{1}{R_{23}}\right)V_3 = 0$$

$$-\left(\frac{1}{R_{23}}\right)V_2 + \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right)V_3 = 0$$

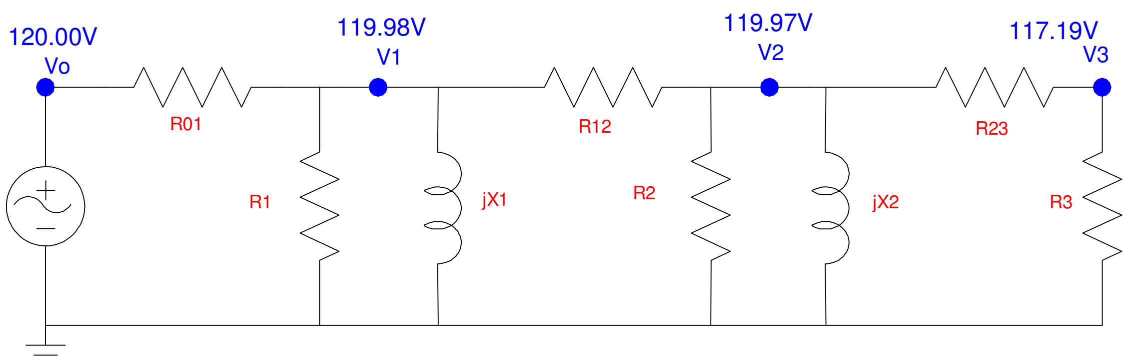
Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{R_{01}}\right) & \left(\frac{1}{R_{01}} + \frac{1}{R_1} + \frac{1}{jX_1} + \frac{1}{R_{12}}\right) & \left(\frac{-1}{R_{12}}\right) & 0 \\ 0 & \left(\frac{-1}{R_{12}}\right) & \left(\frac{1}{R_{12}} + \frac{1}{R_2} + \frac{1}{jX_2} + \frac{1}{R_{23}}\right) & \left(\frac{-1}{R_{23}}\right) \\ 0 & 0 & \left(\frac{-1}{R_{23}}\right) & \left(\frac{1}{R_{23}} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 120 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
R01 = 214 * (120/238e3)^2;
R1 = 10e6 * (120/238e3)^2;
X1 = j*300e3 * (120/238e3)^2;
R12 = 1.07 * (120/13.2e3)^2;
R2 = 40e3 * (120/13.2e3)^2;
X2 = j*10e3 * (120/13.2e3)^2;
R23 = 0.01704;
R3 = 0.72;

b1 = [1,0,0,0];
b2 = [-1/R01,1/R01+1/R1+1/X1+1/R12,-1/R12,0];
b3 = [0,-1/R12,1/R12+1/R2+1/X2+1/R23,-1/R23];
b4 = [0,0,-1/R23,1/R23+1/R3];
B = [b1;b2;b3;b4]
A = [120;0;0;0]
V = inv(B)*A

B =
  1.0e+004 *

   0.0001                  0                  0                  0

  -1.8381             2.9690 - 0.0013i  -1.1308                  0

        0            -1.1308             1.1367 - 0.0001i  -0.0059

        0                  0            -0.0059             0.0060
```

```
A =

   120
     0
     0
     0


V =

  1.0e+002 *

V0   1.2000 + 0.0000i
V1   1.1999 + 0.0009i
V2   1.1997 + 0.0011i
V3   1.1720 + 0.0010i

>> abs(V)

V0  120.0000
V1  119.9866
V2  119.9690
V3  117.1953

>>
```



5) Determine the efficiency of this system

- Ignoring the core losses
  - Assumes a large number of customers share these losses
- Including the core losses
  - Assumes a single customer

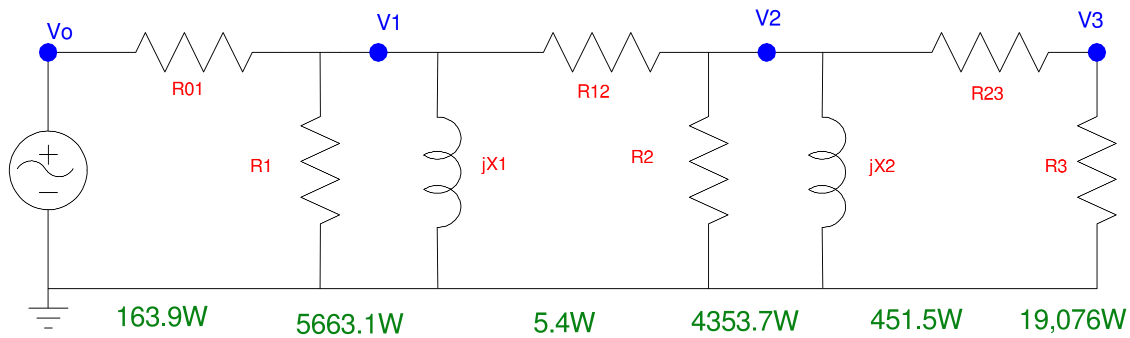
```
V0 = V(1);
V1 = V(2);
V2 = V(3);
V3 = V(4);

P01 = abs(V0 - V1)^2 / R01
P1 = abs(V1)^2 / R1
P12 = abs(V1 - V2)^2 / R12
P2 = abs(V2)^2 / R2
P23 = abs(V2 - V3)^2 / R23
P3 = abs(V3)^2 / R3

eff = P3 / (P1 + P12 + P2 + P23 + P3)

eff = P3 / (P12 + P23 + P3)

P01 =   163.9477

P1 =  5.6631e+003

P12 =    5.3675

P2 =  4.3537e+003

P23 =  451.4662

P3 =  1.9076e+004

eff =    0.6456

eff =    0.9766
```

The efficience including the core losses is 64.56%

The efficiency is you only have one customer

The efficiency excludin the core losses is 97.66%

The efficiency if you have many customers



Power Dissipated by Each Resistor

```
P01 =   163.9477

P1 =  5.6631e+003

P12 =    5.3675

P2 =  4.3537e+003

P23 =  451.4662

P3 =  1.9076e+004
```

Efficiency Including Code Losses

```
eff =    0.6456
```

Efficiency Excluding Core Losses

```
eff =    0.9766
```