ECE 111 - Homework #15

Week #11 - ECE 343 Signals

Problem 1-5) Let x(t) be a function which is periodic in 2π

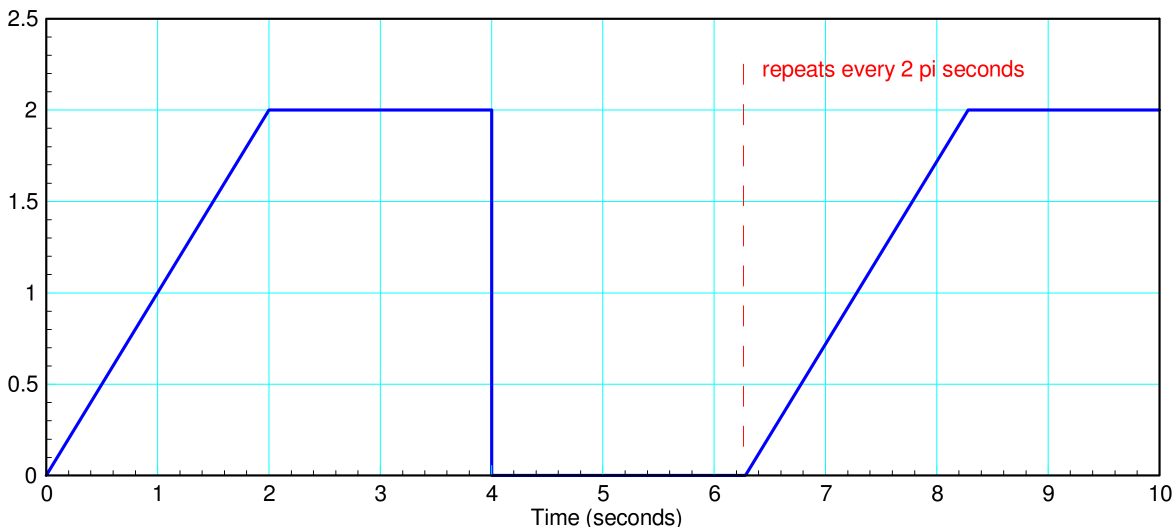
$$x(t) = x(t + 2\pi)$$

Over the interval $(0, 2\pi)$ x(t) is

$$x(t) = \max(0, 5\sin(t) - 3)$$

or in Matlab:

```
t = [0:0.001:2*pi]' + 1e-6;
x = t .* (t<2)   +   2*(t>2).*(t<4);
plot(t,x)
```



x(t)   Note that x(t) repeats repeats every 2π seconds

Curve Fitting with a power series:

1) Using least squares, approximate x(t) over the interval (0, 2π) as

$$x(t) \approx a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5$$

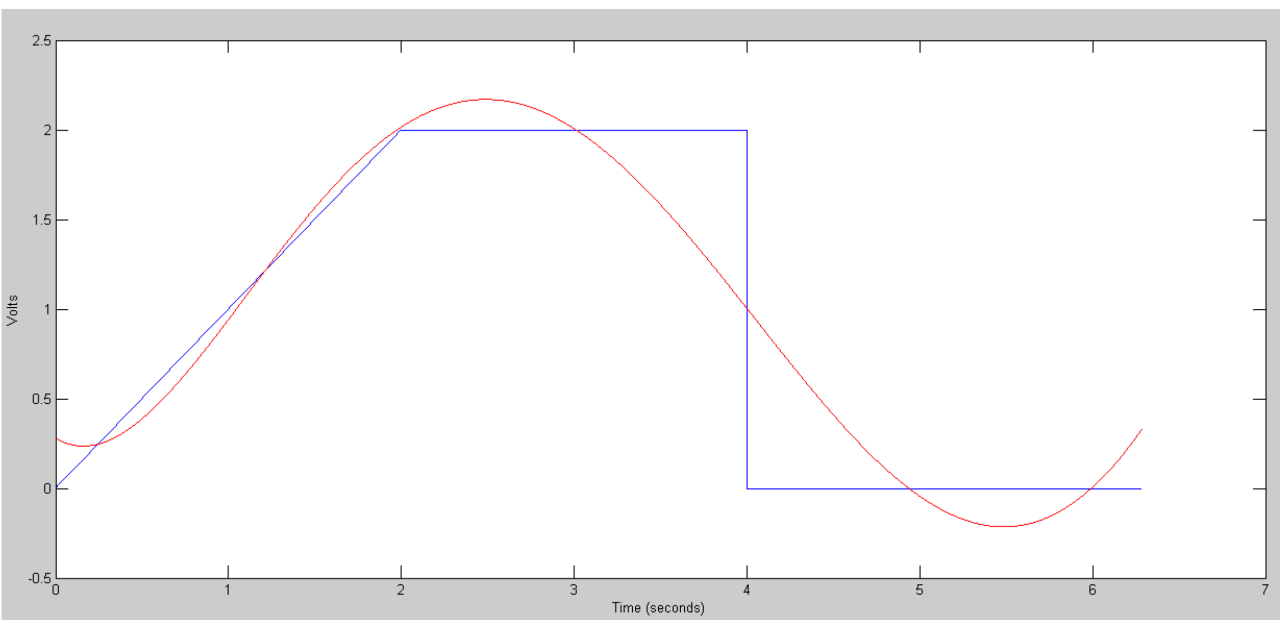
Plot x(t) along with it's approximation.

```
t = [0:0.001:2*pi]' + 1e-6;
x = t .* (t<2)  +  2*(t>2).*(t<4);
plot(t,x)

B = [t.^0, t, t.^2, t.^3, t.^4, t.^5];
A = inv(B'*B)*B'*x

 a0   0.2822
 a1  -0.5765
 a2   1.9626
 a3  -0.8444
 a4   0.1234
 a5  -0.0058

>> plot(t,x,'b',t,B*A,'r');
>> xlabel('Time (seconds)');
>> ylabel('Volts');
>>
```



Comment:

- It's not a great curve fit - add more terms and it will get better
- It's not a useful answer. Given the curve fit, I still don't know how to find y(t)

Curve Fitting using a Fourier Series

2) Using least squares, approximate x(t) over the interval (0, 2π) as

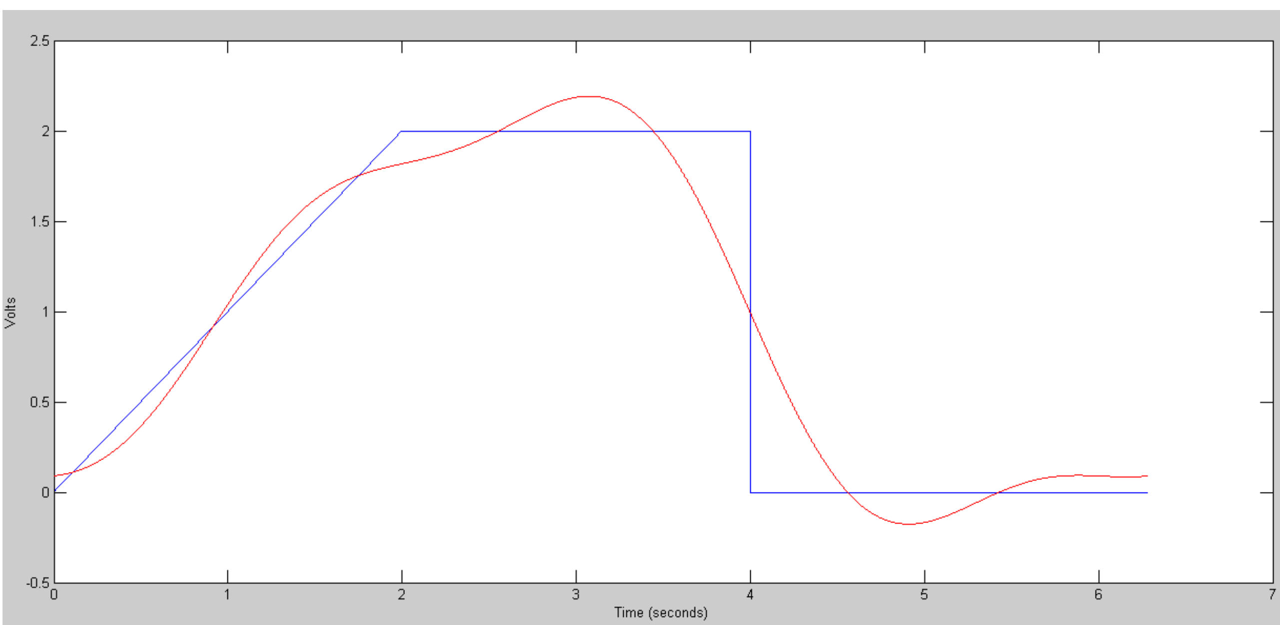
$$x(t) = a_0 + a_1\cos(t) + b_1\sin(t) + a_2\cos(2t) + b_2\sin(2t) + a_3\cos(3t) + b_3\sin(3t)$$

Plot x(t) along with it's approximation.

```
>> t = [0:0.001:2*pi]' + 1e-6;
>> x = t .* (t<2)  +  2*(t>2).*(t<4);
>> B = [t.^0, cos(t), sin(t), cos(2*t), sin(2*t), cos(3*t), sin(3*t)];
>> A = inv(B'*B)*B'*x
```

```
a0    0.9548
a1   -0.9324
b1    0.7058
a2    0.1834
b2   -0.0142
a3   -0.1155
b3   -0.1888
```

```
>> plot(t,x,'b',t,B*A,'r');
>> xlabel('Time (seconds)');
>> ylabel('Volts');
>>
```



Comment:

- This is a slightly better curve fit
- Add more terms and it gets better
- This answer ***is*** useful. Now that x(t) is expressed in terms of sine waves, I know how to find y(t)

Superposition

3) Assume X and Y are related by

$$Y = \left(\frac{0.25}{s^2+0.5s+0.25}\right)X$$

3a) Determine x(t) in terms of its Fourier Transform out to 3 rad/sec

```
>> a0 = mean(x)

a0 =    0.9548

>> a1 = 2*mean(x .* cos(t))

a1 =   -0.9324

>> b1 = 2*mean(x .* sin(t))

b1 =    0.7058

>> a2 = 2*mean(x .* cos(2*t))

a2 =    0.1834

>> b2 = 2*mean(x .* sin(2*t))

b2 =   -0.0142

>> a3 = 2*mean(x .* cos(3*t))

a3 =   -0.1155

>> b3 = 2*mean(x .* sin(3*t))

b3 =   -0.1888

>>
```

Comment:

- This is the same result we got in part a)
- Fourier transforms are nothing more than a curve fit where the basis funciton is composed of sine waves.

3b) Plot x(t) and its Fourier approximation taken out to 3 rad/sec

- same result as problem #2

4) Determine the output, y(t), at DC (w = 0)

$$Y = \left(\frac{0.25}{s^2+0.5s+0.25}\right)X$$

```
>> s = 0;
>> X0 = a0

X0 =    0.9548

>> Y0 = (0.25 / (s^2 + 0.5*s + 0.25)) * X0

Y0 =    0.9548
```

$$y(t) = 0.9548$$

5) Determine the output, y(t), at 1 rad/sec

```
>> s = j*1;
>> X1 = a1 - j*b1

X1 =  -0.9324 - 0.7058i

>> Y1 = (0.25 / (s^2 + 0.5*s + 0.25)) * X1

Y1 =   0.1066 + 0.3063i
```

$$y(t) = 0.1066\cos(t) - 0.3063\sin(t)$$

6) Determine the output, y(t), at 2 rad/sec

```
>> s = j*2;
>> X2 = a2 - j*b2

X2 =   0.1834 + 0.0142i

>> Y2 = (0.25 / (s^2 + 0.5*s + 0.25)) * X2

Y2 =  -0.0112 - 0.0039i
```

$$y(t) = -0.0112\cos(2t) + 0.0039\sin(2t)$$

7) Determine the output, y(t), at 3 rad/sec

```
>> s = j*3;
>> X3 = a3 - j*b3

X3 =  -0.1155 + 0.1888i

>> Y3 = (0.25 / (s^2 + 0.5*s + 0.25)) * X3

Y3 =   0.0041 - 0.0047i
```

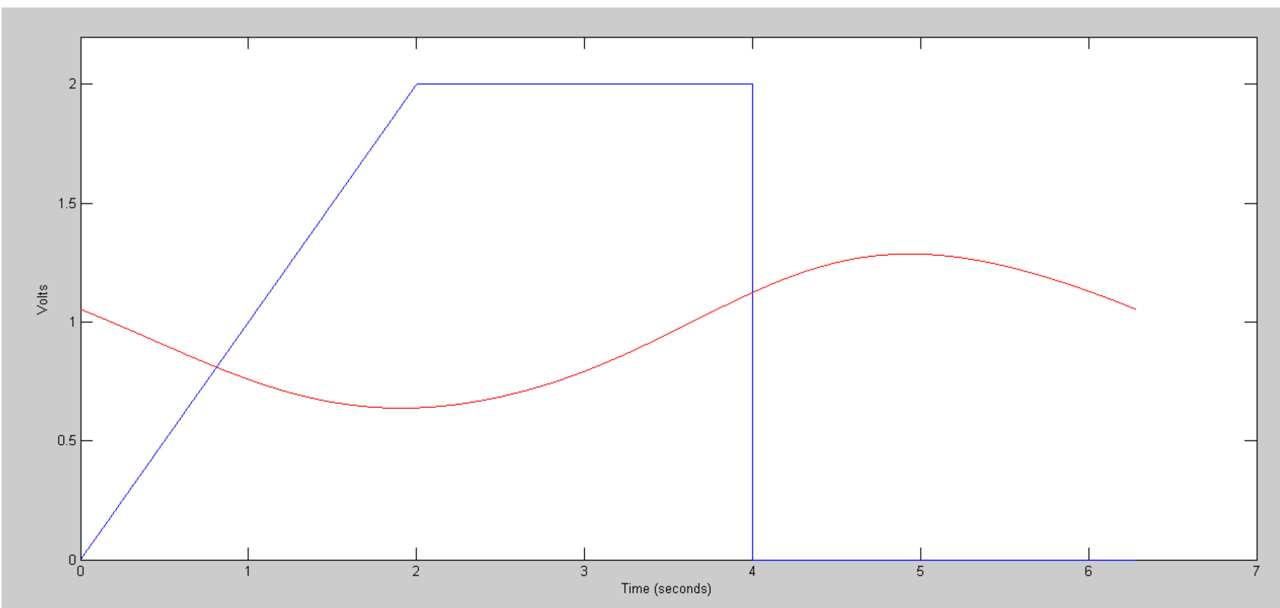
$$y(t) = 0.0041\cos(3t) + 0.0047\sin(3t)$$

8) Determine the total answer, y(t)

- Plot x(t) and y(t)

$$
\begin{aligned}
y(t) = {} & 0.9548 \\
& +0.1066\cos(t) - 0.3063\sin(t) \\
& -0.0112\cos(2t) + 0.0039\sin(2t) \\
& 0.0041\cos(3t) + 0.0047\sin(3t)
\end{aligned}
$$

```
>> >> y = Y0 + real(Y1)*cos(t) - imag(Y1)*sin(t) + real(Y2)*cos(2*t) - imag(Y2)*sin(2*t) + real(Y3)*cos(3*t) - imag(Y3)*sin(3*t);
>> plot(t,x,'b',t,y,'r')
>> xlabel('Time (seconds)');
>> ylabel('Volts');
```



Comment

- In theory, you need an infinite number of terms to determine y(t)
- In practive, the terms get smaller and smaller, meaning that if you only include a few terms, you essentially have y(t)

The result looks like a sine wave with a DC offset

- A good approximation for y(t) would just include the DC term and the 1st harmonic:

$$y(t) \approx 0.9548 + 0.1066\cos(t) - 0.3063\sin(t)$$