ECE 111 - Homework #14

ECE 343 Signals & Systems

Filter Analysis

1) A filter has the following transfer function

$$Y = \left(\frac{400(s+1)}{(s+5)(s+7)(s+9)} \right) X$$

1a) What is the differential equation relating X and Y?

Multiply out and cross multiply. In Matlab

```
>> poly([-5,-7,-9])

ans =      1     21    143    315
```

$$Y = \left(\frac{400s+400}{s^3+21s^2+143s+315} \right) X$$

$$(s^3 + 21s^2 + 143s + 315)Y = (400s + 400)X$$

Note that sY means *the derivative of y*

$$y''' + 21y'' + 143y' + 315y = 400x' + 400x$$

1b) Find y(t) assuming $x(t) = 5$

Use phasor analysis.

s = 0

X = 5

$$Y = \left(\frac{400(s+1)}{(s+5)(s+7)(s+9)} \right)_{s=0} \cdot (5)$$

$$Y = 6.3492$$

$$y(t) = 6.3492$$

In Matlab

```
>> s = 0;
>> X = 5;
>> Y = 400*(s+1) / ( (s+5)*(s+7)*(s+9) ) * X

Y =    6.3492
```

1c) Find y(t) assuming $x(t) = 5\sin(3t)$

s = j3

X = 0 - j5 *real = cosine, -imag = sine*

$$Y = \left(\frac{400(s+1)}{(s+5)(s+7)(s+9)}\right)_{s=j3} \cdot 0 - j5X$$

In Matlab

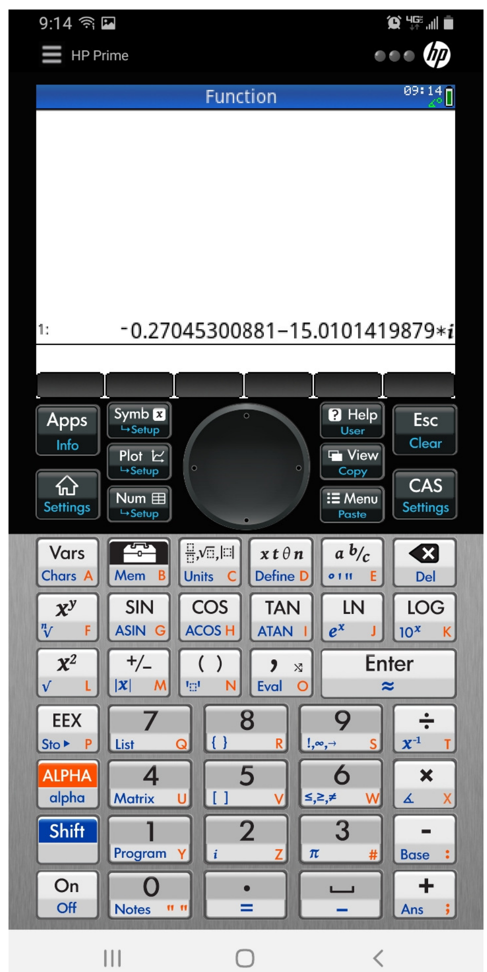
```
>> s = j*3;
>> X = 0 - j*5;
>> Y = 400*(s+1) / ( (s+5)*(s+7)*(s+9) ) * X

Y =  -0.2705 -15.0101i
```

meaning

$$y(t) = -0.2705\cos(3t) + 15.0101\sin(3t)$$

On an HP Prime (in RPN mode)

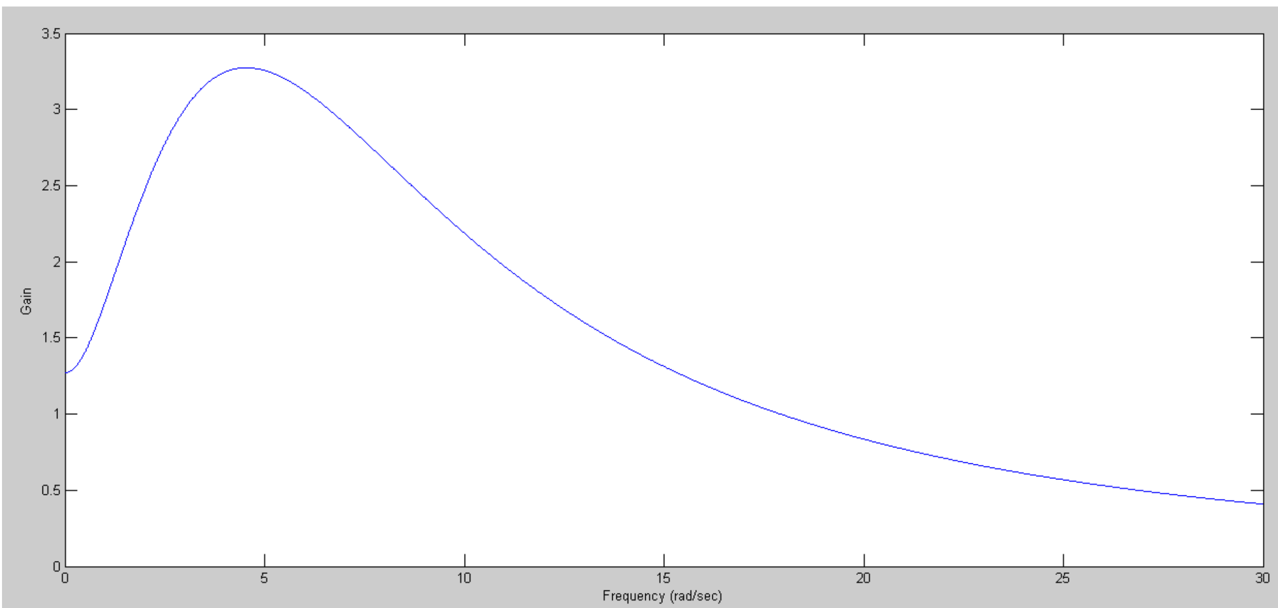
```
400
enter
1i3
*
5i3
/
7i3
/
9i3
/
0i-5
*
```



2) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{400(s+1)}{(s+5)(s+7)(s+9)} \right) X$$

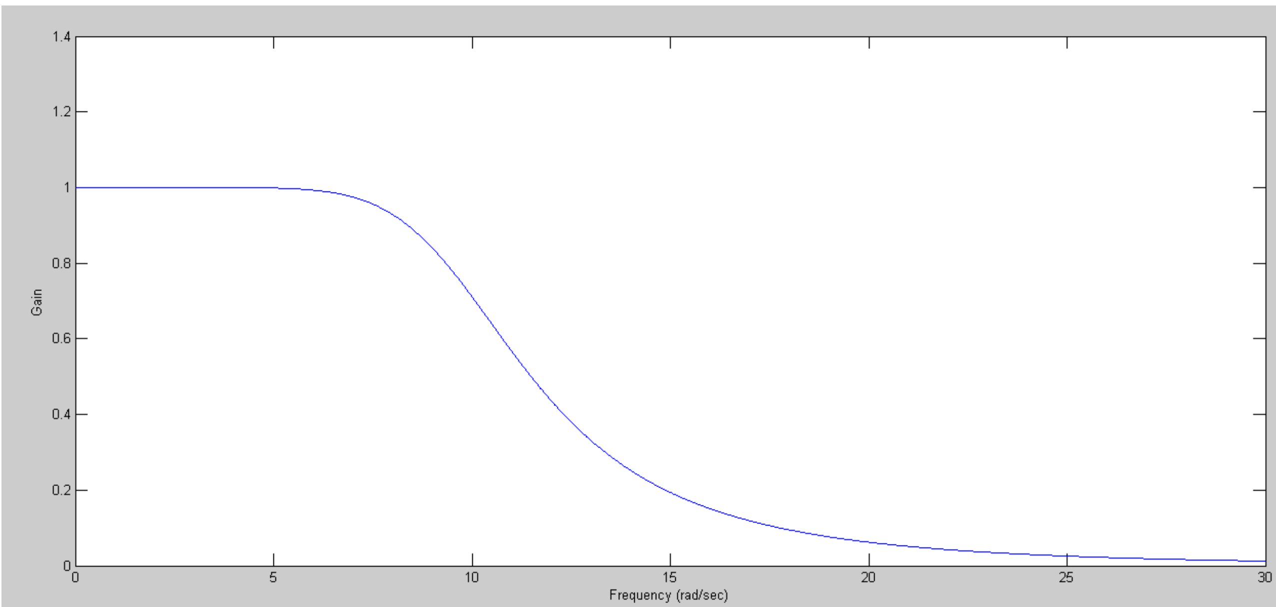
```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 400*(s+1) ./ ( (s+5).*(s+7).*(s+9) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



3) Plot the gain vs. frequency for this filter from 0 to 30 rad/sec.

$$Y = \left(\frac{10{,}000}{\left(s^2+18.5s+100\right)\left(s^2+7.6s+100\right)} \right) X$$

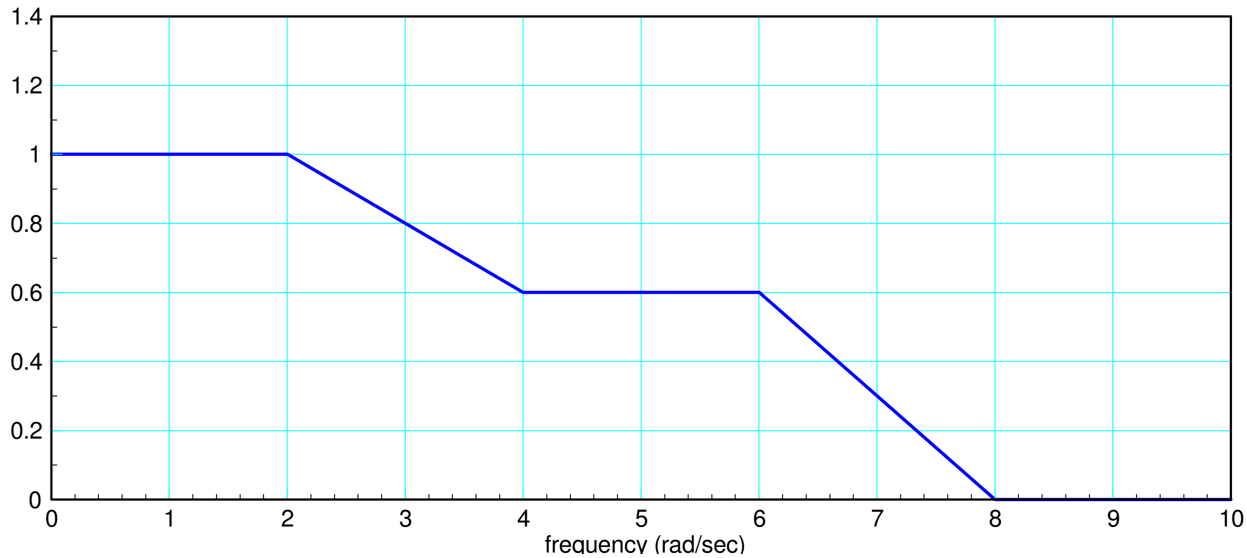
```
>> w = [0:0.01:30]';
>> s = j*w;
>> G = 10000 ./ ( (s.^2 + 18.5*s + 100).*(s.^2 + 7.6*s + 100) );
>> plot(w,abs(G))
>> xlabel('Frequency (rad/sec)');
>> ylabel('Gain');
>>
```



Filter Design

Problem 4-6) Design a filter of the following form so that the gain matches the graph below:

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right)$$



4) Write an m-file, cost.m, which

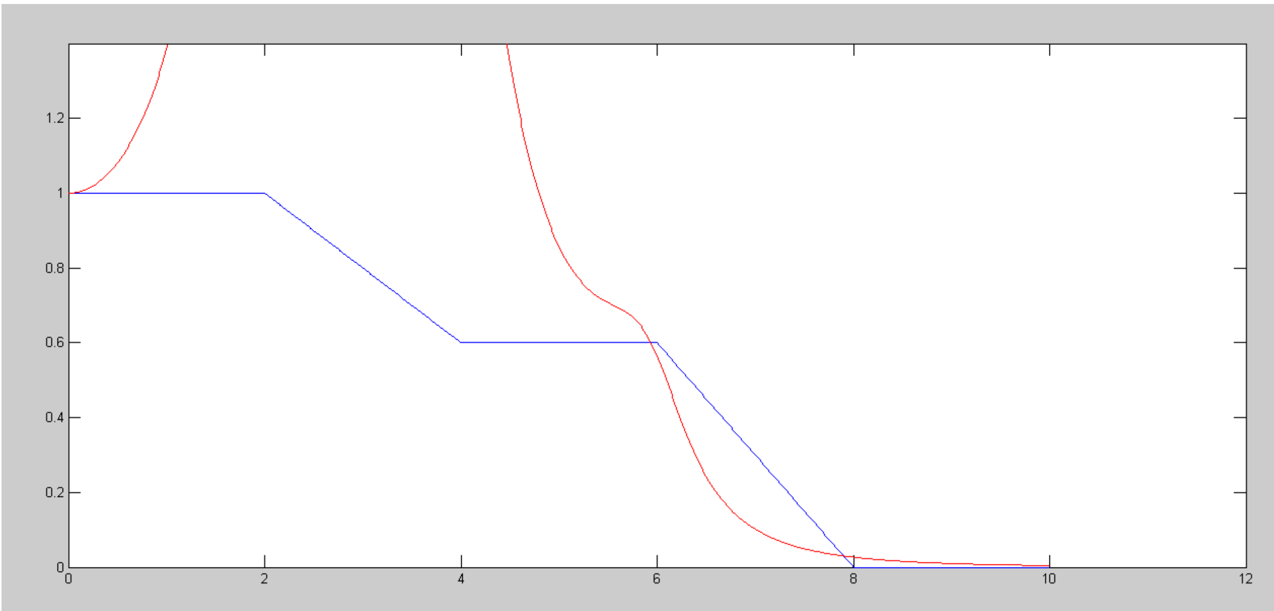
- Is passed an array, z, with each element representing (a, b, c, d, e, f, g)
- Computes the gain, G(s) for this value of (a, b, c, d, e, f, g)
- Computes the difference between the gain, G, and the target (above), and
- Returns the sum-squared error in the gain

```
function [ J ] = costf( z )
 a = z(1);
 b = z(2);
 c = z(3);
 d = z(4);
 e = z(5);
 f = z(6);
 g = z(7);

 w = [0:0.01:10]' + 1e-6;
 s = j*w;

 g1 = 1 * (w < 2);
 g2 = (-0.2*w + 1.4).*(w>2).*(w<4);
 g3 = 0.6 * (w>4).*(w<6);
 g4 = (-0.3*w + 2.4) .* (w>6).*(w<8);
 Gideal = g1 + g2 + g3 + g4;

 G = a ./ ( (s.^2 + b*s + c) .* (s.^2 + d*s + e ).*(s.^2 + f*s + g) );
 G = abs(G);
 E = abs(Gideal) - abs(G);

 J = sum(E .^ 2);

 plot(w,Gideal,w,abs(G),'r');
 ylim([0,1.4]);
 pause(0.01);
end
```

5) Use your m-file to determine how 'good' the following filter is:

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right) = \left(\frac{2304}{(s^2+s+4)(s^2+s+16)(s^2+s+36)} \right)$$

```
>> costF([2304,1,4,1,16,1,36])

ans =   682.0555
```



6) Use fminsearch() to find the 'best' filter of the form

$$G(s) = \left(\frac{a}{(s^2+bs+c)(s^2+ds+e)(s^2+fs+g)} \right)$$

a) Give the resulting (a, b, c, d, e, f, g)

```
>> [Z,e] = fminsearch('costF',[2304,1,4,1,16,1,36])


Z =
      a           b             c             d             e             f
7.1473e+003  3.3233e+000  5.8652e+000  5.4393e+000  3.0569e+001  1.4878e+000

      g
4.0737e+001

e =    0.6354
```

b) Give the resulting filter, and

$$G(s) = \left(\frac{7147.3}{(s^2+3.32s+5.86)(s^2+5.43s+30.56)(s^2+1.48s+40.73)} \right)$$

c) Plot the 'optimal' filter's gain vs. frequency