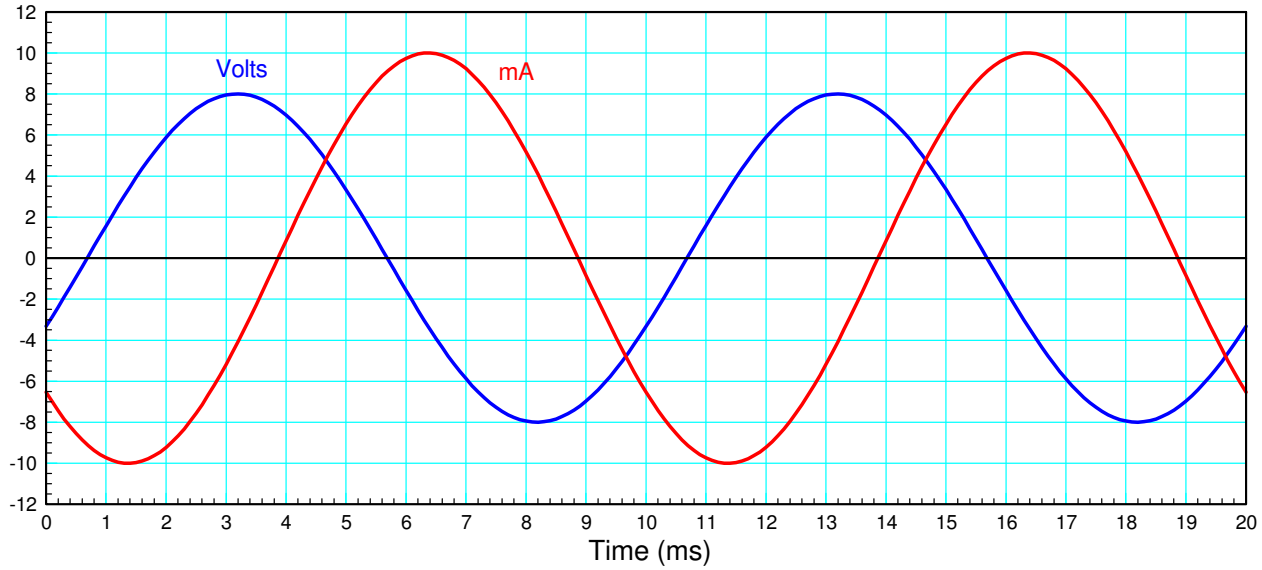
ECE 111 - Homework #13

ECE 311 Circuits II - Phasors

Phasor Voltages

1) Express V and I as phasors (i.e. as complex numbers)

- From this, determine the impedance, Z = V/I



Polar mode is easiest

Volts:

|V| = 8.00V (peak), period = 10ms. delay = 3.2ms

$$\theta = -\left(\frac{3.2ms}{10ms}\right)360^0 = -115.2^0$$

$$V = 8.00\angle -115.2^0$$

Current

|I| = 10ms, period = 10ms, delay = 6.3ms

$$\theta = -\left(\frac{6.3ms}{10ms}\right)360^0 = -226.8^0$$

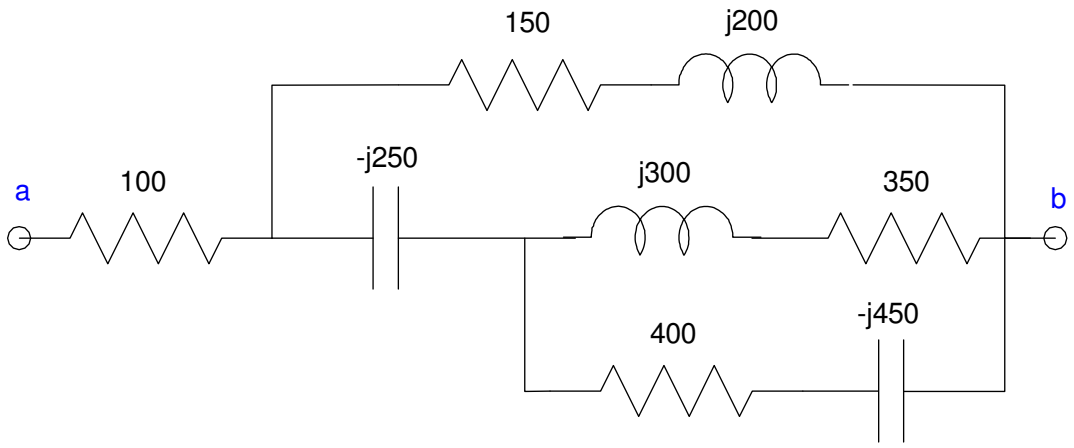
$$I = 10.00\angle -226.8^0 \text{ mA}$$

Impedance

$$Z = \left(\frac{V}{I}\right) = \left(\frac{8.00\angle -115.2^0}{10.00\angle -226.8^0}\right) = 0.80\angle 111.6^0$$

Phasor Impedances

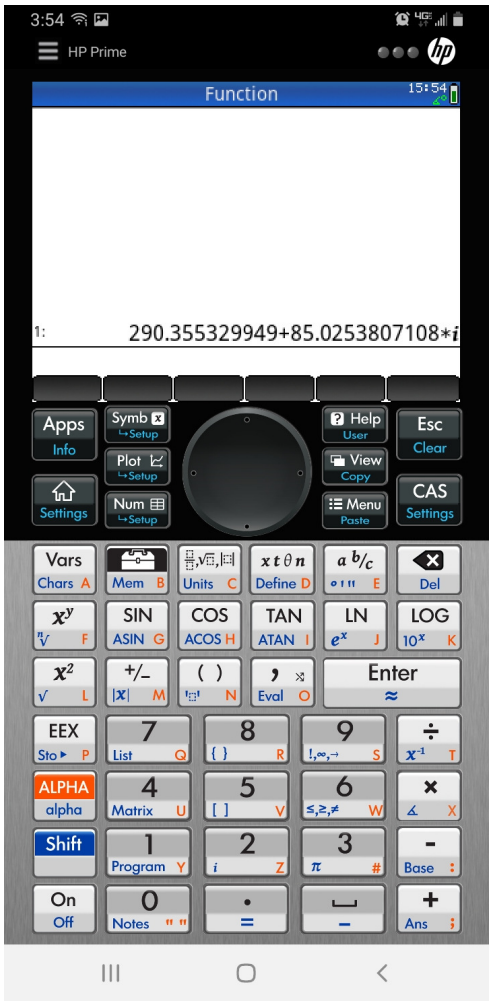
2) Determine the impedance, Zab



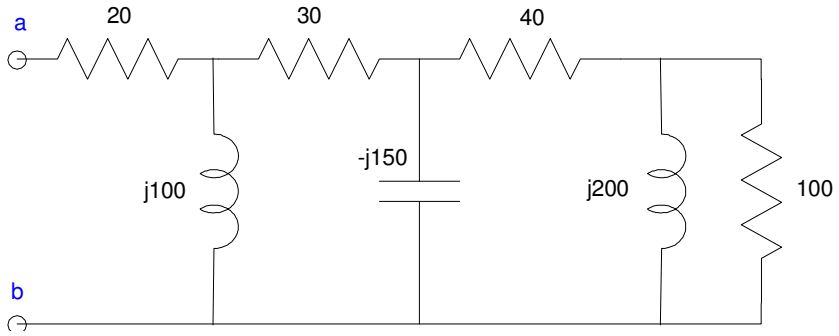
HP Prime: (RPN mode)

- Settings - Entry - RPN

```
350i300
enter
x⁻¹
400i-450
x⁻¹
+
x⁻¹
0i-250
+
x⁻¹
150i200
x⁻¹
+
x⁻¹
100
+
```



3) Determine the impedance, Zab

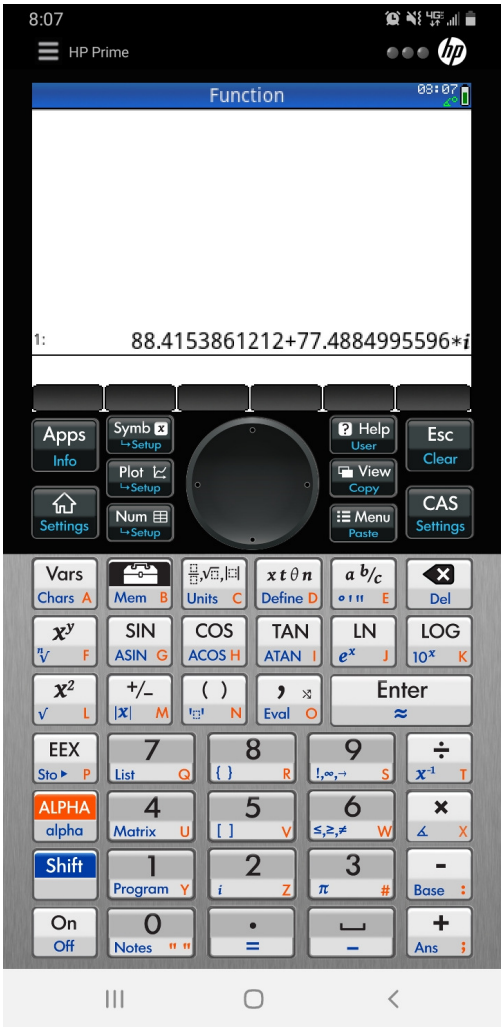


Zab = 88.415 + j77.488

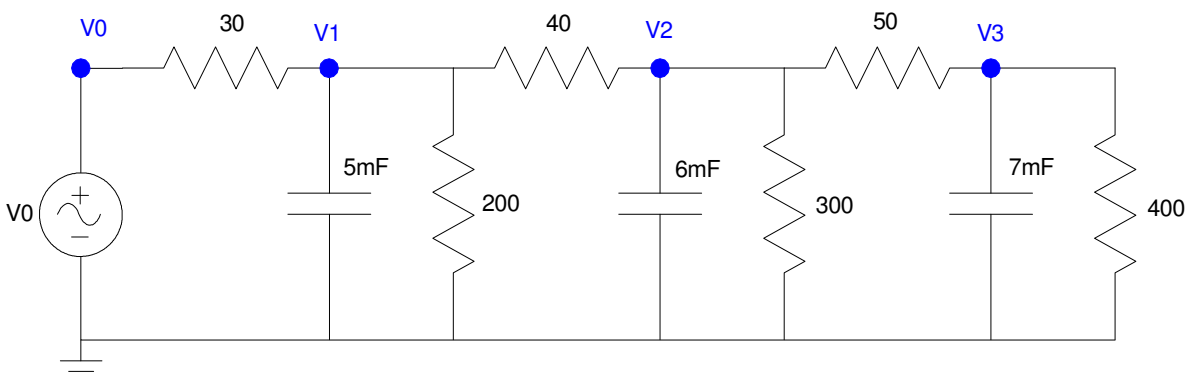
HP Prime (RPN mode)

- Settings - Entry - RPN

```
100
enter
x⁻¹
0i200
x⁻¹
+
x⁻¹
40
+
x⁻¹
0i-150
x⁻¹
+
x⁻¹
30
+
x⁻¹
0i100
x⁻¹
+
x⁻¹
20
+
```



Voltage Nodes with Phasors



4) Assume V0 = 10.

a) Determine the impedances of each element at 0 rad/sec

$$C \rightarrow \frac{1}{j\omega C} = \infty$$

$$I_c = 0$$

you can ignore capacitors at DC: the current is zero

b) Write the votlage node equations

$$V_0 = 10$$

$$\left(\frac{V_1 - V_0}{30}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1 - V_2}{40}\right) = 0$$

$$\left(\frac{V_2 - V_1}{40}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2 - V_3}{50}\right) = 0$$

$$\left(\frac{V_3 - V_2}{50}\right) + \left(\frac{V_3}{400}\right) = 0$$

c) Solve for V1, V2, and V3.

Group terms

$$V_0 = 10$$

$$-\left(\frac{1}{30}\right)V_0 + \left(\frac{1}{30} + \frac{1}{200} + \frac{1}{40}\right)V_1 - \left(\frac{1}{40}\right)V_2 = 0$$

$$-\left(\frac{1}{40}\right)V_1 + \left(\frac{1}{40} + \frac{1}{300} + \frac{1}{50}\right)V_2 - \left(\frac{1}{50}\right)V_3 = 0$$

$$-\left(\frac{1}{50}\right)V_2 + \left(\frac{1}{50} + \frac{1}{400}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{200}+\frac{1}{40}\right) & \left(\frac{-1}{40}\right) & 0 \\ 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{50}\right) & \left(\frac{-1}{50}\right) \\ 0 & 0 & \left(\frac{-1}{50}\right) & \left(\frac{1}{50}+\frac{1}{400}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

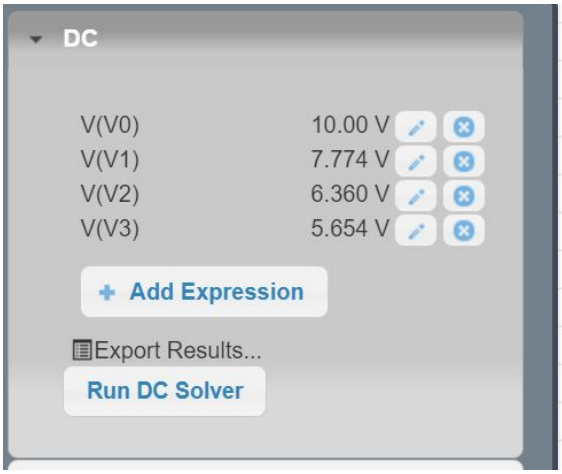
Solve using Matlab

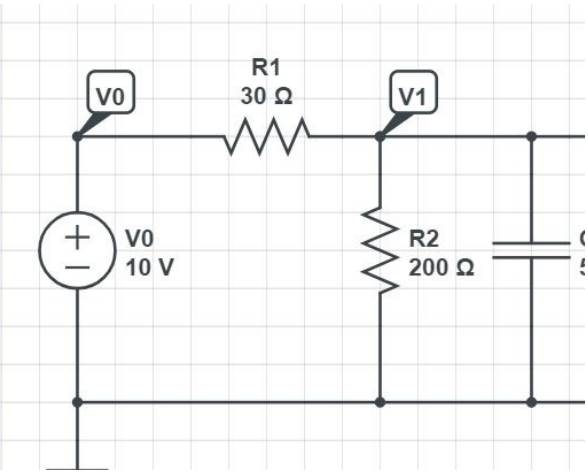
```
>> b1 = [1,0,0,0];
>> b2 = [-1/30,1/30+1/200+1/40,-1/40,0];
>> b3 = [0,-1/40,1/40+1/300+1/50,-1/50];
>> b4 = [0,0,-1/50,1/50+1/400];
>> B = [b1;b2;b3;b4]

    1.0000         0         0         0
   -0.0333    0.0633   -0.0250         0
         0   -0.0250    0.0483   -0.0200
         0         0   -0.0200    0.0225

>> A = [10;0;0;0]

    10
     0
     0
     0

>> V = inv(B)*A

V0   10.0000
V1    7.7739
V2    6.3604
V3    5.6537

>>
```

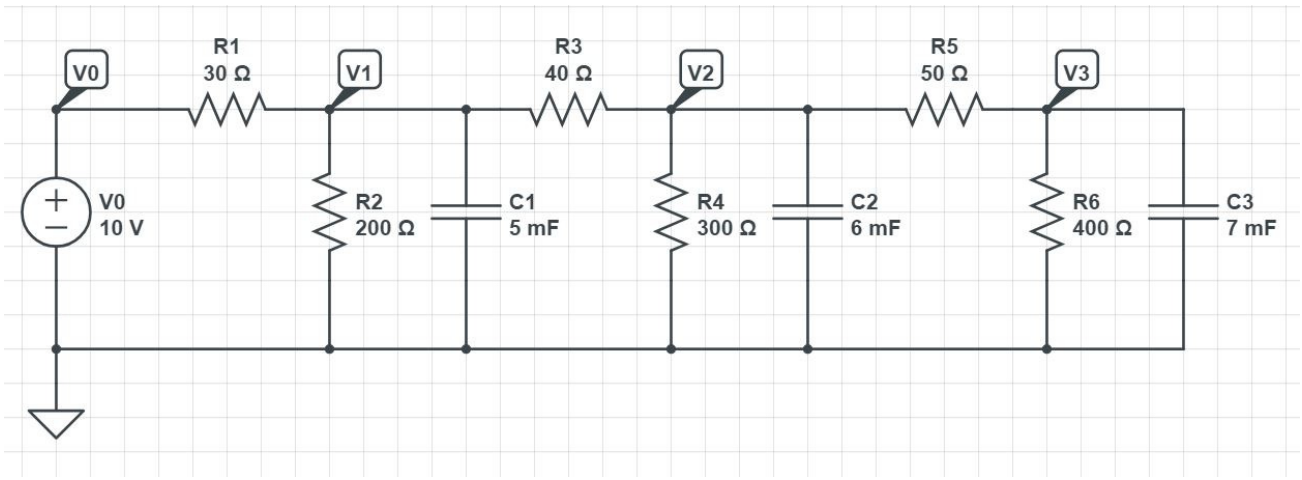
5) Check your results in CircuitLab

The results are the same

```
V0    10.0000
V1     7.7739
V2     6.3604
V3     5.6537
```

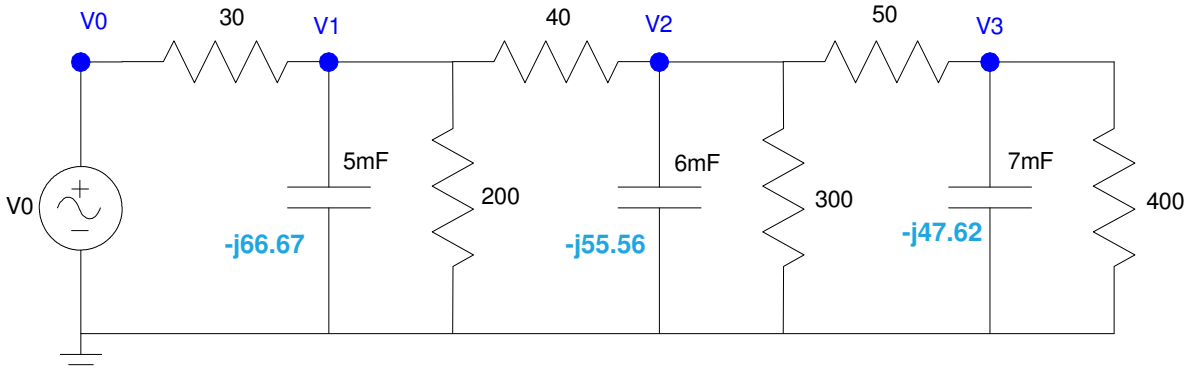






6) Assume $V_0 = 10\sin(3t)$ *10V, 3 rad/sec sine wave (0.478Hz)*

a) Determine the impedances of each element at 3 rad/sec



$$C_1 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(0.005)} = -j66.67\Omega$$

$$C_2 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(0.006)} = -j55.56\Omega$$

$$C_3 \rightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(0.007)} = -j47.62\Omega$$

b) Write the votlage node equations

$$V_0 = 0 - j10$$

$$\left(\frac{V_1 - V_0}{30}\right) + \left(\frac{V_1}{200}\right) + \left(\frac{V_1}{-j66.67}\right) + \left(\frac{V_1 - V_2}{40}\right) = 0$$

$$\left(\frac{V_2 - V_1}{40}\right) + \left(\frac{V_2}{300}\right) + \left(\frac{V_2}{-j55.56}\right) + \left(\frac{V_2 - V_3}{50}\right) = 0$$

$$\left(\frac{V_3 - V_2}{50}\right) + \left(\frac{V_3}{400}\right) + \left(\frac{V_3}{-j47.62}\right) = 0$$

c) Solve for V1, V2, and V3 as complex numbers

Group terms

$$V_0 = 0 - j10$$

$$-\left(\frac{1}{30}\right)V_0 + \left(\frac{1}{30} + \frac{1}{200} + \frac{1}{40} + \frac{1}{-j66.67}\right)V_1 - \left(\frac{1}{40}\right)V_2 = 0$$

$$-\left(\frac{1}{40}\right)V_1 + \left(\frac{1}{40} + \frac{1}{300} + \frac{1}{50} + \frac{1}{-j55.56}\right)V_2 - \left(\frac{1}{50}\right)V_3 = 0$$

$$-\left(\frac{1}{50}\right)V_2 + \left(\frac{1}{50} + \frac{1}{400} + \frac{1}{-j47.62}\right)V_3 = 0$$

Place in matrix form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ \left(\frac{-1}{30}\right) & \left(\frac{1}{30}+\frac{1}{200}+\frac{1}{40}+\frac{1}{-j66.67}\right) & \left(\frac{-1}{40}\right) & 0 \\ 0 & \left(\frac{-1}{40}\right) & \left(\frac{1}{40}+\frac{1}{300}+\frac{1}{50}+\frac{1}{-j55.56}\right) & \left(\frac{-1}{50}\right) \\ 0 & 0 & \left(\frac{-1}{50}\right) & \left(\frac{1}{50}+\frac{1}{400}+\frac{1}{-j47.62}\right) \end{bmatrix} \begin{bmatrix} V_0 \\ V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Solve in Matlab

```
>> b1 = [1,0,0,0];
>> b2 = [-1/30,1/30+1/200+1/40+1/(-j*66.67),-1/40,0];
>> b3 = [0,-1/40,1/40+1/300+1/50+1/(-j*55.56),-1/50];
>> b4 = [0,0,-1/50,1/50+1/400+1/(-j*47.62)];
>> B = [b1;b2;b3;b4]

 1.0000             0                  0                  0
-0.0333         0.0633 + 0.0150i  -0.0250                  0
   0            -0.0250             0.0483 + 0.0180i  -0.0200
   0                 0             -0.0200             0.0225 + 0.0210i

>> A = [0-j*10;0;0;0]

         0 -10.0000i
         0
         0
         0

>> V = inv(B)*A

V0        0 -10.0000i
V1  -2.3048 - 5.3654i
V2  -2.6197 - 1.6418i
V3  -1.9725 + 0.3816i
```

d) Express V1, V2, and V3 in terms of sine and cosine function:

V0 = 10.0000 sin(3t)
V1 = -2.3048 cos(3t) + 5.3654 sin(3t)
V2 = -2.6197 cos(3t) + 1.6418 sin(3t)
V3 = -1.9725 cos(3t) - 0.3816 sin(3t)

7) Check your results in CircuitLab using a transient simulation for 5 seconds (time step = 5ms).

Note: Polar form works better

```
>> abs(V)

V0   10.0000
V1    5.8395
V2    3.0916
V3    2.0090
```

The magnitude of each votlage from CircuitLab matches calculations (move cursor over waveforms)

```
V0   10.000
V1    5.836
V2    3.122
V3    1.938
```