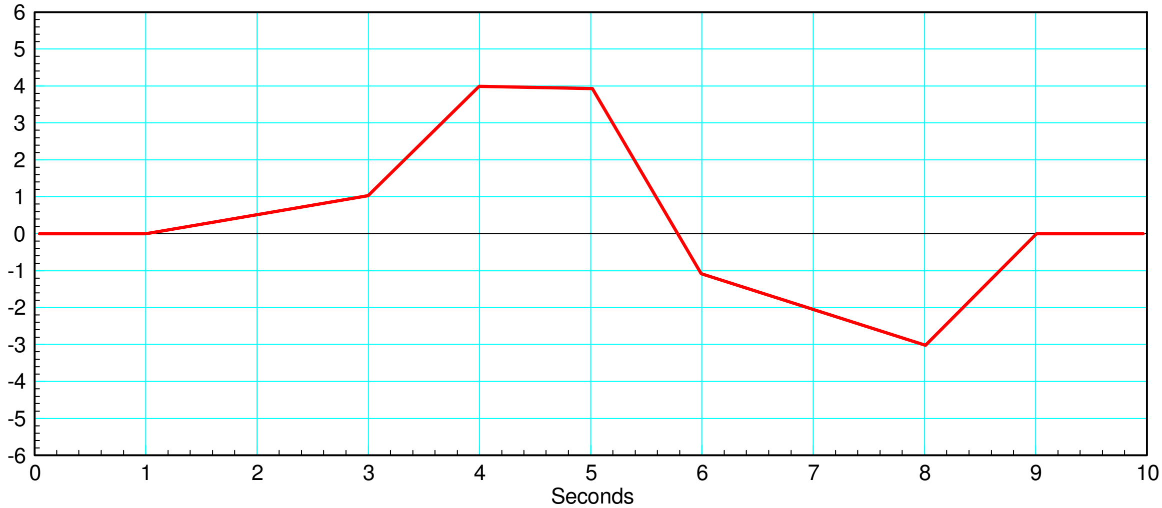
ECE 111 - Homework #11

ECE 351 Electromagnetics - Wave Equation

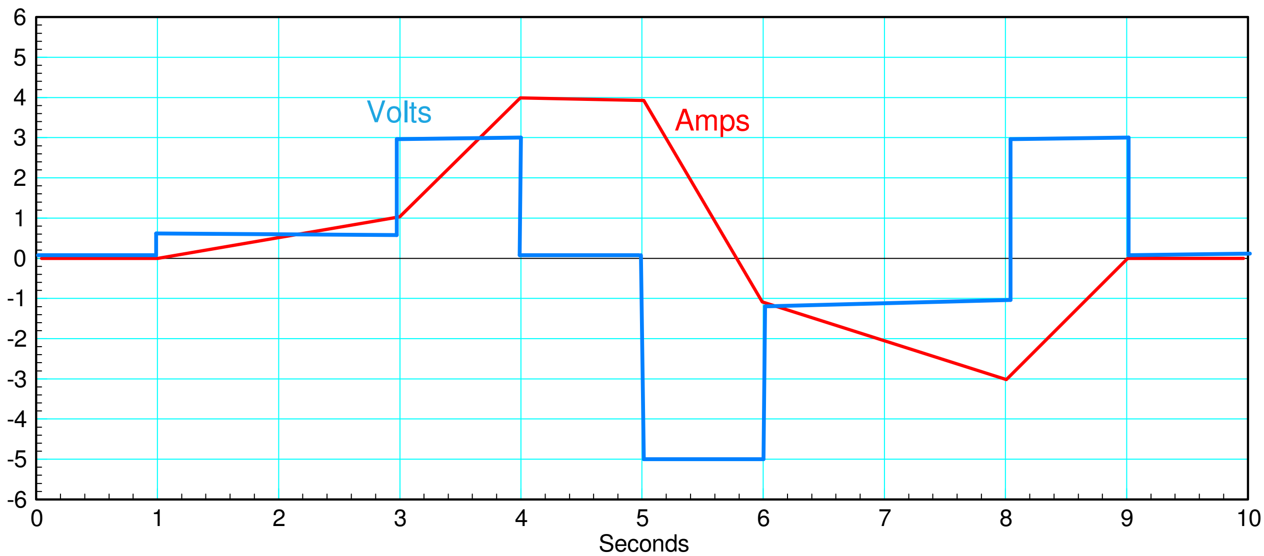
1) Assume the current flowing through a one Henry inductor is shown below. Sketch the voltage.

$$V = L\frac{dI}{dt}$$

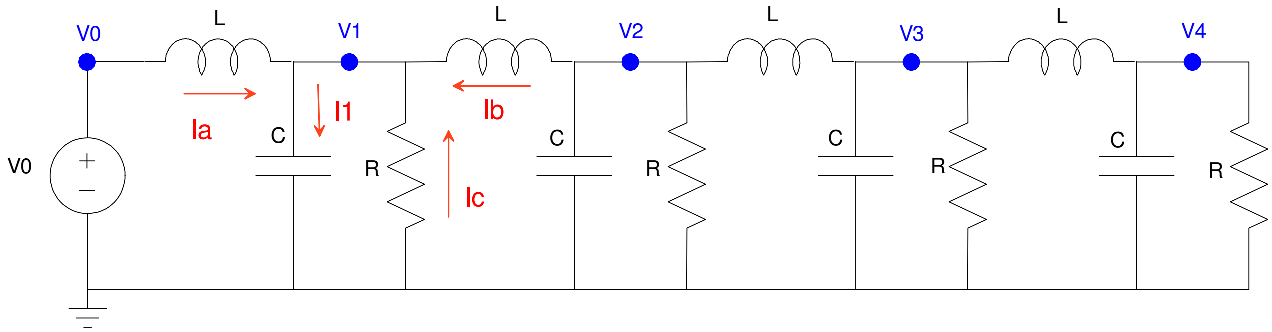


Inductors are differentiators: the voltage is the derivative of the current

Sketching the derivative (slope) gives V



4-Node RLC Circuit



R = 220Ω, C = 0.15F, L = 0.22H. Repeat for 30 nodes for problems 4-6

2) Write the dynamic equations for the following 4-stage RLC circuit. (i.e. write the node equations)

This is kind of tricky. One way to do this is to use conservation of current

$$I_1 = I_a + I_b + I_c$$

The inductor and capacitor equations are

$$I_1 = C \cdot V_1'$$

$$V_0 - V_1 = L \cdot I_a'$$

$$V_2 - V_1 = L \cdot I_b'$$

and

$$I_c = -\tfrac{1}{R} V_1$$

Substitute for I1 and Ic

$$C \cdot V_1' = I_a + I_b - \tfrac{1}{R} V_1$$

Differentiate

$$C \cdot V_1'' = I_a' + I_b' - \tfrac{1}{R} V_1'$$

Substitute for I'

$$C \cdot V_1'' = \left(\tfrac{V_0 - V_1}{L}\right) + \left(\tfrac{V_2 - V_1}{L}\right) - \tfrac{1}{R} V_1'$$

Group terms and solve for V"

$$V_1'' = \left(\tfrac{1}{LC}\right) V_0 - \left(\tfrac{2}{LC}\right) V_1 + \left(\tfrac{1}{LC}\right) V_2 - \left(\tfrac{1}{RC}\right) V_1'$$

Plugging in numbers

$$V_1'' = \left(\frac{1}{LC}\right)V_0 - \left(\frac{2}{LC}\right)V_1 + \left(\frac{1}{LC}\right)V_2 - \left(\frac{1}{RC}\right)V_1'$$

$$V_1'' = 30.30V_0 - 60.60V_1 + 30.30V_2 - 0.03V_1'$$

Same holds for nodes 2 and 3

$$V_2'' = 30.30V_1 - 60.60V_2 + 30.30V_3 - 0.03V_2'$$

$$V_3'' = 30.30V_2 - 60.60V_3 + 30.30V_4 - 0.03V_3'$$

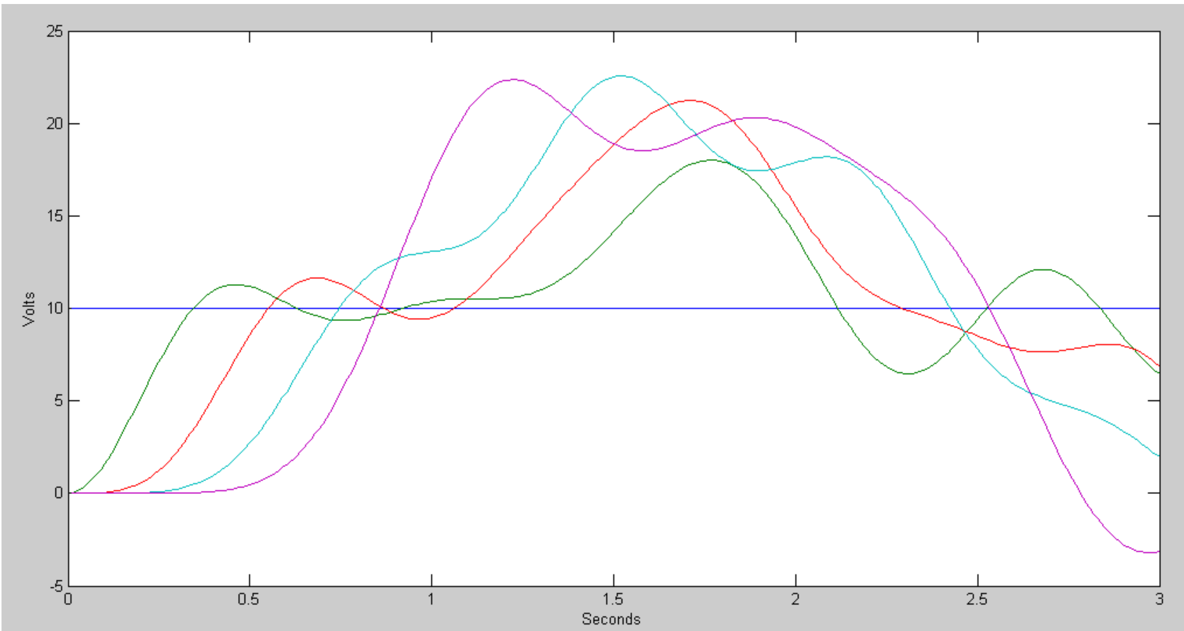
Node 4 is a little different due to only one inductor being attached

$$V_4'' = \left(\frac{1}{LC}\right)V_3 - \left(\frac{1}{LC}\right)V_4 - \left(\frac{1}{RC}\right)V_4'$$

$$V_4'' = 30.30V_3 - 30.30V_4 - 0.03V_3'$$

3) Assume Vin = 10V and the initial conditions are zero ($V_1 = V_2 = V_3 = V_4 = 0$). Solve for the voltages at t = 3 seconds. *Hint: Solve numerically using Matlab*

Voltages vs time



Note:

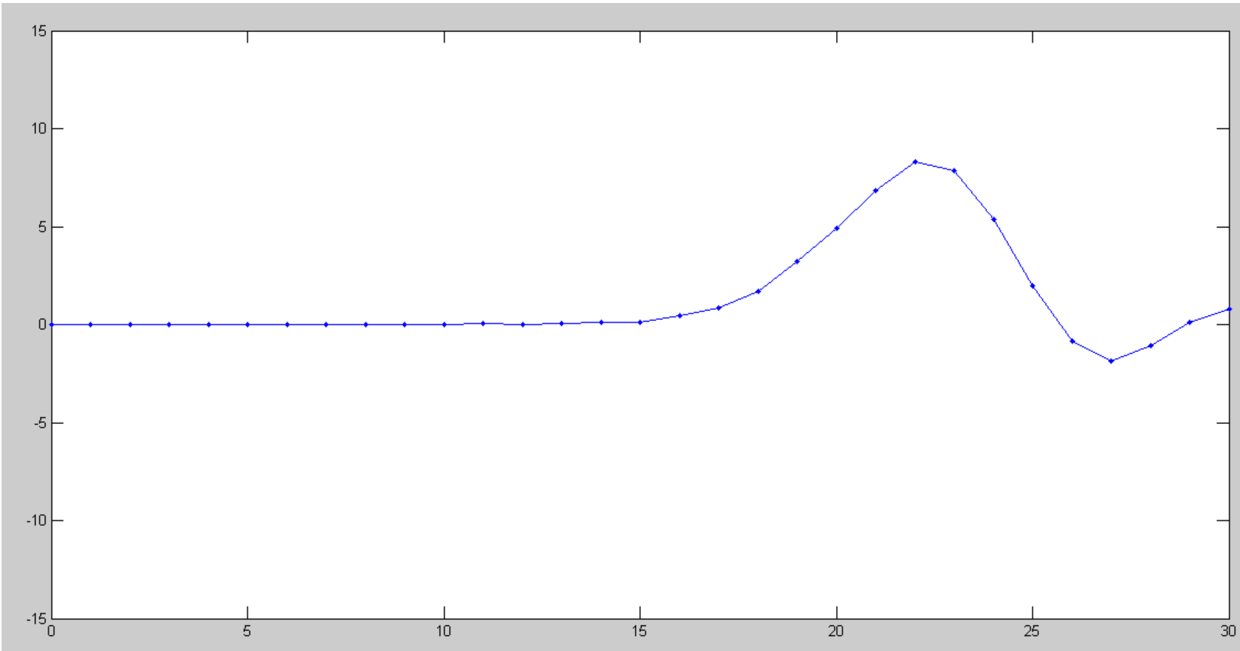
- The wave equation behaves very differently than the heat equation (last week)

Code:

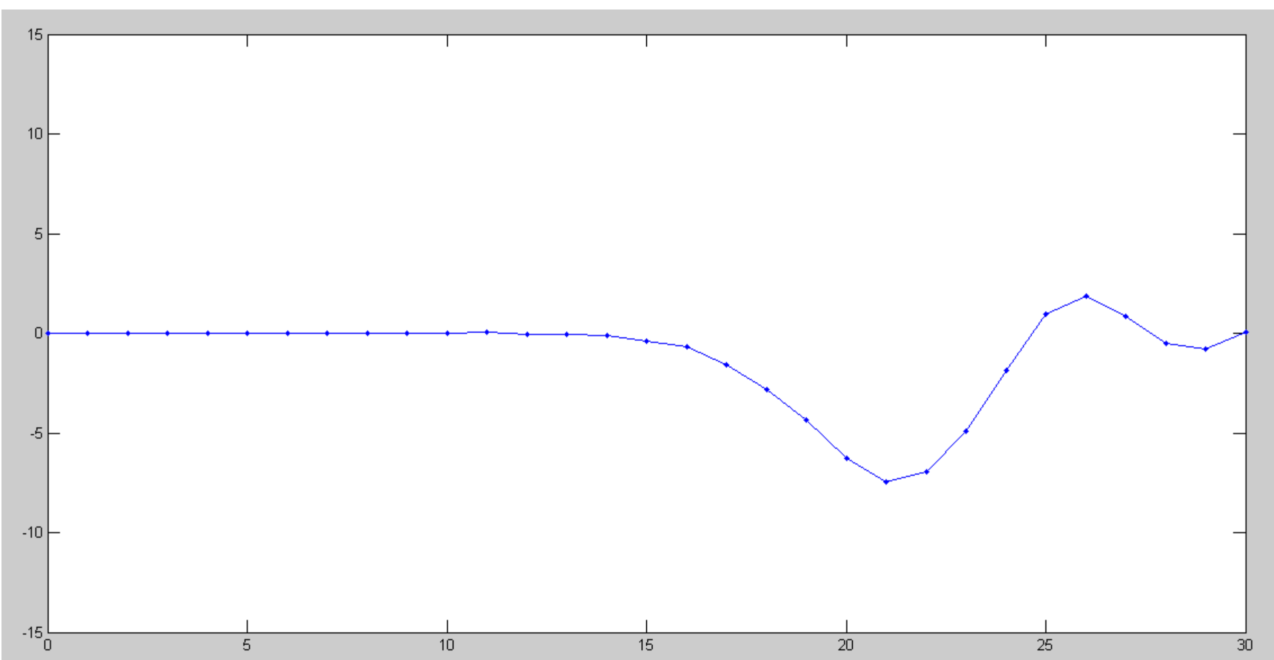
```
V0 = 10;
V1 = 0;
V2 = 0;
V3 = 0;
V4 = 0;

dV1 = 0;
dV2 = 0;
dV3 = 0;
dV4 = 0;

V = [];

t = 0;
dt = 0.01;

while(t < 2.99)

   ddV1 = 30.3*V0 - 60.6*V1 + 30.3*V2 - 0.03*dV1;
   ddV2 = 30.3*V1 - 60.6*V2 + 30.3*V3 - 0.03*dV2;
   ddV3 = 30.3*V2 - 60.6*V3 + 30.3*V4 - 0.03*dV3;
   ddV4 = 30.3*V3 - 30.3*V4           - 0.03*dV4;

   dV1 = dV1 + ddV1*dt;
   dV2 = dV2 + ddV2*dt;
   dV3 = dV3 + ddV3*dt;
   dV4 = dV4 + ddV4*dt;

   V1 = V1 + dV1*dt;
   V2 = V2 + dV2*dt;
   V3 = V3 + dV3*dt;
   V4 = V4 + dV4*dt;

   t = t + dt;

   plot([0,1,2,3,4],[V0,V1,V2,V3,V4],'.-');
   ylim([-20,20]);
   pause(0.01);

   V = [V ; [V0,V1,V2,V3,V4]];

end

pause(2)
t = [1:length(V)]' * dt;
plot(t,V)
```

30-Node RLC Circuit (hint: modify the program Wave.m)

4) Expand the RLC circuit from problem #2 to 30 nodes. Plot the voltage at t = 8 seconds (just after the reflection) for $1 / R_{30}C = 0.01$



5) Plot the voltage at t = 8 seconds for $1 / R_{30}C = 100$

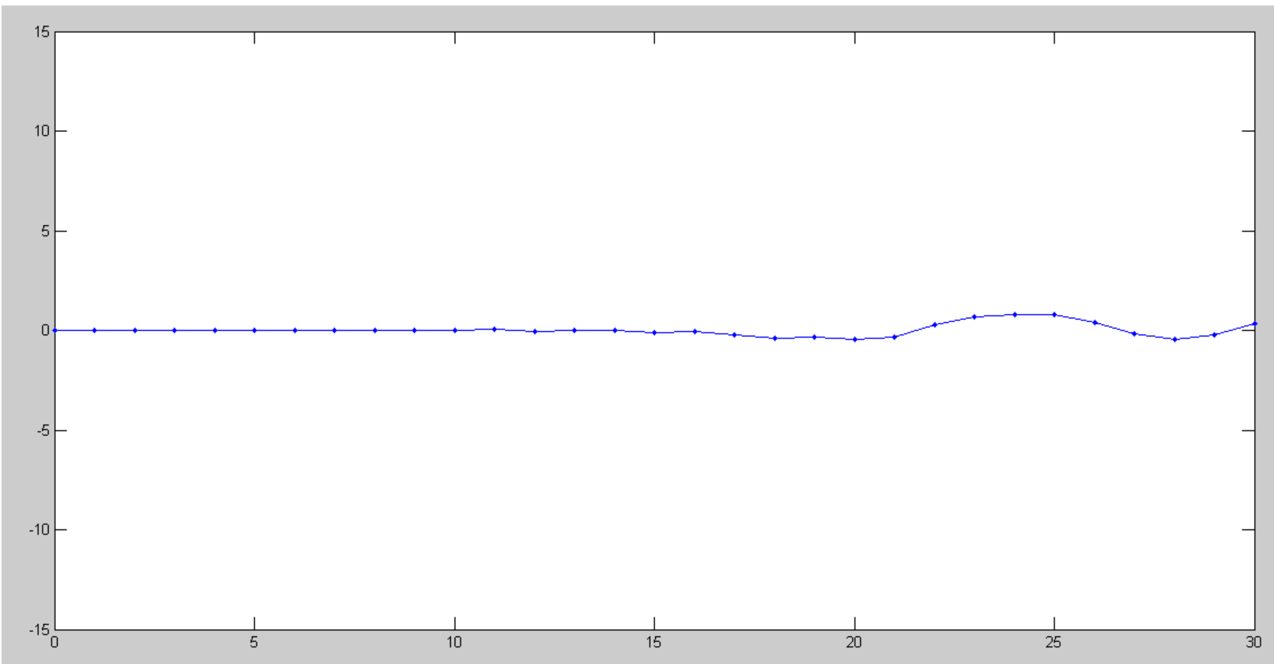


6) Determine experimentally R_{30} so that the reflection is almost zero

$$\left(\frac{1}{R_{30}C}\right) = 5.5$$

$$R_{30} = 1.21\Omega$$

Add a 1.21 Ohm resistor at the right endpoint and you'll remove almost all reflections



Voltage at t = 8 seconds for 1/RC = 5.5 at the right endpoint