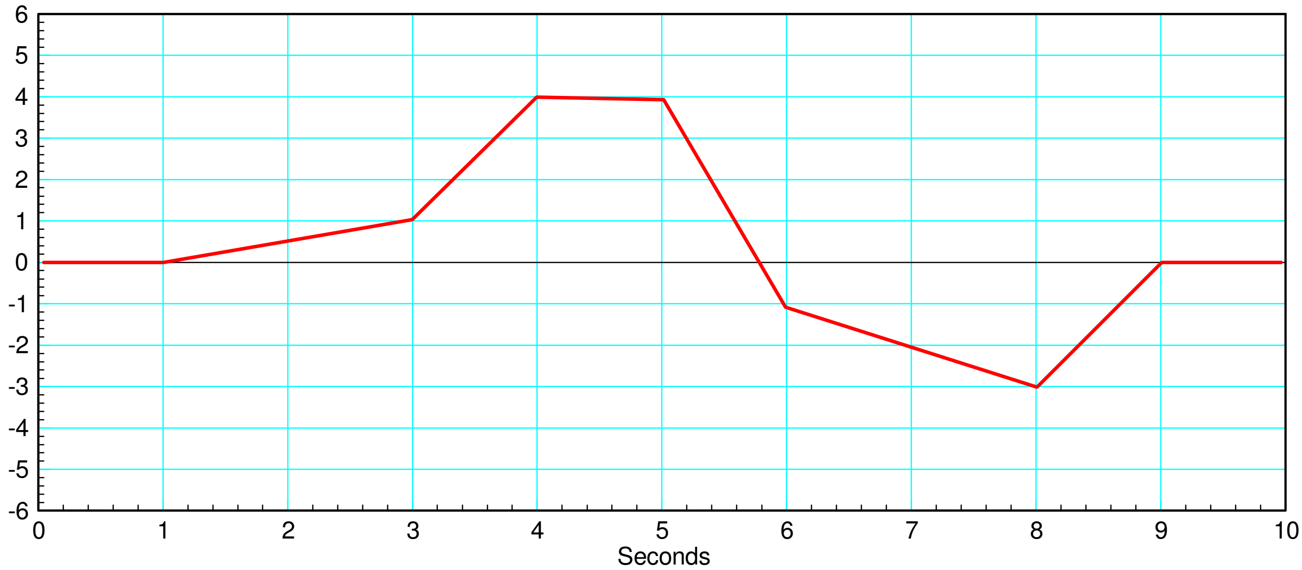
ECE 111 - Homework #7:

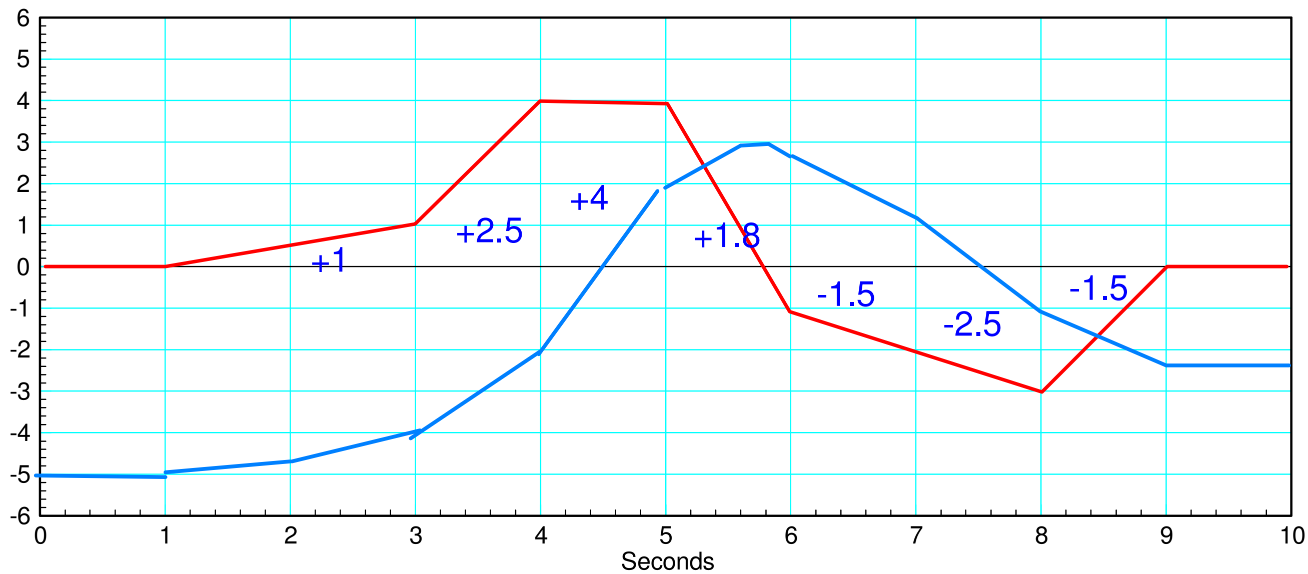
Math 166: Integration

1) Sketch the integral of the following funciton. Assume its initial value is -5



If this is how much money you are depositing (positive) or withdrawing (negative) from your checking account, what is the balance at each instance?

The integral is the area under the curve



Numerical Integration

```
function [y ] = Integrate( x, dy )

npt = length(x);
y = 0*dy;
for i=2:npt
 y(i) = y(i-1) + 0.5*(dy(i) + dy(i-1)) * (x(i) - x(i-1));
end
end
```

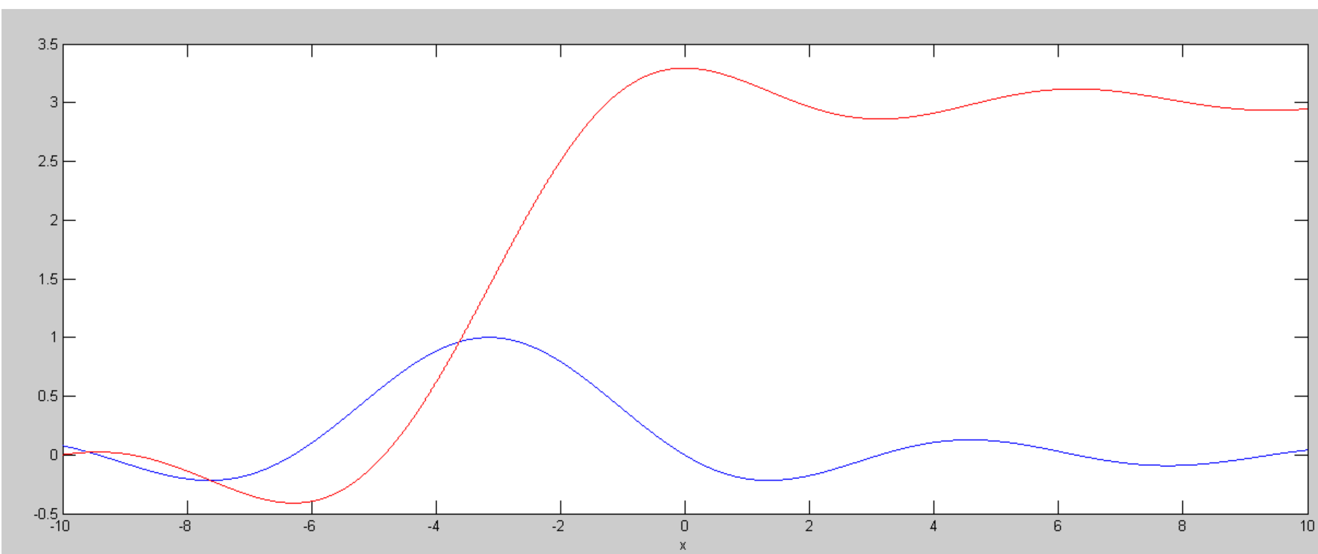
2) Use numerical methods to determine the integral of y

$$y = \left(\frac{\sin(x+\pi)}{x+\pi}\right)$$

$$z = \int y \cdot dx$$

for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

```
>> x = [-10:0.001:10]';
>> y = sin(x+pi) ./ (x+pi);
>> iy = Integrate(x,y);
>> plot(x,y,'b',x,iy,'r');
>> xlabel('x');
>>
```



y(x) (blue) & its integral (red)

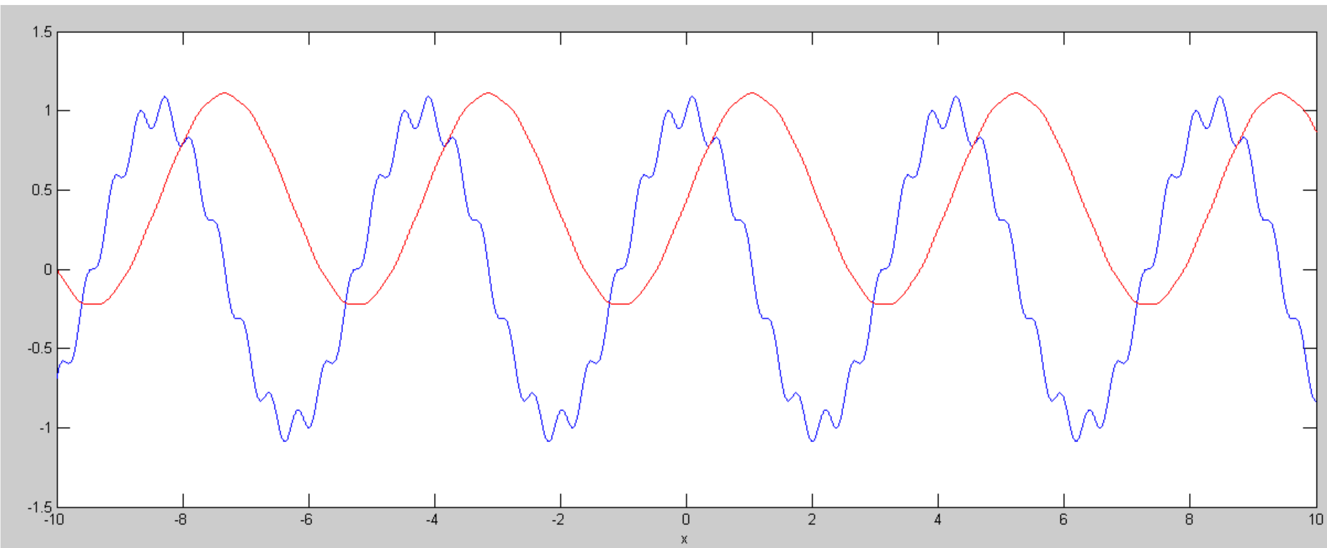
3) Use numerical methods to determine the integral of y

$$y = \cos(1.5x) + 0.1\sin(15x)$$

$$z = \int y \cdot dx$$

for -10 < x < 10. (a plot is sufficient). Assume z(-10) = 0.

```
>> x = [-10:0.001:10]';
>> y = cos(1.5*x) + 0.1*sin(15*x);
>> iy = Integrate(x,y);
>> plot(x,y,'b',x,iy,'r');
>> xlabel('x');
>>
```



y(x) (blue) & its integral (red)

Note: Integration attenuates high frequencies

Animation in Matlab with Numerical Integration

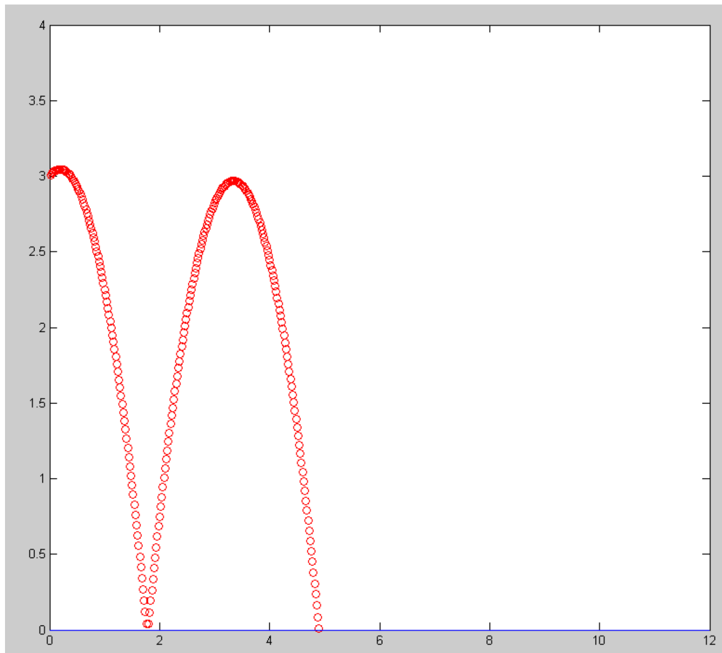
4) Calculate the position of a bouncing ball in freefall:

- The acceleration is y" = -9.8 m/s2
- If the ball hits the ground (y<0) the velocity becomes positive: y' = |y'|
- The initial position is (x = 0, y = 3)
- The initial velocity is (x' = 2, y' =1.0)

Plot the path of the ball for two bounces

- i.e. find the x position of the ball at its 2nd bouce

Matlab Code




```
x = 0;
y = 3;

dx = 2;
dy = 1;

t = 0;
dt = 0.01;


Bounce = 0;

while(Bounce < 2)
    ddx = 0;
    ddy = -9.8;

    dx = dx + ddx * dt;
    dy = dy + ddy * dt;

    x = x + dx * dt;
    y = y + dy * dt;

    t = t + dt;

    if(y < 0)
        dy = abs(dy);
        Bounce = Bounce + 1;
        end

    plot(x,y,'ro',[0,12],[0,0],'b',0,10,'b+');
    xlim([0,12]);
    ylim([0,4]);

    pause(0.01);
end

x

 x = 4.9200
```

5) Determine the initial velocity on x' so that the ball hits a target at (x=10, y=0) on the second bounce

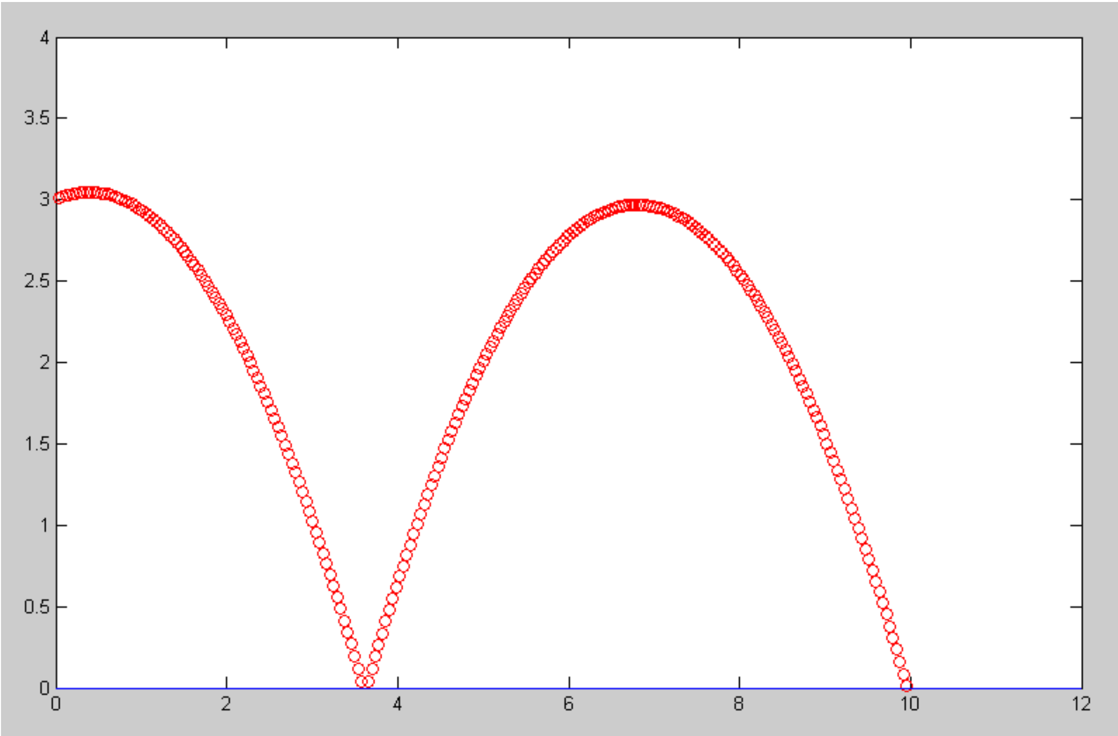
- note: this is a f(x) = 0 problem

Using the California method, scale the initial velocity

$$x' = \left(\frac{10m}{4.92m}\right)2.0\frac{m}{s}$$

$$x' = 4.065$$

```
x =   9.9999
```

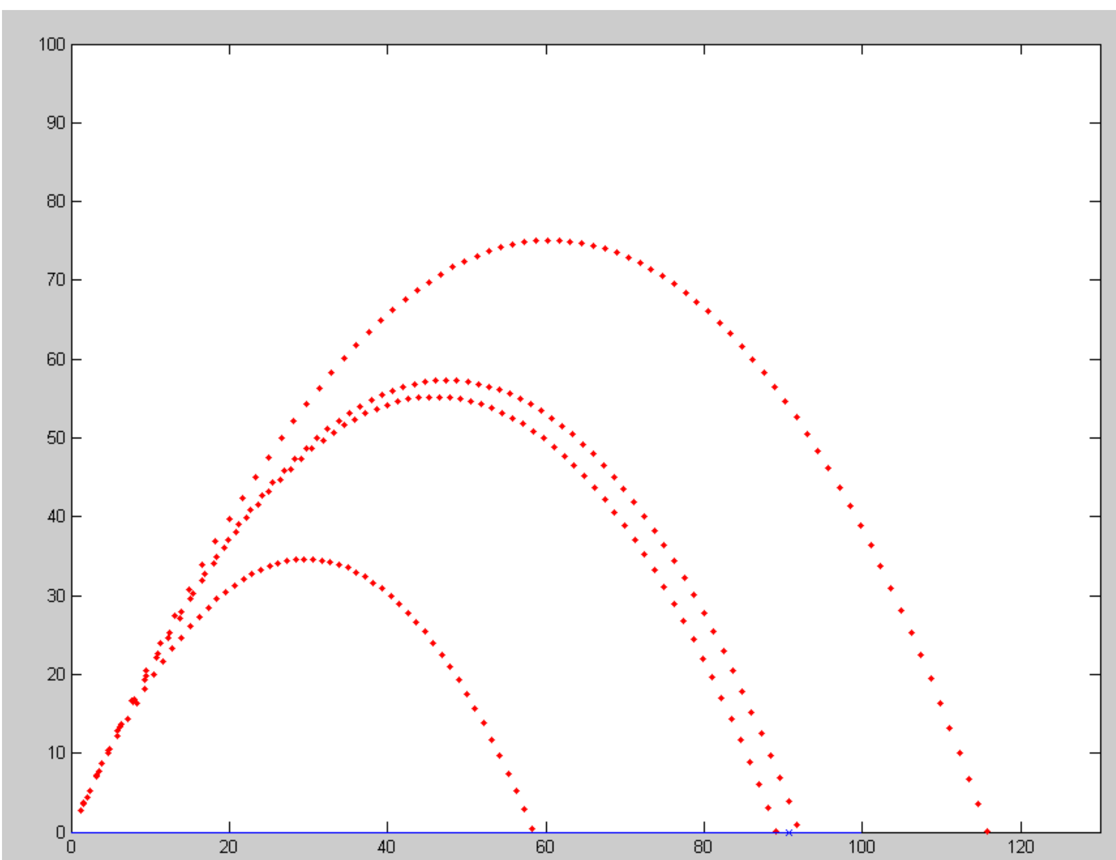


f(x) = 0: Shoot Game

- Pick a random number from 50 to 100 for your target.
- Pick a random number from 30 to 70 for your firing angle

6) Use trial and error to find the initial velocity (X) to fire a tennis ball to hit the target (result is zero)

```
>> Target = 50 + 50*rand

Target =   90.7362

>> Angle = 30 + 40*rand

>> Shoot(30, Angle, Target)

ans =   32.3244

>> Shoot(50, Angle, Target)

ans =  -25.0275

>> Shoot(40, Angle, Target)

ans =    1.5482

>> Shoot(41, Angle, Target)

ans =   -1.3471

>>
```



7) Repeat using Newton's/California method to find the initial velocity (X) to fire the tenis ball to hit the target

Using California method (pretty similar to Newton's method)

```
>> X1 = 30;
>> Y1 = Shoot(X1, Angle, Target)

Y1 =   32.3244

>> X2 = 50;
>> Y2 = Shoot(X2, Angle, Target)

Y2 =  -25.0275

>> X3 = X2 - (X2-X1)/(Y2-Y1)*Y2

X3 =   41.2723

>> Y3 = Shoot(X3, Angle, Target)

Y3 =   -2.1272

>> X4 = X3 - (X3-X2)/(Y3-Y2)*Y3

X4 =   40.4616

>> Y4 = Shoot(X4, Angle, Target)

Y4 =    0.2060

>> X5 = X4 - (X4-X3)/(Y4-Y3)*Y4

X5 =   40.5331

>> Y5 = Shoot(X5, Angle, Target)
```

Y5 =   -0.0013

note: there are better methods than guessing