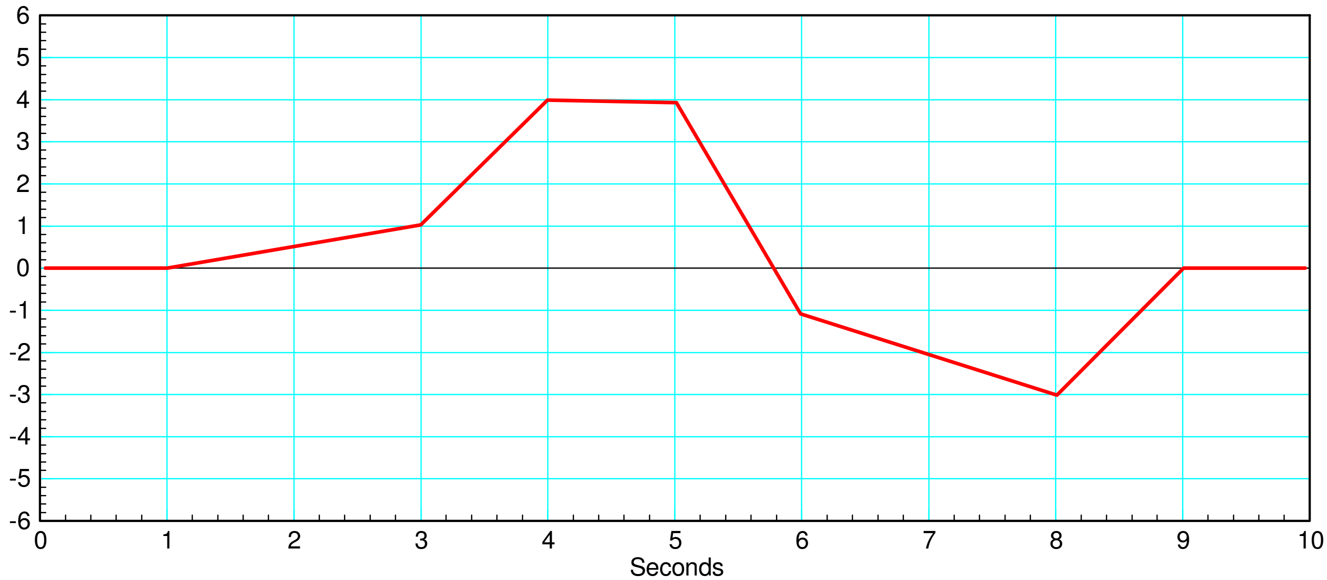
ECE 111 - Homework #6:

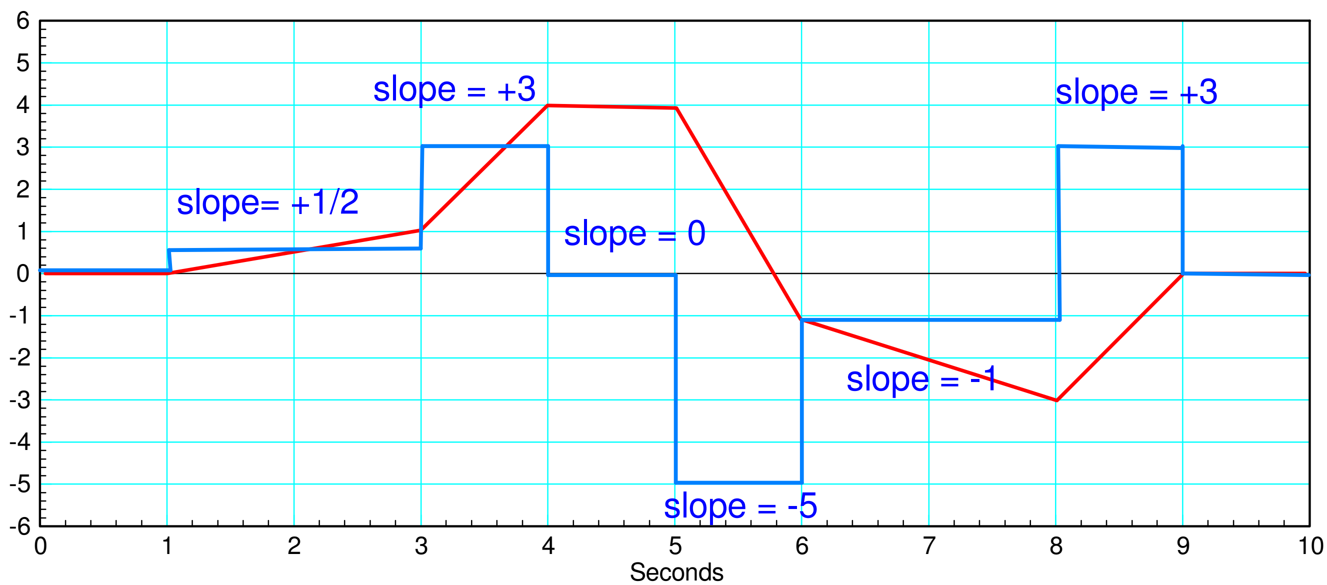
Math 165: Differentiation

1) Sketch the derivative of the following funciton

If this is the balance of your checking account, how much money are you adding (positive) or withdrawing (negative) for the balance to be as shown?



The derivative is the slope



Numerical Differentiation:

```
function [ dy ] = Derivative( x, y )

dx = x(2) - x(1);
dy = 0*y;
n = length(y);
for i=2:n-1
  dy(i) = ( y(i+1) - y(i-1) ) / ( x(i+1) - x(i-1) );
end

dy(1) = (y(2) - y(1))/(x(2) - x(1));
dy(n) = (y(n) - y(n-1)) / (x(n) - x(n-1));

end
```

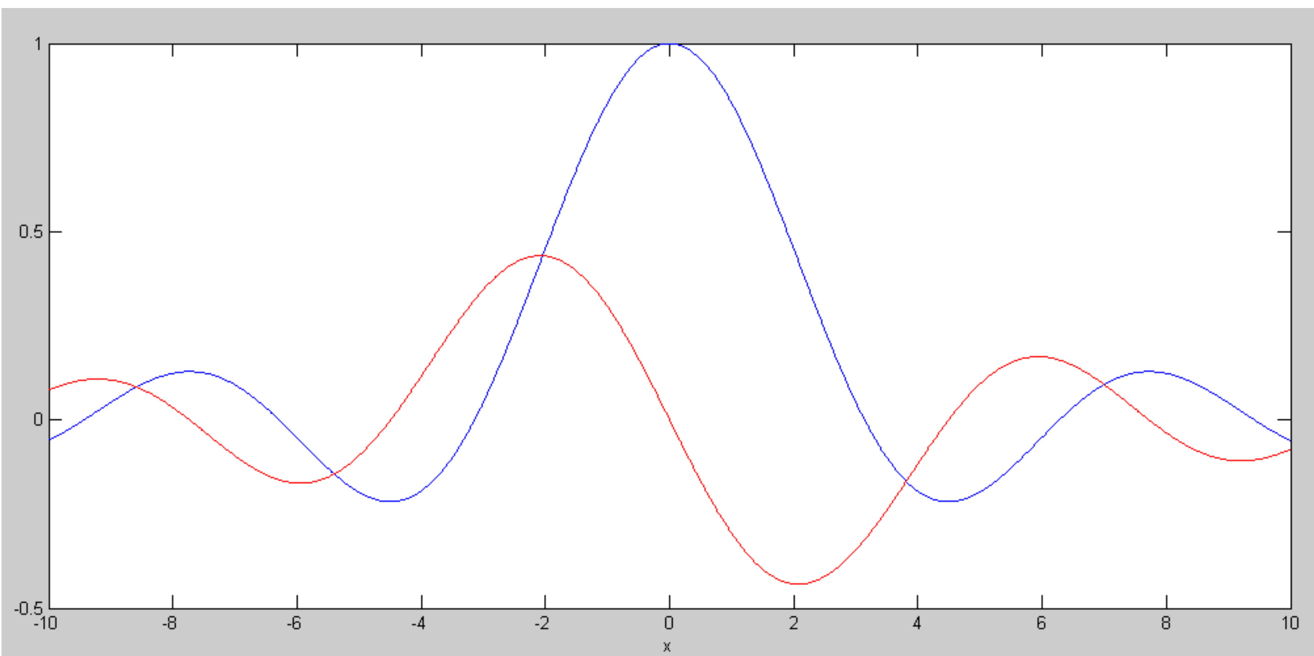
2) Use numerical methods to determine the derivative of y:

$$y = \left(\frac{\sin(x+0.001)}{x+0.001}\right)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> x = [-10:0.01:10]';
>> y = sin(x + 0.001) ./ (x + 0.001);
>> dy = Derivative(x,y);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x');
>>
```



y(x) (blue) & dy/dx (red)

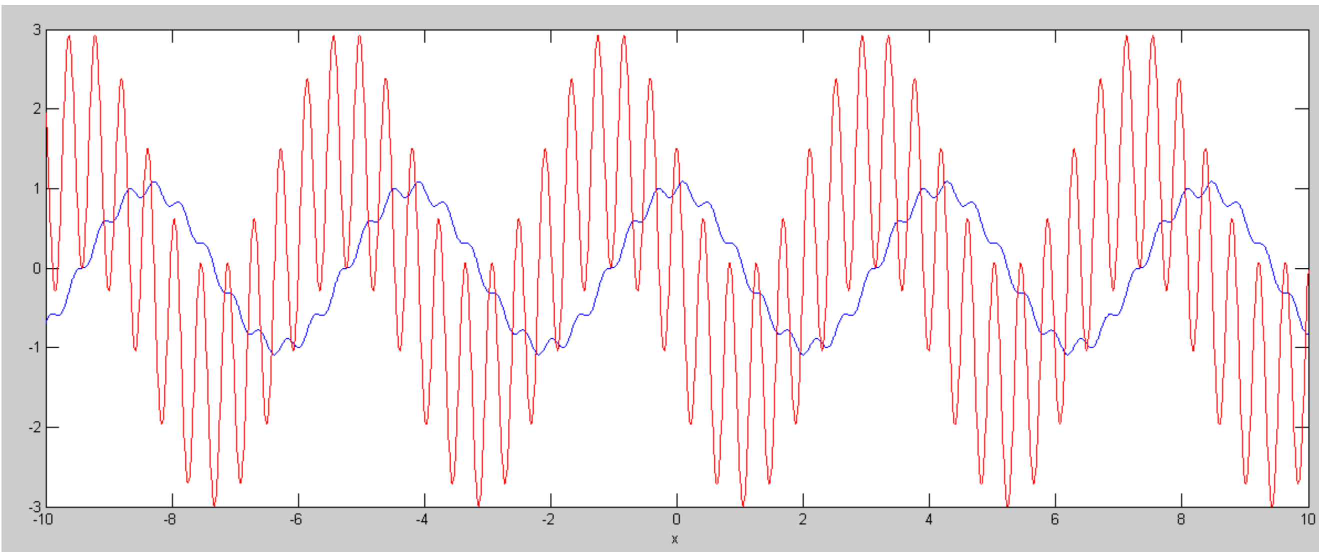
3) Use numerical methods to determine the derivative of y:

$$y = \cos(1.5x) + 0.1\sin(15x)$$

$$z = \frac{d}{dx}(y)$$

for -10 < x < 10. (a plot is sufficient).

```
>> x = [-10:0.01:10]';
>> y = cos(1.5*x) + 0.1*sin(15*x);
>> dy = Derivative(x,y);
>> plot(x,y,'b',x,dy,'r')
>> xlabel('x');
>>
```



y(x) (blue) & dy/dx (red)

note: Differentiation amplifies high frequency signals

Path Planning

4) Assume a motor's angle is as follows:

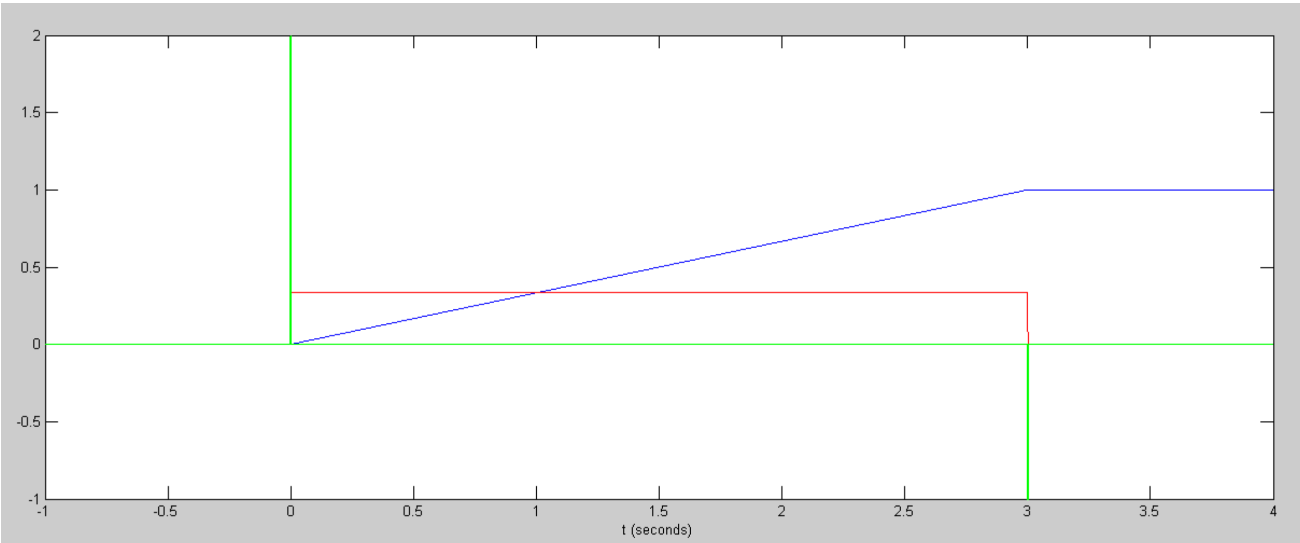
$$\theta = \begin{cases} 0 & t < 0 \\ t/3 & 0 < t < 3 \\ 1 & t > 3 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.001:4]' + 1e-6;
>> q4 = (t/3).*(t>0).*(t<3) + 1*(t>3);
>> dq4 = Derivative(t,q4);
>> ddq4 = Derivative(t,dq4);

>> plot(t,q4,'b',t,dq4,'r',t,ddq4,'g')
>> ylim([-1,2])
>> xlim([-1,4])
>> xlabel('t (seconds)');
>>
```



angle (blue), velocity (red), & acceleration (green)

5) Assume a motor's angle is as follows:

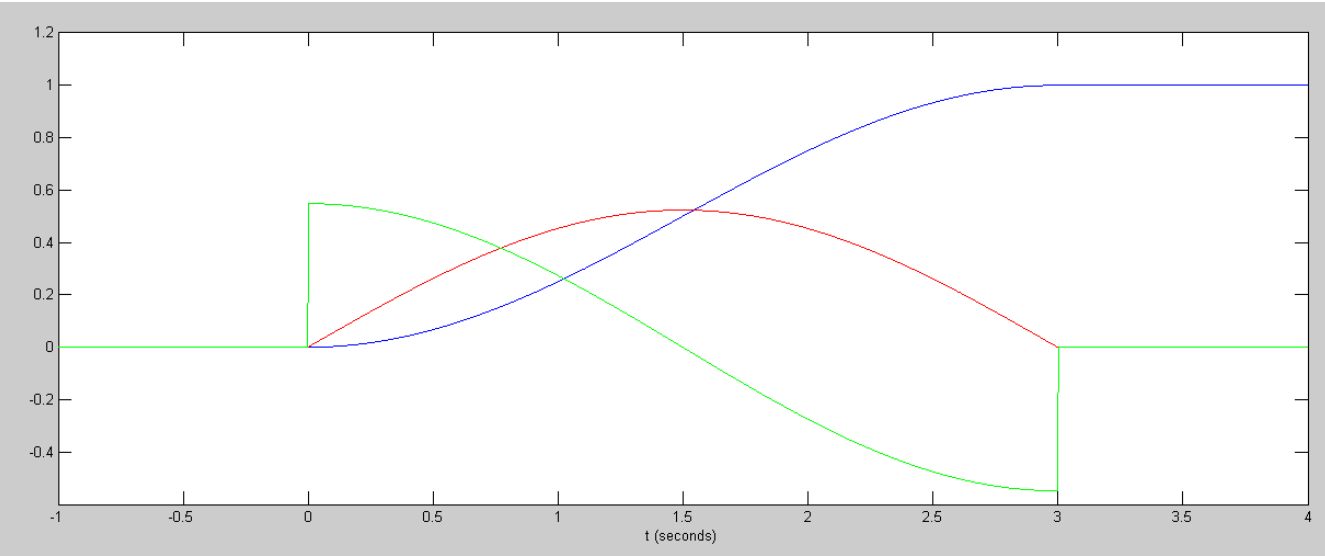
$$\theta = \begin{cases} 0 & t < 0 \\ \frac{1}{2} - \frac{1}{2}\cos\left(\frac{1}{3}\pi t\right) & 0 < t < 3 \\ 1 & t > 3 \end{cases}$$

Calculate and plot using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.001:4]' + 1e-6;
>> q = (0.5-0.5*cos(pi*t/3)).*(t>0).*(t<3) + 1*(t>3);
>> dq = Derivative(t,q);
>> ddq = Derivative(t,dq);

>> plot(t,q,'b',t,dq,'r',t,ddq,'g')
>> ylim([-0.6,1.2])
>> xlim([-1,4]);
>> xlabel('t (seconds)');
>>
```



6) Assume a motor's angle is as follows:

$$\theta = \begin{cases} 0 & t < 0 \\ \frac{2}{9}t^2 & 0 < t < 1.5 \\ 1 - \frac{2}{9}(t-3)^2 & 1.5 < t < 3 \\ 1 & t > 3 \end{cases}$$

Calculate using Matlab and numerical differentiation:

- The velocity vs. time (i.e. the voltage to the motor), and
- The acceleration vs. time (i.e. the current to the motor).

```
>> t = [-1:0.001:4]' + 1e-6;
>> q = (2/9*t.^2).*(t>0).*(t<1.5) + (1-2/9*(t-3).^2).*(t>1.5).*(t<3) + 1*(t>3);
>> dq = Derivative(t,q);
>> ddq = Derivative(t,dq);

>> plot(t,q,'b',t,dq,'r',t,ddq,'g')
>> ylim([-0.6,1.2])
>> xlim([-1,4]);
>> xlabel('t (seconds)');
>>
```