ECE 111 - Homework #4

Math 129 Linear Algebra.

1) Solve for {x, y}

$$-14x + 5y = -2$$

$$-2x + 14y = 3$$

Express this problem in matrix form

$$\begin{bmatrix} -14 & 5 \\ -2 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

Solve in Matlab

```
>> B = [-14,5 ; -2,14]

   -14     5
    -2    14

>> A = [-2;3]

    -2
     3

>> inv(B)*A
```

x 0.2312
y 0.2473

2) Solve for {x, y, z}

$$2x + 7y + 2z = 1$$

$$16x + 8y - 12z = 7$$

$$-8x - 2y - 11z = 26$$

Express in matrix form

$$\begin{bmatrix} 2 & 7 & 2 \\ 16 & 8 & -12 \\ -8 & -2 & -11 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 26 \end{bmatrix}$$

Solve in Matlab

```
>> B = [2,7,2 ; 16,8,-12 ; -8,-2,-11]

     2     7     2
    16     8   -12
    -8    -2   -11

>> A = [1;7;26]

     1
     7
    26

>> inv(B)*A

x   -1.2620
y    0.9667
z   -1.6216
```

3) Solve for {a, b, c, d}

$$-7a - 4b + c + 5d = -21$$

$$2a - 18b - 5c + 7d = -8$$

$$-a + 8b + 3c + 17d = 14$$

$$19a + 9b + 6c + 2d = 11$$

Express in matrix form

$$\begin{bmatrix} -7 & -4 & 1 & 5 \\ 2 & -18 & -5 & 7 \\ -1 & 8 & 3 & 17 \\ 19 & 9 & 6 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -21 \\ -8 \\ 14 \\ 11 \end{bmatrix}$$

Solve in Matlab

```
>> b1 = [-7,-4,1,5];
>> b2 = [2,-18,-5,7];
>> b3 = [-1,8,3,17];
>> b4 = [19,9,6,2];
>> B = [b1;b2;b3;b4]

    -7    -4     1     5
     2   -18    -5     7
    -1     8     3    17
    19     9     6     2

>> A = [-21;-8;14;11]

   -21
    -8
    14
    11

>> inv(B)*A

a    1.2067
b    2.5671
c   -6.0925
d    0.7616
```

Comments:

- With Matlab, solving N equations for N unknowns is easy
- This requires using matricies (why Math 129 is required for ECE majors)
- What matters most with Matlab is get the equations right - you don't really care how many equations you get.

This is a trick that you're going to see over and over in ECE

- Given a problem, convert it to N equations and N unknowns

Global CO2 Levels

Problem 4) Determine a parabolic curve fit for this data in the form of

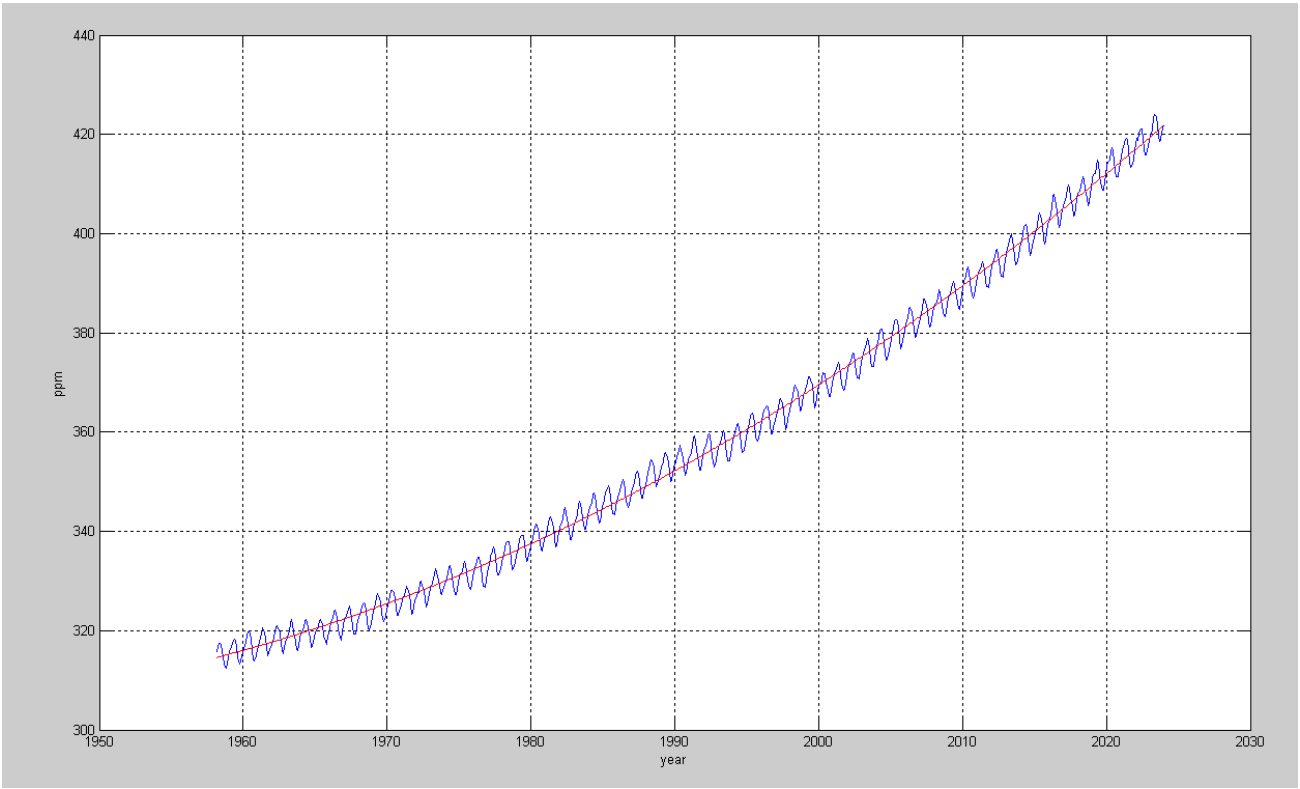
$$CO_2 \approx ay^2 + by + c$$

where 'y' is the year. From this data, when do you predict that we will hit

- 500ppm?
- 2000 ppm of CO2? (the same as what was observed during the Permian extinction)

Note: Column #3 of the data set is year, #4 is CO2

```
>> year = DATA(:,3);
>> CO2 = DATA(:,4);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*CO2

a   1.3175e-002
b  -5.0837e+001
c   4.9343e+004

>> plot(year,CO2,'b',year,B*A,'r');>> xlabel('year');
>> ylabel('ppm');
>>
```



When will CO2 levels reach 500ppm?

One way is find this is use the *roots()* command

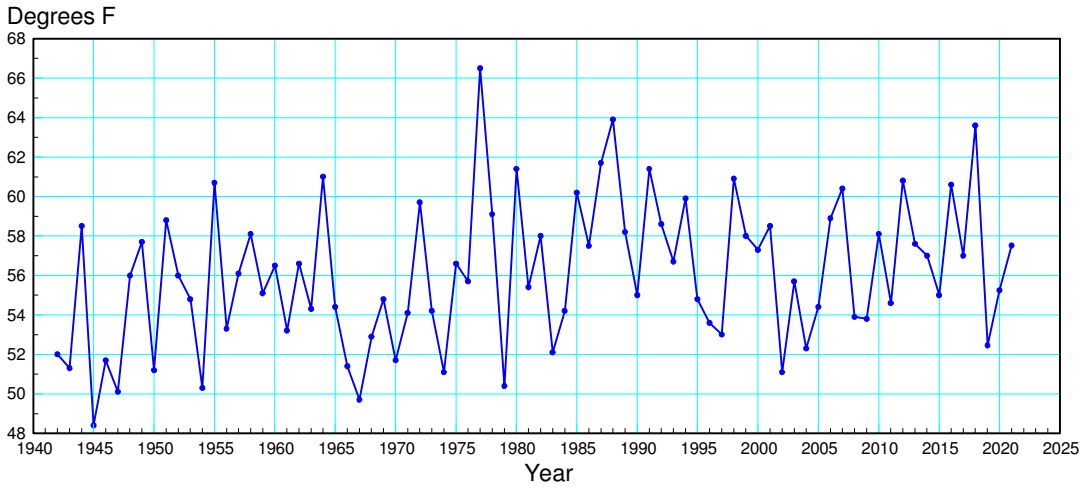
```
>> roots(A - [0;0;500])

  2.0514e+003
  1.8072e+003
```

If nothing changes, we'll hit 500ppm in the year 2051

```
>> roots(A - [0;0;2000])

  2.2881e+003
  1.5704e+003
```

If nothing changes, we'll hit 2000ppm in the year 2288

Fargo Temperatures



Average temperatre in Fargo
http://www.bisonacademy.com/ECE111/Code/Fargo_Weather_Monthly_Avg.txt

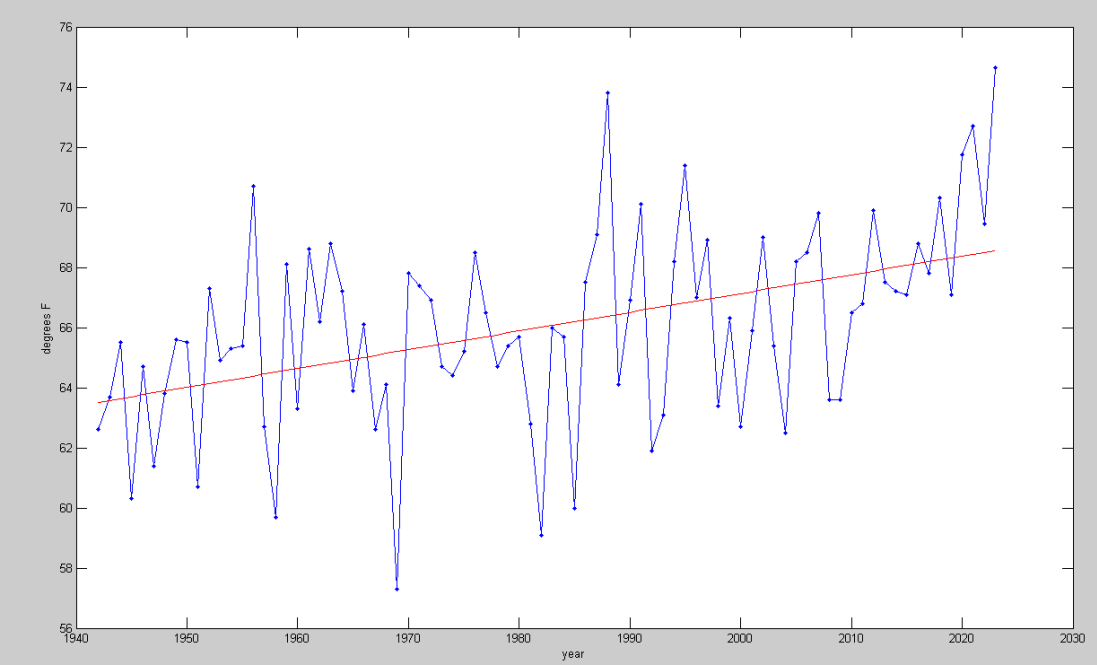
note: Column #1 of the data set is year, column #7 is average temeperature of June in degrees F

```
year = DATA(:,1);
F = DATA(:,7);
```

5) Using the average temperature in Fargo from 1942 to 2023:

5a) Determine a curve fit of the form of $T = ay + b$

```
>> year = DATA(:,1);
>> June = DATA(:,7);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*June

A =

  6.2359e-002
 -5.7587e+001


>> plot(year,June,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('degrees F');
```



June average temperature in Fargo vs. year

5b) How much has Fargo warmed up over the past 80 years?

```
>> a = A(1);
>> b = A(2);
>> a*80

   4.9887e+000
```

Over the past 80 years, Fargo's gotten almost 5F degrees warmer

5c) What will the average temperature in Fargo be in June

- In the year 2050?
- In the year 2100?

```
>> y = 2050;
>> F = a*y + b

F =   70.249     average temperature in 2050

>> y = 2100;
>> F = a*y + b

F =   73.367     average temperature in 2100
```

Problem 6-7) Sea Ice: The area covered by sea ice is recored by the National Snow and Ice Data Center:

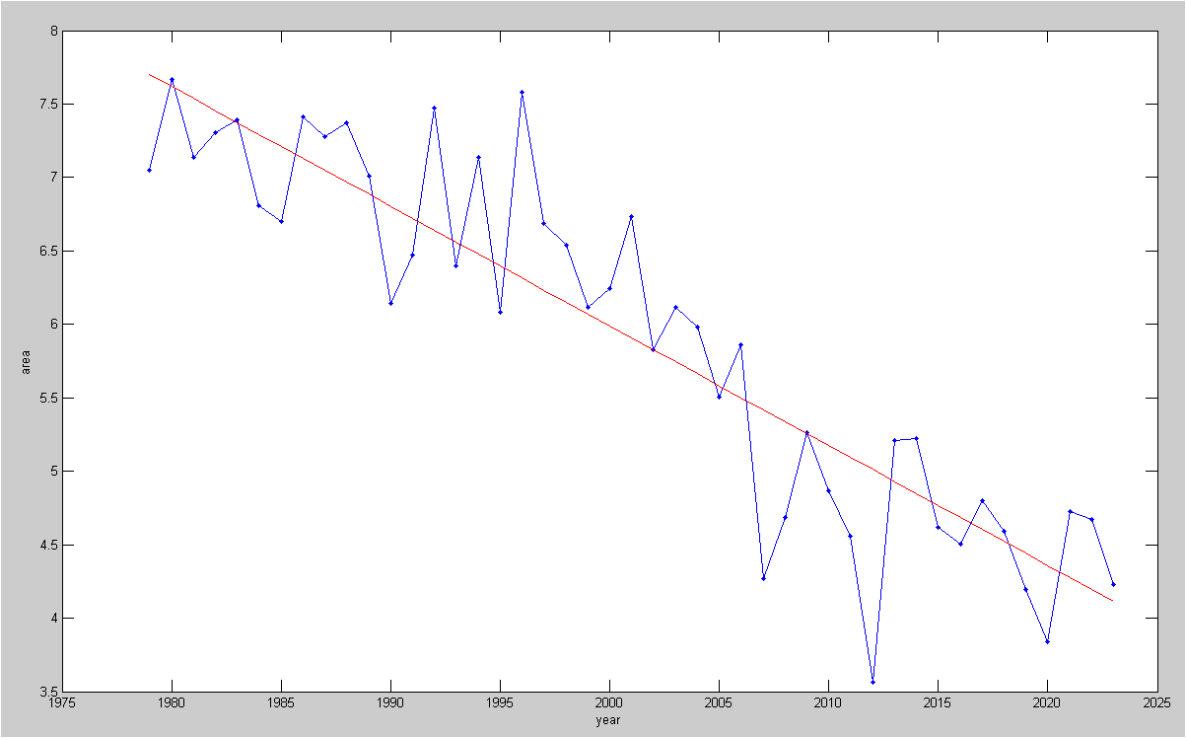
6) Approximate this data from the years 1979 - 2022 with a line

$$Area \approx ay + b$$

From this curve fit, when do you expect the Arctic to be ice free? (First time in 5 million years)

```
>> year = DATA(:,1);
>> ICE = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*ICE

 -8.1471e-002
  1.6893e+002

>> roots(A)

ans =  2073.5

>> plot(year,ICE,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('area');
>>
```

Assuming a linear curve fit, the Arctic should be ice free in the year 2073



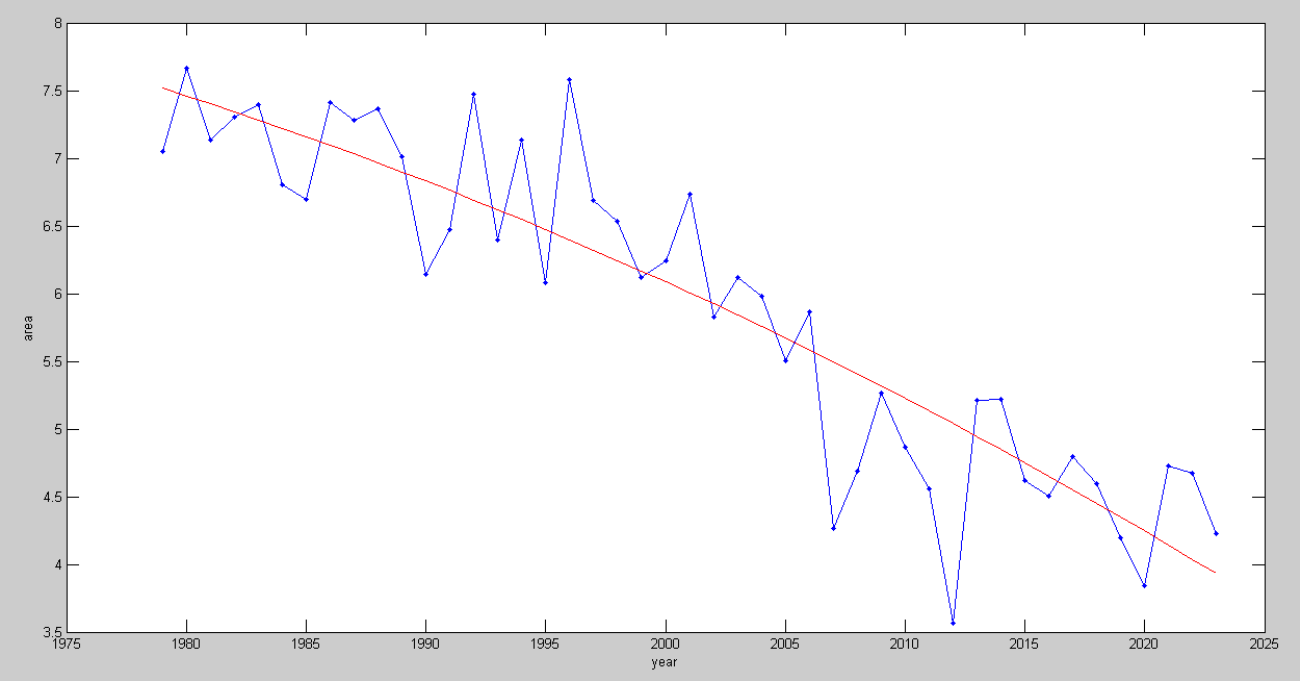
7) Approximate this data with a parabolic curve fit:

$$Area \approx ay^2 + by + c$$

From this curve fit, when do you expect the Arctic to be ice free?

```
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*ICE

 -5.8159e-004
  2.2461e+000
 -2.1597e+003

>> roots(A)

  2.0544e+003
  1.8075e+003

>> plot(year,ICE,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('area');
```

Assuming a parabolic curve fit (constant rate of increase), the Arctic should be ice free in the year 2054



Problem 8-9: World Temperatures. NASA Goddard has been keep records since 1880 (139 years of data).

8) Determine a least-squares curve fit for this data from the year 1880 - 1910 in the form of

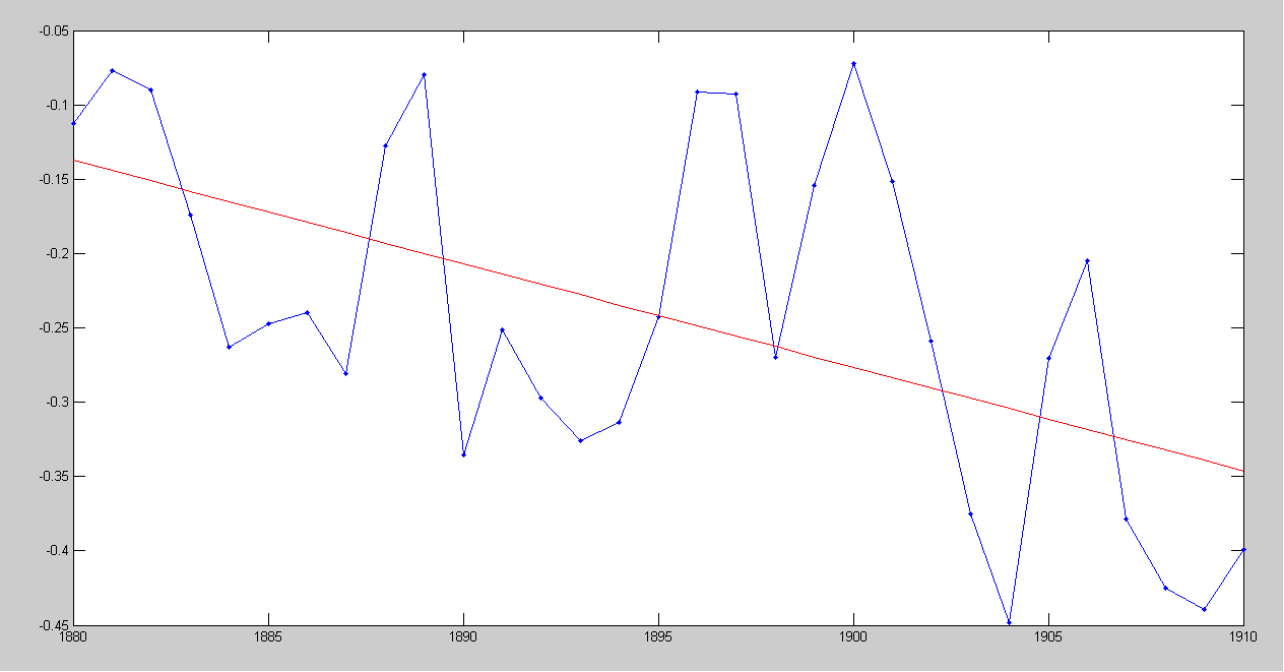
$$\delta T = ay + b$$

Based upon this data, what *should* the temperature deviation be in the year 2023?

```
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year, year.^0];
>> A = inv(B'*B)*B'*dT

a -6.9656e-003
b  1.2958e+001

>> a = A(1);
>> b = A(2);
>> y = 2023;
>> a*y + b

 -1.1333e+000
```

Based upn 1880 - 1910, it should be 1.1333 degrees colder this year

- It's actually 1.01C warmer, for a net difference of 2.14C



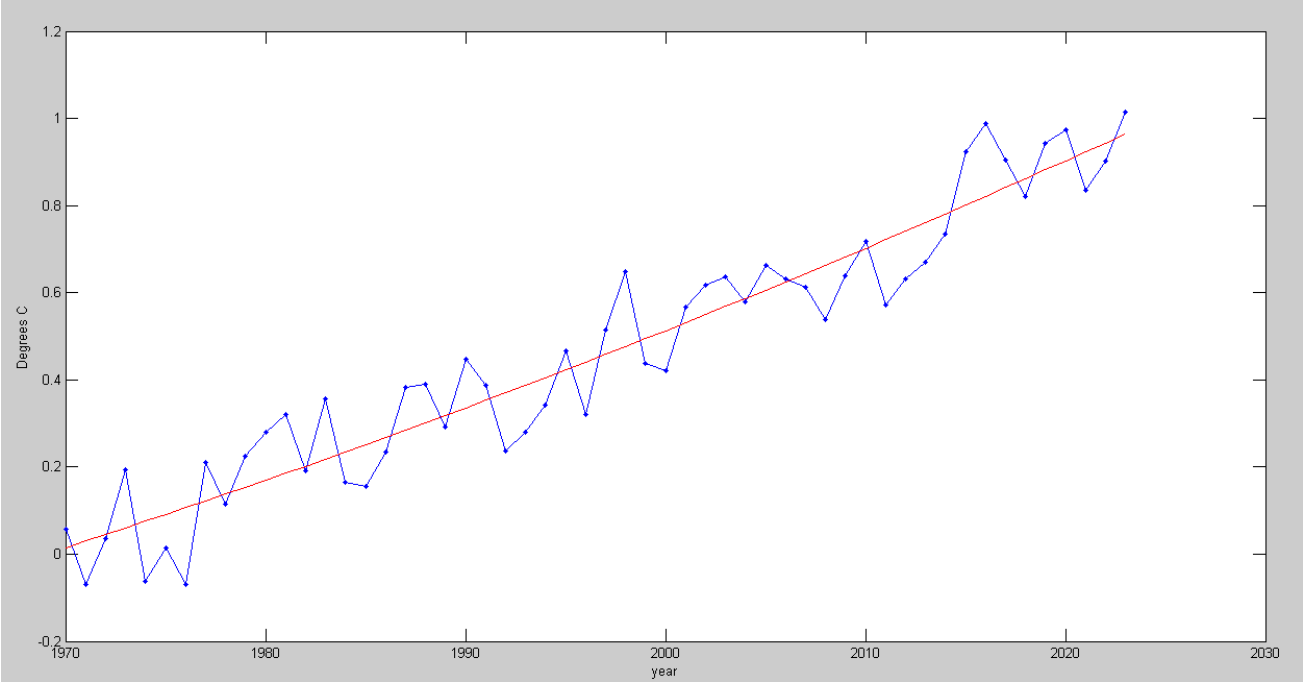
9) Determine a least-squares curve fit for this data from the year 1970 - 2022 in the form of

$$\delta T \approx ay^2 + by + c$$

Based upon this data, predict when we will see a 10 degree temperature increase if nothing changes?

```
>> year = DATA(:,1);
>> dT = DATA(:,2);
>> B = [year.^2, year, year.^0];
>> A = inv(B'*B)*B'*dT

a   5.7553e-005
b  -2.1189e-001
c   1.9409e+002

>> plot(year,dT,'b.-',year,B*A,'r')
>> xlabel('year');
>> ylabel('Degrees C');

>> roots(A - [0;0;10])

   2.2769e+003
   1.4048e+003
```

Assuming a parabolic curve fit (constant rate of change), we should hit +10C in the year 2276



10) What does a temperature rise of 10 degrees mean for the planet?

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not graded - too political

The Permian Extinction Event suggests that it's not good: no animals larger than a mouse survived the Permian Extinction - which was triggered by CO2 levels at 2000ppm and a +10 degree C temerature rise.

One Degree: 2024 Summers like 2003 where a heat wave in France caused 10,000 deaths become the norm. Flows of the Po and Rhine river decrease. Crop production drops.

```
-->roots(A - [0;0;1])

  2024.7
  1657.1
```

Two Degrees: 2067. Oceans absorb less CO2 (too hot) and soils start to release CO2. Vacations to the Mediterranean in the summer are just too hot. Crop failures in Africa and Central America cause mass migration. Coastal cities flood. 1/3rd of species face extinction.

```
-->roots(A - [0;0;2])

  2067.1
  1614.7
```

Three Degrees: 2102. Crop failures in China cause the migration of more than 1 billion people. Collapse of equatorial governments.

```
-->roots(A - [0;0;3])

  2102.7
  1579.1
```

Four Degrees: 2134. Spain becomes a desert. Mass migration to Northern latitudes. Rain forests burn up.

```
-->roots(A - [0;0;4])

  2134.0
  1547.8
```

Six Degrees: 2138. Ice caps are gone. Methane hydrates become unstable raising temperatures in a positive-feedback loop. Ocean circulation stops. Hydrogen sulfide producing bacteria flourish poisoning the air. The Ozone layer dissipates leaving the land sterilized with UV radiation. End-Permian-like conditions make life nearly impossible.

```
-->roots(A - [0;0;0;6])

  2188.2
  1493.5
```

Scary? Yes. That's why the rest of the world sees the Paris Climate Accord as being important. That's why the United Nations sees Global Warming as the #1 threat - far greater than terrorism. Far greater than COVID.