ECE 111 - Homework #2

Math 103 - Algebra, Functions & Solving f(x) = 0.

Newton's Method

1) Let x and y be related by:

$$y = x \cdot \ln(x)$$

Use Newton's method to solve for x when

- y = 5
- y = 10

y = 5

Start with a function which returns the error

```
function [e] = Prob1(x)
   y = x * log(x);
   e = y - 5;
end
```

Find the zero using Newton's Method

- Initial guess is 3
- Itterate ten times

```
x3 = 3;
for i=1:10
    x1 = x3;
    y1 = Prob1(x1);

    x2 = x1 + 0.001;
    y2 = Prob1(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([x1, y1]);
    pause(0.1);
end
```

Result

x	error
3.000000000000000	-1.704163133995671
3.811978386145567	0.100992457380318
3.768787459362492	0.000251277513011
3.768679470610399	0.000000015872597
3.768679463788925	0.000000000000905
3.768679463788536	0
3.768679463788536	0
3.768679463788536	0
3.768679463788536	0
3.768679463788536	**0**

y = 10

Change the funciton

```
function [e] = Prob1(x)
   y = x * log(x);
   e = y - 10;
end
```

Solve using Newton's method

- nothing changes

```
x3 = 3;
for i=1:10
    x1 = x3;
    y1 = Prob1(x1);

    x2 = x1 + 0.001;
    y2 = Prob1(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([x1, y1]);
    pause(0.1);
end
```

Result

x	error
3.000000000000000	-6.704163133995671
6.194315997927543	1.296153472957442
5.735291458491899	0.017481273313585
5.728927054129022	0.000004087383264
5.728925565434333	0.000000000130115
5.728925565386943	0.000000000000004
5.728925565386942	0.000000000000002
5.728925565386941	0
5.728925565386941	0
5.728925565386941	**0**

Comment:

- y = x ln(x) has no closed-form solution
- using numeric methods, you can solve regardless
- Newton's method converges really fast

2) Let x and y be related by

$$y = \cos(3x)$$

$$y = (x+1)(x-2)$$

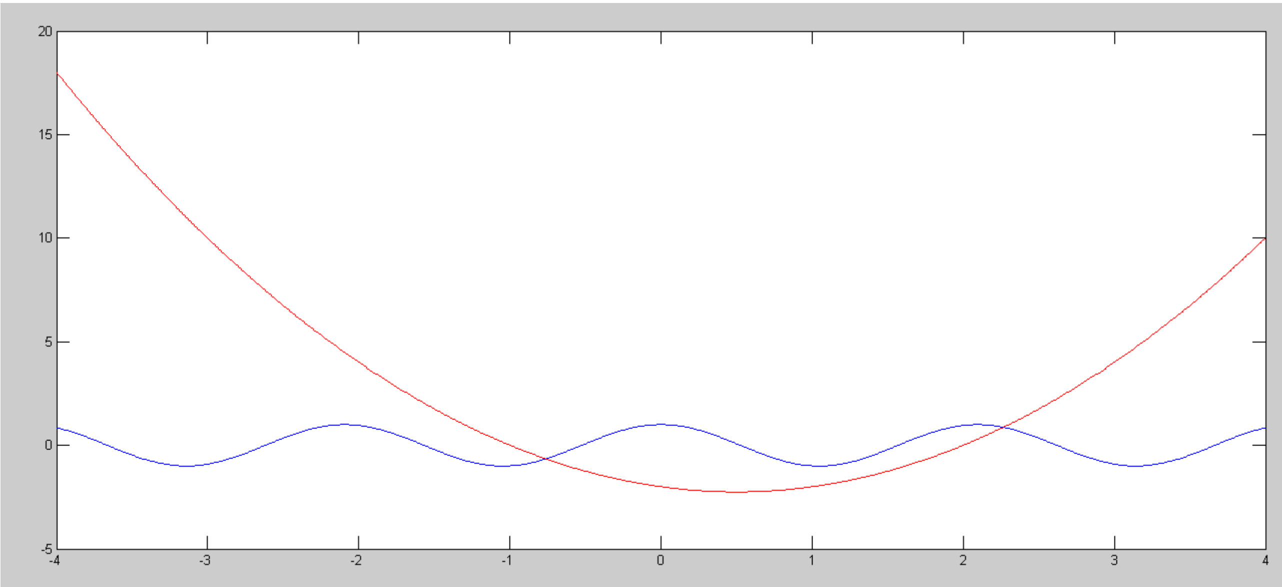
Find all solutions using graphical methods. (Plot both functions on the same graph. The solution is when the two functions intersect.)

In Matlab

```
>> x = [-4:0.01:4]';
>> y1 = cos(3*x);
>> y2 = (x+1) .* (x-2);
>> plot(x,y1,'b',x,y2,'r');
>>
```

From the graph, there are two solutions

- x = -0.75, y = -0.70
- x = +2.26, y = 0.86



3) Find the solutions to problem #2 using Newton's method.

Let

$$y_1 = \cos(3x)$$

$$y_2 = (x+1)(x-2)$$

$$e = y_1 - y_2$$

Find the solutions for f(x) = 0 using Netwon's method. In Matlab, start with a function

```
function [e] = Prob3(x)
   y1 = cos(3*x);
   y2 = (x+1)*(x-2);
   e = y1 - y2;
end
```

Solve using Newton's method. The initial guess determines which solution you converge to.

```
x3 = -1;
for i=1:10
    x1 = x3;
    y1 = Prob3(x1);

    x2 = x1 + 0.001;
    y2 = Prob3(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([x1, y1]);
    pause(0.1);
end
```

Starting with x = -1:

x	error
-1.000000000000000	-0.989992496600445
-0.711104134948823	0.249910500025049
-0.761475584511851	0.004096703052352
-0.762330334912776	0.000003083702626
-0.762330978753865	0.000000001256905
-0.762330979016292	0.000000000000512
-0.762330979016399	-0.000000000000000
-0.762330979016399	0.000000000000000
-0.762330979016399	0.000000000000000
-0.762330979016399	**0.000000000000000**

Starting with x = +2

x	error
2.000000000000000	0.960170286650366
2.443071907374465	-1.024518008364445
2.285106671104962	-0.095858725289637
2.266669742208026	-0.001729046718695
2.266325352688896	-0.000002273673034
2.266324899516075	-0.000000002227148
2.266324899072175	-0.000000000002181
2.266324899071740	-0.000000000000002
2.266324899071740	0.000000000000000
2.266324899071740	**0.000000000000000**

Newton's Method with a Thermistor

Assume the temperature - resistance relationship of a thermistor is:

$$R = 2000 \cdot \exp\left(\frac{4200}{T+273} - \frac{4200}{298}\right)\Omega$$

$$e = R - R_0$$

```
T = [-20:0.2:20]';
R = 2000*exp( 4200./(T+273) - 4200/298 );
plot(T,R);
```

4) Write a Matlab funciton which

- Is passes the temeprature T, and
- Returns e (the difference between R and R0)

```
function [e] = Therm(T)
   R0 = 5000;
   R = 2000 * exp(4200 / (T+273) - 4200/298);
   e = R - R0;
end
```

5) Use Newton's method to find the temperature when

- R0 =5,000 Ohms
- R0 = 10,000 Ohms

Newton't Method

- same program as before
- Newton's method is pretty useful

R = 5000:

```
       T                  error
   2.000000000000000   1500.731582640176
   6.156909065538900   178.3653701271878
   6.796023432171404   3.425267385358893
   6.808784118962809   0.001224228571118
   6.808788683313439  -0.000000037041900
   6.808788683175333  -0.000000000004547
   6.808788683175316   0.000000000013642
   6.808788683175367  -0.000000000004547
   6.808788683175350  -0.000000000004547
   6.808788683175333  -0.000000000004547
```

If R = 5000 Ohms, the temperature is 6.8087 degrees C

- same answer as homework #1, just a lot more accurate

R = 10,000

```
function [e] = Therm(T)
  R0 = 10000;
  R = 2000 * exp(4200 / (T+273) - 4200/298);
  e = R - R0;
end
```

Iteration results:

```
        T                       error
  0.002000000000000  -3.499268417359823  * 1000
 -0.007692699730678   1.357591781639152  * 1000
 -5.689396694259486  86.990855448777438
 -5.542669894801186   0.418489021811183
 -5.541957140268218  -0.000003978389941
 -5.541957147044380   0.000000000152795
 -5.541957147044120  -0.000000000025466
 -5.541957147044164   0.000000000010914
 -5.541957147044145   0.000000000010914
 -5.541957147044126  -0.000000000025466
```

If R = 10,000 Ohms, the temperature is -5.541957 degrees C

- same answer as homework #1, just much more accurate

Newton's Method and a Voltage Divider

Assume

$$V = \left(\frac{R}{R+4000}\right) \cdot 10V$$

$$e = V - V_0$$

6) Write a Matlab function which

- Is passed the temperature, T, and
- Returns the error, e.

```
function [e] = Volt(T)
   V0 = 8;
   R = 2000 * exp(4200 / (T+273) - 4200/298);
   V = R / (R + 4000) * 10;
   e = V - V0;
end
```

7) Use Netwon's method to determine the temperature when

- V0 = 8.00V
- V0 = 6.00V

Use the same Newton's method routine as before:

```
x3 = 2;
for i=1:10
    x1 = x3;
    y1 = Volt(x1);

    x2 = x1 + 0.001;
    y2 = Volt(x2);

    x3 = x1 - (x2-x1)/(y2-y1) * y1;
    disp([x1, y1]);
    pause(0.1);
end
```

V0 = 8V:

```
          T                    error
   2.000000000000000  -1.809258401207783
 -11.814408415301466  -0.152757541261541
 -13.283110322186735  -0.003115819475259
 -13.314348815923390  -0.000001487943436
 -13.314363747524231  -0.000000000022405
 -13.314363747749068   0.000000000000004
 -13.314363747749033   0.000000000000004
 -13.314363747748997  -0.000000000000003
 -13.314363747749024   0.000000000000004
 -13.314363747748988  -0.000000000000003
```

If V0 = 8.00V, the temperature is -13.314363 degrees C

- same results as homework #1 just more accurate

If you read 6.00V

```
function [e] = Volt(T)
   V0 = 6;
   R = 2000 * exp(4200 / (T+273) - 4200/298);
   V = R / (R + 4000) * 10;
   e = V - V0;
end
```

Results from Newton's method:

```
        T                    error
   2.000000000000000   0.190741598792217
   3.456388068030666  -0.000725658307831
   3.450886356278794  -0.000000008847584
   3.450886289197826  -0.000000000000022
   3.450886289197658  -0.000000000000002
   3.450886289197644   0.000000000000007
   3.450886289197698  -0.000000000000002
   3.450886289197685  -0.000000000000002
   3.450886289197671  -0.000000000000002
   3.450886289197658  -0.000000000000002
```

If you read 6.00V, the temperature is +3.4508862 degrees C

- same result as homework #1, just way more accurate